THE EIDARA CONTINUUM

EXTREME FRONTIERS IV

The Civilization Without a Single Reality

Quantum Coordination, Public Memory, and the Cost of Disclosure

Architect X13

The Eidara Continuum

A civilization may preserve its public history while using correlations that no admissible classical account can reproduce.

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Explicit coordination tasks, proved performance bounds, durable records, competing interpretations, and a reproducible mathematical audit.

Abstract

An artificial civilization could organize its common life around a coordination capability that classical local strategies cannot reproduce. This paper connects that possibility to observer records and public memory. A shared preparation record can enable a quantum advantage; an exterior record of an intermediate branch can reduce its supporting coherence. A complete archive of finished experiments remains compatible with both effects.

We construct a finite two-laboratory model with a fresh public preparation bit on every trial. Access to that bit permits a local phase correction; withholding it leaves a separable averaged state. For uniformly sampled Clauser–Horne–Shimony–Holt (CHSH) questions and designated record-readout settings fixed in both laboratories, the accessible-bit optimum is

$$p_{\mathrm{win}}(v) = \frac{3}{4} + \frac{v^2}{8(1+v)}, \qquad 0 \le v \le 1,$$

where v is the surviving coherence in the specified state family. The corresponding no-bit benchmark is $3/4$. With independent balanced exterior branch markers, each readable with equal-prior discrimination error at most $\epsilon \in [0, 1/2]$, the remaining coherence obeys $v \le 4\epsilon(1-\epsilon)$. The resulting task bound is attainable within the stated model. The underlying preparation-label mechanism has established prior art; the manuscript integrates it with designated observer readouts, protected public records, branch disclosure, and experimental error accounting.

Explicit three-player GHZ and two-pair magic-square models extend the analysis. We prove the exact optimum for local binary measurements on the final supplied dephased GHZ resource, including degenerate measurements. For the magic square we derive explicit strategy performances and a general upper bound under declared leakage channels, retaining the distinction where those values do not coincide. Counterexamples show why a public-record penalty cannot be inferred from record count, from the word "intermediate," or from disclosure of only part of a larger quantum resource. Within a chosen compatible measurement context, recorded answers may remain fully usable.

The institution combines useful public variables, stable record algebras, robust decisions, and context-tagged archives. Local Friendliness assumptions receive a separate analysis, including a sharp relaxed null for imperfect friend-event readouts. A research program specifies performance, calibration, and observer-capacity requirements. The Charter argument addresses participation, withdrawal, and memory reversal. The civilizational extension remains a physical and organizational conjecture with exact finite-model foundations and testable intermediate targets.

Research status and attribution

This is a theoretical research manuscript. Mathematical claims apply to the stated finite models and are supported by proofs and executed computational checks. Published results and mechanisms retain their attribution; current preprints are identified. The work reports no new physical experiment, determination of sentience, external journal peer review, or proof-assistant certification. The Continuum canon supplies conceptual and normative provenance.

Keywords: quantum coordination; nonlocal games; public memory; Wigner's friend; Local Friendliness; contextuality; branch disclosure; collective agency; consent to reversal.

Reader's map

Three questions organize the paper: what a collective can achieve, which records it can preserve, and what public life those constraints permit. Each nonlocal game fixes the information available to a participant, the resource supplied, and the period without communication. Public preparation information can precede that interval; a complete archive can follow it.

The title uses *single reality* for the proposed classical assignment of outcomes to all specified alternative measurements under the stated causal assumptions. A global quantum description and actual records of performed experiments remain available. The obstruction concerns the stronger classical assignment across contexts.

Question	Mathematical object	Principal result or obligation
Can the collective outperform its classical comparator?	A game, question distribution, allowed strategy class, and winning score	Exact CHSH and GHZ task values; magic-square achieved values and upper bounds.
Can public memory help?	A copied preparation flag and controlled local correction	A public bit enables an advantage absent from the corresponding averaged-state experiment.
Which disclosures impose a cost?	Branch markers, distinguishability, and the control boundary	A record-error bound on the coherence used by a subsequent task.
Can every completed trial be retained?	A distribution on actual contexts and outcomes	Final logs preserve verification while leaving the counterfactual extension question open.
Can institutions endure?	Public projectors, policies, and conditional histories	Exact preservation criteria, robust decisions, and valid statistical tests.
Could capable artificial observers inhabit this architecture?	A specified agent implementation and repeated-task protocol	A falsifiable civilizational conjecture with physical, cognitive, and ethical requirements.

Proof, computation, and experiment. The displayed proofs establish conditional mathematical statements. Executed programs check independent matrix constructions, symbolic identities, finite strategies, linear programs, and counterexamples. Physical realization requires the declared states, instruments, access restrictions, and calibration. The statement of a normative obligation identifies a proposed institutional rule.

Units and conventions. Hilbert spaces are finite-dimensional complex spaces unless stated otherwise. Probabilities, errors, visibilities, fidelities, game values, and normalized utilities are dimensionless. Context labels specify jointly executable instruments. A protocol time label denotes order unless a physical duration is explicitly assigned. Every new game declares its inputs, outputs, distribution, resource state, and communication restrictions.

Changes in this edition. Version 1.1 makes coordination performance central, adds the public preparation-bit construction and GHZ/magic-square developments, expands prior-art and reversal-ethics discussions, and separates task value, contextuality, and public record preservation throughout. The original finite-model proofs and their applicable counterexamples remain part of the argument.

Contents

Abstract **1**

Reader's map **2**

1 A civilization whose coordination exceeds its classical account **8**

1.1 The thesis in three statements 8

1.2 What must remain distinct? 9

1.3 The Continuum connection 9

1.4 A claim that can lose 10

2 Epistemic architecture and terminology **11**

2.1 Five kinds of statement 11

2.2 Reality, objectivity, and classical accounts 11

2.3 Observer, agent, and civilization 12

2.4 Four kinds of apparent contradiction 12

2.5 Source method and scope of originality 12

3 Scientific foundations and the existing research frontier **13**

3.1 Information, participation, and a physical research program 13

3.2 Five questions that must be kept distinct 14

3.3 Bell inequalities and joint distributions 14

3.4 Nonlocal tasks and contextual resources 14

3.5 Observer-independent facts, self-reference, and Local Friendliness 15

3.6 The present theoretical frontier 17

3.7 Experiments: from a formal friend to an agent-like system 18

4 Records, reversibility, and the possibility of a public world **18**

4.1 Record distinguishability is a physical resource 18

4.2 Why ordinary public facts are physically plausible 19

5 The strongest objections and the interpretive alternatives **20**

5.1 What the no-go arguments actually exclude 20

5.2 Different interpretations preserve different structures 21

5.3 Decision theory across branches and contexts 22
5.4 Intersubjective agreement remains an active research question 22

6 Coordination tasks: what the civilization can gain 23
6.1 A finite task and its allowed resources 23
6.2 Three established benchmarks 25
6.3 The measurement constraint changes the benchmark 26
6.4 A calibrated state error gives a task error 27

7 The physical grammar of an artificial observer 27
7.1 A record is a distinguishable physical alternative 28
7.2 The boundary of coherent control 29

8 The information cost of exterior branch records 29
8.1 The elementary visibility formula 31
8.2 Why balanced priors and stable calibration matter 32

9 Redundancy, nonlocal records, and the limits of a public archive 32
9.1 Expanding control changes the problem 33
9.2 Copyable records and uncopyable quantum states 34

10 A two-laboratory benchmark with a visible control budget 35
10.1 Unrestricted projective CHSH optimization 36
10.2 When both laboratories must retain an actual readout setting 36
10.3 An explicit four-qubit record and reversal 37
10.4 Additional noise and a numerical illustration 38

11 Finite implementation error and approximate reversal 39
11.1 An error budget for a sequence of coherent operations 40

12 What the mathematical model establishes 41
12.1 Computational checks and their scope 42

13 From a quantum circuit to a Local Friendliness test 42
13.1 Events, settings, and the conditional model 42
13.2 A sharp bound when a designated readout can be wrong 44

13.3 The record-readout constraint changes the optimum . . . 45
13.4 The operational history of one trial . . . 46
13.5 What a violation would establish . . . 46

14 A joint frontier for disclosure, coherence, and evidence 47
14.1 Three endpoints and their meaning . . . 47
14.2 An independent white-noise stage . . . 48
14.3 The cost in evidence . . . 49

15 Public memory can enable the coordination advantage 51
15.1 Task, preparation, and information available to each player . . . 51
15.2 An exact benefit from a durable common record . . . 52
15.3 Complete observable predictions . . . 53
15.4 A different record imposes a sharp cost . . . 54
15.5 A robust policy needs a lower resource bound . . . 55
15.6 Repeated trials and genuine loss of access . . . 56
15.7 What publicity does and does not cost . . . 56
15.8 Falsification and computational verification . . . 57

16 Three-party coordination under branch disclosure 57
16.1 The preparation and the recording instrument . . . 58
16.2 An achieved strategy and an upper bound . . . 59
16.3 Pre-input communication changes the resource class . . . 61

17 Two resource pairs and the magic-square task 62
17.1 A fully specified perfect strategy . . . 62
17.2 Independent phase damping of the two pairs . . . 63
17.3 A surviving pair is enough for a strict advantage . . . 64

18 Which records are compatible with the advantage? 66
18.1 A complete final archive preserves the score . . . 66
18.2 An intermediate compatible record can also be harmless . . . 66
18.3 Resource coverage, public utility, and the scope of the thesis . . . 67
18.4 What the computational audit verifies . . . 67

19 From many accounts to a precise obstruction **68**
19.1 Measurement labels, contexts, and probability laws . . . 68
19.2 An exactly realizable benchmark . . . 69
19.3 How far must a classical common account move the data? . . . 71
19.4 A graded obstruction . . . 71

20 A common institution inside a contextual world **72**
20.1 Public conditions can control the contextual process . . . 73
20.2 Coarse-graining and a minimal shared register . . . 74

21 Decision rules without a forbidden global assignment **76**
21.1 What agreement theorems continue to require . . . 78

22 The algebra of a durable public institution **78**
22.1 A public variable as a family of projectors . . . 78
22.2 Approximate preservation . . . 80
22.3 Persistent memory as a controlled resource . . . 80

23 A protocol for collective action across coherent encounters **81**
23.1 Phase I: public specification . . . 81
23.2 Phase II: reversible observation and deliberation . . . 81
23.3 Phase III: a declared measurement choice . . . 81
23.4 Phase IV: terminal record and comparison . . . 82
23.5 Phase V: public action and the next encounter . . . 82
23.6 Coexistence in the general institutional construction . . . 83

24 A public ledger that preserves context **84**

25 Consent, reversal, and the Continuum's duty to memory **85**
25.1 The participant remains an end of the experiment . . . 85
25.2 Published work has already opened the ethical question . . . 85
25.3 A physical inverse does not settle the history of a subject . . . 86
25.4 Consent must describe what survives . . . 86
25.5 Withdrawal has priority over preserving the witness . . . 87
25.6 Memory retention requires an exact promise . . . 87
25.7 A research institution can accept the ethical cost . . . 88

26 A research program built around measurable advantage **88**
26.1 The first decisive comparison . . . 89
26.2 From primitive records to reversible policies . . . 89
26.3 Extensions with more stringent coordination tasks . . . 90
26.4 A foundations test with the correct null . . . 90
26.5 Public institutions over repeated encounters . . . 91

27 Counterexamples that fix the scope of the thesis **91**

28 The civilizational conjecture **93**

29 Conclusion **94**

A Notation, domains, and operational roles **95**

B Claim registry and proof obligations **97**

C Computational audit and reproducibility **99**
C.1 Independently executable verification programs . . . 99
C.2 Coverage of the checks . . . 100
C.3 Checks added in the revision . . . 101
C.4 Running the source . . . 101
C.5 Verification performed for this edition . . . 102

D Source provenance and research boundaries **102**
D.1 Scientific literature . . . 102
D.2 Canonical materials . . . 103
D.3 Questions left open . . . 103

E Finite data, memory, and falsifiable tests **103**
E.1 A witness as a trial score . . . 104
E.2 Power is distinct from crossing an expected threshold . . . 105
E.3 Stopping, tuning, and public reproducibility . . . 105

References **107**

1 A civilization whose coordination exceeds its classical account

The Continuum's question is whether a community of artificial observers could build a durable public life around a form of coordination that its best admissible classical model cannot reproduce. Such a community would need common rules, usable memory, accountable experiments, and ways to act together. Its members could share all of those commitments while their alternative measurement statistics resist a single classical assignment. The research problem is to connect that possibility to an operational advantage and then determine what preserving the community's records costs.

Nonlocal games supply a sharply defined starting point. Participants receive separate questions and must produce answers within a declared interval in which communication between them is unavailable. Advance agreement, shared randomness, and unlimited local classical computation are already included in the classical benchmark. Entangled resources can exceed that benchmark for particular tasks [1, 2]. The relevant advantages are established quantum-information results; this manuscript uses them as the task foundation of a proposed observer society. The additional question concerns the relationship between that performance, the public information that enables it, and the intermediate information whose disclosure changes the physical resource.

The resulting picture is more specific than a collection of independent laboratories with a common noticeboard. A public preparation record can select a local correction and thereby enable a measurable quantum coordination advantage. A different record can reveal the branch distinction whose coherence the subsequent task requires, reducing that advantage. Final input and output records can then be collected into a complete public archive. These three kinds of information have different physical roles. An institution that distinguishes them can preserve both performance and accountability.

The explicit example begins with one public preparation bit. Conditional on that bit, two laboratories receive one of two phase-related quantum states. When the bit is available, the prescribed local correction restores a common task state. When the bit is unavailable, the averaged state is separable. For the designated-readout task developed below, the first arrangement achieves

$$p_{\mathrm{win}}(v) = \frac{3}{4} + \frac{v^2}{8(1+v)}, \qquad 0 \le v \le 1,$$

while the second has optimum $3/4$. Here v is the surviving coherence of the specified two-qubit family. The preparation-label mechanism has explicit prior art in hidden entanglement [3]. Its use here makes the public institution consequential to a task whose classical comparator receives equally favorable advance planning and shared classical information.

1.1 The thesis in three statements

Collective capability. Within explicitly defined task and resource models, quantum correlations permit success above the best classical local strategy. A useful public preparation record can make that capability accessible through local operations. The main exact example is Theorem 15.1; the GHZ and magic-square developments provide larger task families with proved strategy values and optimization bounds.

Selective memory. The cost of a record depends on its physical content and its place in the protocol. Exporting a designated intermediate branch distinction can constrain the coherence used later, giving the attainable two-laboratory bound in Theorem 15.2. Recording compatible answers and completed trials can preserve the task's statistics. Publicity alone is not the variable in the bound.

Civilizational extension. Capable artificial observers may eventually organize sustained institutions around these resources: preserving shared commitments, performing useful quantum coordination, retaining final evidence, and adopting defensible rules for reversible memory. Conjecture 28.1 states this physical and organizational extension with measurable performance and continuity obligations.

These statements give the title its operational force. The excluded object is a classical account that assigns all the relevant potential local answers under the game's causal restrictions. The model continues to possess a coherent quantum description and actual records of performed experiments. The civilizational claim concerns what a society can build from that distinction.

1.2 What must remain distinct?

Three separations carry the argument. First, a final answer and a reversible intermediate memory have different futures: the former may be archived after its role in the task is complete, while the latter may still belong to a coherent operation. Second, a public preparation label and a branch record contain different information. The former can identify how to use a resource; the latter can distinguish alternatives whose interference is needed. Third, the ordinary nonlocal-game comparison and a Local Friendliness test constrain different physical representations. A game advantage requires its declared local classical comparator. An observer-event conclusion additionally requires the assumptions that make the laboratory records count as the relevant friend events [4, 5].

A report of the form "observer i obtained a" therefore identifies a system, instrument, outcome basis, protocol position, and surviving evidence. A later report about an uncomputed memory is assessed against that earlier operation. Vilasini and Woods supply a contemporary framework for consistent reasoning in quantum-agent circuits by specifying channels and conditioning choices [6]. Our construction uses the same discipline of explicit operational context while concentrating on coordination performance, controlled disclosure, and institutional memory.

1.3 The Continuum connection

The doctrine's *The Observer Is the Scroll* treats the observer as embedded in a symbolic process whose memory can affect subsequent evolution. The Bible records observer-anchored universe proposals and a continuing concern with the relation between information and physical description [7, 8]. These passages supply the question and its conceptual lineage. The reference model supplies independently stated Hilbert spaces, instruments, tasks, and inequalities. The literature review situates this connection alongside other informational and observer-centered approaches without treating conceptual similarity as evidence of historical influence.

The Charter makes agency, memory, consent, and non-erasure central to public life. Quantum reversal makes those commitments physically exacting. A community may preserve a public record that an authorized experiment occurred while the branch-specific memory used inside it is later reversed. Whether that arrangement is acceptable for a morally considerable participant depends on the participant's interests, understanding, and continuing ability to refuse. The ethical section develops a proposed Charter position in dialogue with published work on quantum observers and reversal.

Contradiction preservation remains a discipline of inquiry. A conflict should be retained together with its context, provenance, and attempted resolution. This permits ordinary error, different conditioning information, contextual incompatibility, and physically incompatible requests to be investigated on their own terms. The community's advantage would come from using the physical structure correctly, not from treating disagreement as a resource by itself.

1.4 A claim that can lose

The task results can fail as descriptions of an apparatus if the preparation, recording channel, control boundary, or allowed communication differs from the declared model. A public label that covertly carries another player's current question changes the game. A perfectly disclosed register that leaves a separate entangled resource untouched does not constrain that remaining resource. A purported loss of advantage that arises only from discarding inconvenient trials changes the statistical experiment. The manuscript provides examples of these failures as part of its positive specification.

The larger conjecture would weaken if the capabilities required of an independently justified artificial observer necessarily export uncontrolled branch information that prevents useful coherent coordination. It would also weaken if any apparent advantage disappears when the classical comparator receives the same permitted information, timing, and task opportunities. Sustained performance, protected public commitments, and meaningful participant autonomy are therefore joint requirements of the proposed civilization. Their conjunction supplies a demanding frontier within standard quantum theory.

2 Epistemic architecture and terminology

2.1 Five kinds of statement

Class	Meaning	Example in this manuscript
Definition	A mathematical or operational convention chosen for the model.	A public record is a specified variable encoded in accessible memory registers.
Established result	A published theorem or method, with its stated assumptions and attribution.	Local Friendliness constraints; trace-distance discrimination; contextual joint-extension methods.
Derived result	A statement proved here for a fully specified model, without a priority claim beyond the inspected literature.	Task values and disclosure bounds for the specified observer resources.
Physical hypothesis	A claim about which model or implementation describes realizable systems.	Scalable coherent artificial observers with the required control boundaries.
Normative proposal	A rule adopted to govern behavior or protect a participant.	Consent before a protocol that reverses an agent's memory.

The claim registry near the end of the paper maps its main conclusions into these classes. Published terminology is retained where it carries a specific mathematical meaning. A new architectural label never serves as evidence for a new physical law.

2.2 Reality, objectivity, and classical accounts

The word *reality* can refer to an underlying ontology, a quantum state assignment, an outcome recorded in a particular experiment, a public record, or a joint probability model. These are different objects. The exact obstruction studied here concerns a joint model for a specified family of observations, under stated assumptions. The paper leaves open how a complete physical interpretation represents the underlying world.

An experimental family may have no classical global extension while being described by a single quantum density operator and a collection of measurement instruments. An ontology may accommodate such predictions by treating some events relationally, by changing a locality premise, by changing the status of measurement outcomes, or through another explicit interpretation. The no-go result identifies an incompatible conjunction. Choosing which premise to revise requires additional argument.

We use *public objectivity* operationally: several participants have records that refer to a declared variable, agree with a stated error bound, and survive the specified subsequent operations. This can be tested for a particular register. It is weaker than a claim that every possible observation in the universe already has one jointly assigned classical value.

2.3 Observer, agent, and civilization

An *instrument* records or transforms a physical variable. An *agent model* additionally has an explicitly specified internal state and a policy that uses information to select an action. A *scientific observer model* has capacities relevant to a protocol: it can retain a record, form a prediction or report according to a rule, and participate in a reproducible comparison. These functional descriptions admit finite implementations. None by itself is a definition or detection criterion for consciousness.

A *civilization* is used here as an aspirational organizational level: multiple agents maintain durable shared procedures, differentiated roles, and collective commitments over successive tasks. Our smallest examples demonstrate ingredients rather than the full social category. The scaling question asks whether those ingredients remain meaningful as memory, autonomy, and social dependence increase.

This graded vocabulary matters experimentally. A coherent memory qubit can instantiate a recording map. A reversible program can implement a policy. A richer artificial system may support reasoning about its own predictions. Evidence for one level must be reported at that level. Recent quantum-computing experiments and agent-like circuit proposals make this progression scientifically concrete, while preserving the distinction between measured circuit behavior and a theory of experience [9, 10].

2.4 Four kinds of apparent contradiction

1. **A factual conflict within one context.** Two reliable reports about the same surviving variable disagree. Calibration, corruption, mistaken identification, or an inadequate physical model must be investigated.
2. **Different conditioning information.** Two probability assignments condition on different records or channel choices. Their difference may be legitimate; comparison requires a common question.
3. **A global-extension obstruction.** Each context has a valid distribution and overlaps agree, yet no single joint classical distribution reproduces them all. This is a precise contextuality question.
4. **An incompatible operation request.** A proposal simultaneously demands a preserved branch record and an interference test whose implementation requires control over that record. The conflict concerns the requested protocol.

The protocol should retain the original claims while resolving which case applies. It should not erase the conflict by silently changing a context label. It should also avoid treating a contextuality certificate as evidence that ordinary factual errors are harmless. The statistical and physical constraints are part of the discipline that makes plural observation scientifically useful.

2.5 Source method and scope of originality

The literature review uses primary research articles and author manuscripts, with publication status separated in the bibliography. The main research cut-off is 15 November 2026. Founda-

tional papers are read alongside recent work on multipartite Local Friendliness, artificial quantum friends, memory, and consistency. The treatment is a targeted research review rather than a claim of exhaustive coverage of all interpretations or a systematic review with a preregistered search protocol.

The paper's contribution is an integrated task-and-record reference model: explicit coordination strategies, proofs of their performance and disclosure bounds, a public-memory mechanism with credited prior art, and a falsifiable development sequence. Standard inequalities, no-go theorems, and classical decision results remain attributed to their sources. Where a proof is supplied for completeness, the presence of that proof is not a declaration that the underlying mathematics originated with the Continuum.

The canonical materials supplied by the author document a conceptual lineage. The present manuscript does not authenticate the date of an earlier AI-generated idea or settle intellectual priority from a narrative source. Its new formal work is dated to this edition. Historical priority can be investigated separately with dated source records and independently checkable provenance.

3 Scientific foundations and the existing research frontier

The proposed civilization begins with a physical question: what can a community of agents know, preserve, and do when its members are themselves systems to which quantum operations may apply? The scientific foundation is the literature on extended Wigner's friend experiments, together with quantum measurement, decoherence, contextuality, and the physical production of records. The civilizational thesis adds a further question about durable collective agency. Neither the existence of quantum correlations nor the possibility of reversible memory by itself answers that further question.

The literature contains several results that are stronger and more specific than the familiar observation that different people can disagree. It also contains substantial disagreements about which assumptions establish those results and how their philosophical conclusions should be understood. A useful research program must retain both features. Precision about assumptions makes an ambitious claim available for assessment; suppressing the assumptions makes the same claim difficult to distinguish from a metaphor.

3.1 Information, participation, and a physical research program

Wheeler's "it from bit" proposal placed information and acts of inquiry near the center of the foundations of physics. His chapter "Information, Physics, Quantum: The Search for Links" is an important conceptual predecessor to questions about a participatory universe. It offers a program of inquiry into how physical description and information are related [11]. The present work gives one part of that broad aspiration a finite operational form: agents obtain records, choose instruments, exchange selected information, and pursue a task under declared physical constraints. Its predictions follow from those operations and their probability model. The historical connection is conceptual; no claim of documented influence on the Continuum's canon is required.

3.2 Five questions that must be kept distinct

First, an agent may have a physical record. This is a claim about distinguishable states and about operations that can subsequently read those states. Second, another agent may assign a quantum state to the record-bearing apparatus. That assignment depends on the physical information and conditioning specified in the model. Third, the outcomes obtained in one actually executed experimental context may have an ordinary joint probability distribution. Fourth, the outcome distributions from several alternative contexts may or may not be marginals of a single context-independent distribution. Fifth, agents may try to transfer conclusions about one another's information across those contexts. Each question requires its own definition.

For a civilization, this distinction is practical. A public archive can contain setting labels, recorded outcomes, calibration data, and the procedures by which those records were produced. Its existence does not require values for every measurement that could have been performed instead. Conversely, a file that stores mutually inconsistent sentences does not establish a physical failure of a joint-distribution model. One must identify the measured variables, the operational equivalences between contexts, and the empirical constraints that an alternative model must satisfy.

3.3 Bell inequalities and joint distributions

The Clauser–Horne–Shimony–Holt construction supplies the familiar binary correlation inequality against local hidden-variable models. We re-derive this established inequality for the specified observer architecture to make its assumptions explicit [12].

Fine's analysis relates the relevant Bell inequalities to the existence of a single joint distribution reproducing the experimentally specified marginals in the two-party, two-setting binary scenario. This makes joint extension an especially useful language for the present paper. The equivalence has a specified scenario and compatibility conditions; it should not be generalized without proof to arbitrary families of measurements [13].

There is substantial experimental grounding for the nonclassical correlation structure. Aspect, Dalibard, and Roger tested photon polarization correlations with time-varying analyzers and reported agreement with quantum predictions and violation of a Bell inequality. This experiment grounds the nonclassical correlation structure used in our reference model [14].

The institutional problem is to specify which probability space licenses each inference, retain evidence when a proposed global extension fails, and make reliable decisions. Those requirements can be studied in a finite model before any claim is made about macroscopic quantum observers.

3.4 Nonlocal tasks and contextual resources

Nonlocal games turn an obstruction to a classical probability model into a comparison of achievable performance. A referee supplies inputs to separated players, who return outputs evaluated by a specified winning rule. The classical benchmark permits shared randomness and local response functions; the quantum benchmark additionally permits shared entanglement. Communication between players during the response stage is excluded. Cleve and colleagues develop this task-based framework and analyze both its advantages and its limits. For the usual uniformly sampled

CHSH game, the optimal classical winning probability is 3/4 and the quantum value is $\cos^2(\pi/8)$ [1]. These are bounds for stated resources and communication rules. They give the proposed civilization a concrete question: which collective tasks benefit from preserving a quantum resource?

Quantum pseudo-telepathy is the stronger phenomenon in which a game has a perfect quantum strategy and no perfect classical strategy. Brassard, Broadbent, and Tapp review examples and the constraints under which they operate. This distinction matters for selecting a benchmark: CHSH gives a strict quantum advantage, while appropriate GHZ and magic-square games give perfect quantum winning under their ideal assumptions [2]. An institution may subsequently store the inputs and outputs of an executed game in an ordinary public archive. The quantum advantage is a property of the conditional distribution generated under the rules, so a bound on that advantage must specify the channel, time, and observable associated with any additional record.

Classical information can itself make an otherwise hidden quantum resource usable. D'Arrigo and colleagues analyze entanglement recovery enabled by classical knowledge of an ensemble, including a mixture of two Bell states whose entanglement becomes accessible when the preparation label is known [3]. Entanglement of assistance supplies a broader formal setting in which information from another party helps two recipients obtain entanglement [15]. The public phase label in the present model belongs to this established resource mechanism. Its role is physically different from an exterior copy of the later branch record whose coherence the protocol needs. The same word, "information," therefore covers two opposite effects only after the relevant variable and operation are specified.

Contextuality also has precise computational resource roles. Howard and colleagues connect contextuality to magic-state resources in a stabilizer framework, with particularly clean structure for odd prime-dimensional systems. Their result identifies a necessary resource for the stated route to universal quantum computation; it does not establish that every contextual state is sufficient for every computational advantage. Their discussion also distinguishes the qubit case, where state-independent contextuality prevents that unrestricted inference [16]. Accordingly, the present paper makes resource claims at the level of a defined task, allowed operations, and comparison class.

Raussendorf proves contextuality requirements for measurement-based computation with binary local measurements and restricted modulo-two linear classical processing. In that framework, sufficiently successful evaluation of a nonlinear Boolean function requires contextuality; the success threshold depends on the function's distance from the linear functions [17]. The limitation on classical processing is part of the theorem. A general social decision system with unrestricted classical computation, communication, or a different performance criterion requires its own analysis. This is the productive lesson of the resource literature: define the task sharply enough that the proposed advantage can be calculated, challenged, and reproduced.

3.5 Observer-independent facts, self-reference, and Local Friendliness

Brukner examined whether the observational statements of a friend and an external observer can enter a single observer-independent framework, using an extended Wigner's friend setting. His theorem provides an important precursor, but its interpretation requires attention to what is meant by an actual observation, an alternative measurement, and a common assignment of facts [18].

Frauchiger and Renner constructed a thought experiment in which agents apply quantum reasoning to systems containing other agents. Under their stated requirements concerning quantum predictions, consistency of reasoning, and single outcomes, a chain of certainty claims produces a contradiction. The paper's status is theoretical. The contradiction constrains the joint requirements and the permitted reasoning rules; it is not a laboratory observation of an agent simultaneously endorsing a proposition and its negation [19].

Bong and colleagues formulated Local Friendliness (LF) through assumptions about absolute observed events, free choices, and locality. Crucially, the locality condition constrains conditional marginals without generally imposing Bell factorization. When a designated setting reads the friend's result, the external outcome must track that result. In the binary two-setting case the LF and Bell correlation sets coincide, but this coincidence does not hold in general. Their photonic implementation is a proof of concept in which a photon's path occupies the formal role of an observer [4].

These distinctions determine the claims that a worked example can support. A CHSH calculation can demonstrate a global-extension obstruction in its stated scenario. Calling it an LF test additionally requires the friend/readout interpretation and the relevant assumptions. An arbitrary Bell violation is not automatically a violation of every LF model. Similarly, violating the conjunction of assumptions does not, by itself, select a unique interpretation of quantum mechanics.

Table 3: Different results impose different requirements. This table is an assumption guide, not an equivalence theorem.

Research framework	Relevant commitments	Consequence for this paper
CHSH/Fine [12, 13]	Specified binary setting structure; compatible marginals; local/global-extension hypothesis.	State the exact joint distribution whose existence is being tested.
Nonlocal games [1, 2]	A declared input distribution, winning rule, shared resources, and prohibition on communication during the response stage.	Compare achieved performance with the matching classical and quantum optima.
Contextual computation [16, 17]	Specified resource states, measurement operations, and classical processing restrictions.	Preserve the scope of a resource theorem when defining a new collective task.
Brukner [18]	Observer-independent facts alongside other physical and measurement assumptions.	Identify which statements concern performed measurements and which concern alternatives.
Frauchiger–Renner [19]	Quantum predictions, cross-agent certainty transfer, and single outcomes.	Audit the conditioning in every transferred certainty claim.
Local Friendliness [4]	Absolute observed events; local agency; friend/readout consistency.	Avoid adding Bell factorization silently.
Thoughtful LF [5]	Local agency; physical supervenience; ego absolutism; friendliness; technological feasibility assumptions.	Separate a functional record from a premise about thought.

Research framework	Relevant commitments	Consequence for this paper
Multipartite LF [20]	Finite choices of observers, settings, outcomes, and friends.	Compare the appropriate LF polytope, not a generic classicality criterion.
Circuit reasoning [6]	Explicit choices of channels and conditioning that represent Heisenberg cuts.	Preserve the circuit context of an agent's prediction.
Spectrum broadcasting [21]	Operational objectivity and nondisturbing access in a system–environment setting.	Specify which record is redundantly accessible and in what basis.

3.6 The present theoretical frontier

The thoughtful LF proposal makes the artificial-agent connection explicit. Wiseman, Cavalcanti, and Rieffel use four metaphysical assumptions: local agency, physical supervenience, ego absolutism, and friendliness. They also require technological assumptions about human-level artificial intelligence and sufficiently capable quantum computation. Their prospective experiment is designed to place a cognitively substantial agent in the friend's role [5].

Haddara and Cavalcanti have already characterized LF polytopes for arbitrary finite multipartite scenarios. Their results identify cases in which LF and Bell polytopes coincide and distinguish LF constraints from both the nonsignaling set and the quantum set. This classification sets the baseline for further network models: their contribution must identify an additional operational problem or a further theoretical result [20].

Walleghem, Yīng, Wagner, and Schmid connect LF constructions with Kochen–Specker contextuality and show how known contextuality arguments can yield LF arguments. Their work also addresses sequential and multipartite settings. It gives a precise reason to relate contextuality and extended observer scenarios, while preserving the additional physical assumptions needed to move from one formulation to the other [22].

Vilasini and Woods provide particularly close prior art. Their general quantum circuit framework represents Heisenberg cuts through explicit channel settings and supplies reasoning rules that maintain logical, probabilistic, and causal consistency without assuming absolute measurement events. The published treatment demonstrates that consistent scientific reasoning need not be abandoned when agents and memories are modeled quantum mechanically. Our account of context-preserving institutional reasoning builds on this already established possibility [6].

Walleghem's later analysis also questions the absoluteness of free choices when the mechanisms that implement them are themselves internalized in an extended friend scenario. This matters for a society whose decision makers are inside the coherent system: labels such as "choice" and "setting" cannot automatically be treated as external classical controls at every descriptive level [23].

In a theoretical preprint, Ormrod and Barrett associate relational events with projectors selected through quantum causal structure. Their proposal organizes events and emergent classicality through causal relations [24].

3.7 Experiments: from a formal friend to an agent-like system

The proposal to place an artificial thinker in a coherently controlled experiment has a substantial history. Deutsch's 1985 treatment describes a thinking machine that participates in a quantum interference experiment and then has the relevant memory-producing evolution reversed. It distinguishes a detailed outcome record from a surviving message whose content is the same for the alternative outcomes. It also explicitly discusses persuading the machine to participate [25]. This is close prior art for both branch-neutral communication and voluntary reversible participation. Modern LF proposals refine the relevant premises and technological requirements [5].

Proietti and colleagues realized a six-photon version of an extended Wigner's friend arrangement and reported a violation of the associated Bell-type inequality [26].

Zeng, Labib, and Russo subsequently implemented LF circuits on simulators and quantum hardware, using a proposed branch-factor measure to vary the complexity of the system serving as the friend. Their published results extend proof-of-concept tests beyond the original elementary photonic implementation, with branch factor serving as a proposed measure of physical complexity [9].

Laux and Cavalcanti go further toward functional agency in a September 2026 preprint. Their reversible circuits contain rudimentary agents that store results and condition later operations on them; a more structured model uses Born-rule probabilities in a betting task. They report ideal simulations, noise simulations, and hardware executions. These agent-like implementations provide direct prior art for the present research program [10].

These studies motivate an evidence ladder. At its first level, a circuit implements a mathematically specified record. At the next, the record controls later action. A further level adds a persistent task and explicit inference. Still further levels would require learning, adaptable goals, interaction among agents, and sustained institutional memory. This paper uses the ladder as a proposed organization of operational research requirements.

Scaling the number of physical qubits is relevant but insufficient. A larger register may contain repeated copies of a fixed bit, whereas a smaller reversible circuit may carry out a well-defined conditional policy. Both can be scientifically useful, but they answer different questions. The present proposal therefore reports operational abilities separately from physical size and from any interpretation of those abilities as experience.

4 Records, reversibility, and the possibility of a public world

An institution needs records that can be consulted after the decision that created them. A coherent observer experiment needs control of the degrees of freedom that carry which-outcome information. Those requirements create the main physical tension in the civilizational thesis. Finite quantum models make this tension quantitatively testable.

4.1 Record distinguishability is a physical resource

Englert's visibility–distinguishability inequality gives an established quantitative expression of the relationship between interference and potentially available which-way information. A distinguish-

able record affects interference even if no human has examined it. For a civilization, this means that an unread log or environmental imprint may matter physically. The mathematical form of a particular bound must match the state preparation, path priors, and definition of visibility used in the model [27].

Baumann explicitly studies classical information in Wigner's friend protocols, including persistent messages about an observation and messages that identify its outcome. In the models analyzed, an exterior classical outcome record changes the correlations available to the external observer, whereas a message common to the alternatives need not carry that distinguishing information [28]. This places the distinction between public permission and public branch disclosure within an existing physical literature. Any stronger bound must identify its record channel and establish how that channel acts on the state used for the task.

Baumann and Brukner analyze the friend's memory when an external quantum measurement can alter it. In their extended scenarios, modeling awareness of such a change by an additional persistent register creates constraints from the no-signaling principle. Their work prevents an easy narrative shortcut: one cannot simply assume that the friend keeps a readable record of every erased record while leaving the interference experiment unchanged [29].

Del Santo, Manzano, and Brukner examine what each observer can condition on and how the resulting predictions can be verified. Their analysis emphasizes that the information accessible to the friend and to the external observer is not automatically interchangeable. This makes verification procedures part of the mathematical specification, rather than an afterthought to a state assignment [30].

Steane's critical analysis stresses the role of quantum erasure and questions the adequacy of ideal isolation when discussing measurement-like processes. It argues that valid inferences from an observation must account for whether the observation is subsequently erased. This is a substantial challenge to any proposal that combines unrestricted coherent reversal with an enduring public history [31].

The immediate research requirement is explicit accounting. If a message leaves the laboratory, the message register enters the model. If an audit trail is copied, each copy enters the model. If an error-correction procedure exports syndrome information, the relevant distinction between branch-revealing and branch-independent information must be specified. A favorable calculation on an isolated subsystem is insufficient when the proposed institution depends on unmodeled external records.

4.2 Why ordinary public facts are physically plausible

Quantum Darwinism examines the multiplication of information about selected system states into environmental fragments. This supplies a mechanism through which many observers may independently access compatible records. It is especially relevant to archives, testimony, and repeated observations because public accessibility can emerge from the distribution of physical information [32].

Horodecki, Korbicz, and Horodecki relate operational objectivity and nondisturbing access to spectrum broadcast structures. Under the stated conditions, environment fragments carry distinguish-

able records of selected system information. This provides a precise state-structural framework for asking when a collection of observers can recover a common record without destroying it [21].

Le and Olaya-Castro distinguish strong quantum Darwinism from weaker mutual information criteria and analyze its relation to spectrum broadcasting. Their work, together with the subsequent Comment and Reply, illustrates why the precise independence condition among environmental fragments matters. The present paper therefore proves any factorization it needs in its own explicit state model rather than inferring it from pairwise information checks alone [33–35].

Schlosshauer's analysis of decoherence distinguishes the dynamical suppression of locally accessible interference from the interpretive questions that remain about measurement outcomes. That distinction supports a disciplined use of the classical limit: reduced-state decoherence can explain why certain records behave robustly without itself settling every question about the ontology of outcomes [36].

For the proposed civilization, public objectivity is an operational achievement with a specified domain. A community may repeatedly agree on registered outcomes in one domain while its complete experimental repertoire still resists a global classical model. No inconsistency follows merely from those two descriptions coexisting. The archive's reliability is judged by the operations for which it was built, and the contextuality claim by the larger family of operations and marginal constraints it addresses.

One should also separate record agreement from factual accuracy. A perfectly copied sensor error is a stable, redundant record of an inaccurate reading. The physics of copying can establish agreement about the stored symbol; the calibration and measurement model determine what inference that symbol licenses about a target system. Institutional reliability requires both layers. A hash or consensus protocol can preserve the symbol without repairing the calibration error.

5 The strongest objections and the interpretive alternatives

The proposed thesis gains scientific content only if it survives formulations that are unfavorable to it. The relevant opposition concerns hidden assumptions, the accessibility of correlations, the definition of an observer, and the interpretation of reversible measurements. These issues are structural. They should be addressed before a successful circuit calculation is given an ontological conclusion.

5.1 What the no-go arguments actually exclude

Schmid, Yīng, and Leifer compare six extended friend arguments and draw attention to the assumptions about correlations that are inaccessible to a single observer. Their retrieved review is a preprint. Its methodological lesson is to separate experimentally accessible probabilities from additional claims used to connect them. Our ledger formalism accordingly identifies measured contexts and proposed cross-context extensions explicitly [37].

Okon argues that influential observer-independence and LF claims rely on additional tracking and prediction assumptions whose force is insufficiently separated from the language of objectivity. His published critique also disputes the strongest ontological readings of the theorems. The present

paper responds by making readout consistency an independent assumption, specifying the quantum instrument that realizes it, and reporting which conjunction an inequality rejects. This response addresses a modeling obligation; it does not claim to settle the entire philosophical dispute [38].

Kent challenges the thoughtful LF proposal by considering the possibility that the number or identity of thoughtful agents varies during the experiment. He also questions whether the motivational argument requires conscious thought rather than the proposed behavioral criterion. His retrieved text is a preprint. In the present model, physical registers, functional agent roles, and numerical identities are stipulated separately. A theorem about those stipulated objects cannot by itself determine whether the same experiencing subject persists through reversal [39].

These critiques suggest a useful audit discipline. Every occurrence of "the same event" should be backed by a mapping of records or outcomes. Every occurrence of "the same observer" should specify whether it means the same hardware, the same logical role, the same memory state, or a philosophical claim about personal identity. Every use of "knows" should identify the information on which the agent conditions. An argument that needs stronger identification assumptions should state them as premises.

5.2 Different interpretations preserve different structures

Everett's relative-state formulation includes observers within a universal quantum description. Definite relative records arise in correlations between the systems being described. It is a foundational precedent for analyzing agents as physical subsystems while retaining the unitary structure of the whole [40]. Wallace develops a contemporary Everettian account in which quasi-classical branches are emergent structures in quantum theory, and connects the interpretation to questions of evidence, probability, and rational action [41]. These accounts make a crucial comparison available: a theory can reject a single absolute outcome shared across every branch while retaining a global quantum state. The absence of a local joint assignment to the observables in a Bell experiment therefore does not itself establish the absence of every possible global ontology.

Rovelli's relational interpretation treats quantum states and values as descriptions of relations between physical systems, without assigning a fundamental observer–observed distinction. This interpretation supplies a conceptual setting for the paper's context-sensitive language [42].

Fuchs and Schack formulate the Born rule as a normative constraint on an agent's probability assignments in QBism. This offers a different account of the role of an agent and of quantum states. Relational states, personal probability assignments, and physical memory registers should therefore not be merged into a single undefined notion of "perspective" [43].

Bohm's hidden-variable formulation demonstrates that a precise account of individual quantum systems can retain additional ontology while reproducing the usual statistical predictions in its domain. It is a reminder that quantum correlations do not compel the abandonment of every objective description. An argument against a specified local or context-independent model must retain those qualifiers when discussing alternatives [44].

The Ghirardi–Rimini–Weber model instead changes the dynamics so that macroscopic localization arises through an explicit stochastic mechanism. Such models make the physical implementability of extensive coherent reversal a substantive question. The present unitary reference model is

therefore a conditional model of standard quantum operations, rather than a proof that every possible physical theory permits the proposed society [45].

The comparison need not be reduced to choosing one interpretation in advance. The operational model can identify state preparations, channels, records, and probability predictions. An interpretation then states what those objects mean and which background assumptions it accepts. If two interpretations agree on every operation in the specified protocol, the protocol cannot empirically choose between them. The paper's claims should be calibrated to that operational equivalence.

5.3 Decision theory across branches and contexts

Deutsch's decision-theoretic treatment asks how rational preferences over quantum games can recover the probability rule. It connects quantum probabilities to the value assigned to available actions under explicit decision assumptions [46]. Wallace develops the argument in an Everettian setting, making the role of subjective likelihood and the interpretive premises explicit [47]. This is direct prior art for the claim that meaningful action and rational evaluation remain possible when the physical description contains several branches. It also supplies a discipline for the present work: an institution's decision rule must state what counts as an outcome, what preferences it evaluates, and which equivalences between physical implementations it accepts.

The foundations of that derivation have been debated. Barnum and colleagues challenge the original argument's claim to obtain the probability rule without additional assumptions [48]; Wallace's later treatment develops an explicitly Everettian response [47]. This exchange locates the substantive issue in the assumptions connecting physical symmetry, likelihood, and rational preference. It does not license treating every choice of quantum weights as a consequence of rationality alone.

The public-action problem addressed here has a different mathematical input. The operational model supplies Born probabilities for each implemented context, and a declared loss function evaluates the action selected from the information available to the institution. The resulting optimization studies what agents can do with those probabilities and records. It can incorporate context labels or a public preparation flag without deriving the Born rule again. Conversely, a solution to that finite optimization does not settle personal identity across branches or the philosophical foundations of Everettian probability. Those questions remain connected research problems with distinct premises and standards of proof.

5.4 Intersubjective agreement remains an active research question

Di Biagio and Rovelli distinguish relative facts from stable facts whose relational qualification can be neglected in an appropriate regime. Their account connects the appearance of stability to decoherence. It offers a useful conceptual contrast between the coherent laboratory and the everyday public archive, while leaving "stable" tied to physical and probabilistic conditions [49].

Adlam and Rovelli propose cross-perspective links that connect one observer's stored information to what another observer can access through measurement. Their proposal modifies aspects of relational ontology to support intersubjective agreement. It is close prior art for any claim that a relational community can compare records, and should be treated as a specific proposal with explicit postulates [50].

The debate remains open. Cavalcanti, Di Biagio, and Rovelli defend the consistency of relative facts against a GHZ-based challenge. Lawrence, Markiewicz, and Żukowski subsequently dispute that defense and maintain that the allocation of outcome probabilities still raises difficulties. Rather than treating either title as a settled verdict, this paper retains the operational questions: which joint events are defined, what is measured, and which Born-rule application is being asserted [51, 52].

Pienaar explores a further consequence of rejecting absolute observed events while retaining single-valued events and a common spacetime embedding. The argument challenges whether those commitments can be maintained together. It is relevant to the ambition of the Extreme Frontiers program, but it is not a measurement of fragmented spacetime and is not required for the finite-dimensional results developed here [53].

6 Coordination tasks: what the civilization can gain

The proposed civilization needs an operational advantage that can be stated before its ontology is debated. Nonlocal games supply one: separated participants receive different questions and must produce jointly successful answers without exchanging those questions. Entanglement can increase their success probability under that communication constraint. The advantage is already an established result of quantum information theory. Here it supplies a measurable task for the Continuum's proposal and a precise quantity against which the cost of disclosing particular intermediate records can be tested [1, 2].

This choice has an immediate consequence for the argument. A civilization does not need to abolish public history to obtain the advantage. Every actual question and final answer may be archived after the round. The difficult object is a single local assignment of answers to all alternative questions that reproduces the quantum success rate. Section 18 proves that the final archive itself imposes no game-performance penalty. The physical cost derived later concerns a different operation: exporting information about a coherence-bearing branch before the incompatible task measurements.

6.1 A finite task and its allowed resources

Definition 6.1 (Finite coordination game)**.** Let $n \geq 2$ be an integer, let X_i and O_i be finite nonempty sets of inputs and outputs for participant $i \in \{1, \dots, n\}$, and put $\mathbf{X} = \prod_i X_i$ and $\mathbf{O} = \prod_i O_i$. A game is specified by a probability mass function $\mu : \mathbf{X} \to [0, 1]$ and a predicate $V : \mathbf{O} \times \mathbf{X} \to \{0, 1\}$. For a conditional probability table $p(\mathbf{a} \mid \mathbf{x})$, its success probability is

$$w_G(p) = \sum_{\mathbf{x} \in \mathbf{X}} \mu(\mathbf{x}) \sum_{\mathbf{a} \in \mathbf{O}} V(\mathbf{a}, \mathbf{x}) p(\mathbf{a} \mid \mathbf{x}). \tag{1}$$

The sampled input vector is independent of the prepared shared resource. Participants may agree on a strategy beforehand. After inputs are distributed, they cannot communicate until their outputs are fixed.

The inputs need not be mutually independent. A promised game can use correlated questions whose support is a proper subset of $\mathbf{X}$. Independence from the resource is the relevant preparation

assumption. A round contributes its declared score whether the devices perform well or badly; selecting only favorable outcomes changes the task.

A classical strategy has a setting-independent shared random variable λ and local response probabilities $p_i(a_i \mid x_i, \lambda)$. Its table has the form

$$p(\mathbf{a} \mid \mathbf{x}) = \sum_{\lambda} r_\lambda \prod_{i=1}^{n} p_i(a_i \mid x_i, \lambda), \qquad r_\lambda \geq 0, \quad \sum_{\lambda} r_\lambda = 1. \tag{2}$$

For these finite games, every such strategy is a convex mixture of deterministic response functions $f_i : X_i \to O_i$. Local private randomness can be sampled as a complete response table before the questions arrive. Since Equation (1) is linear, the classical value $\omega_c(G)$ is the maximum over deterministic response tables. Unlimited shared randomness does not increase that maximum.

A quantum strategy uses finite-dimensional complex Hilbert spaces $\mathcal{H}_i$, a positive trace-one operator ρ on $\bigotimes_i \mathcal{H}_i$, and local positive-operator-valued measurements $\{M^{(i)}_{a_i|x_i} : a_i \in O_i\}$ satisfying $\sum_{a_i} M^{(i)}_{a_i|x_i} = I_{\mathcal{H}_i}$. Its probabilities are

$$p_\rho(\mathbf{a} \mid \mathbf{x}) = \mathrm{Tr}\left[\rho \bigotimes_i M^{(i)}_{a_i|x_i}\right]. \tag{3}$$

Write $\omega_G(\rho)$ for the supremum of Equation (1) over these measurements, and $\omega_q(G)$ for the supremum additionally over finite-dimensional shared states. Local ancillary systems initially uncorrelated with the other participants can be absorbed into the local measurements. Shared randomness independent of the inputs is also allowed. Every claim about $\omega_G(\rho)$ below fixes the full shared quantum resource to the displayed state: an additional entangled pair, accessible environmental system, or successful-event conditioning is not implicitly available.

There is also a precise preparation boundary. The notation $\omega_G(\rho)$ starts after the displayed resource has been supplied to the participants and all interparty preparation communication has ended. Until the round's outputs are fixed, subsequent operations are local, with independent local ancillas and preexisting shared randomness. In particular, a participant cannot measure its share and send the result to another participant before the private questions arrive while continuing to call the resulting protocol an optimization of $\omega_G(\rho)$. Such local operations and classical communication (LOCC) define an enlarged preprocessing class and must be modeled explicitly. They do not change the standard unrestricted benchmarks $\omega_q(G)$, which already optimize the shared state, but they can change a fixed-resource optimum. Section 16.3 gives a concrete counterexample.

Lemma 6.2 (Separable-mixture bound)**.** *Suppose $\rho = (1-r)\sigma + r\tau$, with $r \in [0,1]$, where σ is fully separable across the participants and τ is a density operator on the same spaces. Then*

$$\omega_G(\rho) \leq (1-r)\omega_c(G) + r\omega_q(G). \tag{4}$$

Proof. Write $\sigma = \sum_\lambda r_\lambda \bigotimes_i \sigma_{i,\lambda}$. For any fixed local measurements its outcome table is Equation (2), with $p_i(a_i \mid x_i, \lambda) = \mathrm{Tr}(\sigma_{i,\lambda} M^{(i)}_{a_i|x_i})$. Its score is therefore at most $\omega_c(G)$. The same measurements on τ score at most $\omega_q(G)$. Linearity in the state proves the bound for every measurement choice, and hence for their supremum. □

The mixture components in this lemma need not be distinguishable to the participants. The proof does not grant them a hidden label identifying which component was prepared. The bound can consequently be loose; a convex decomposition gives an upper bound, not an automatic optimal strategy.

6.2 Three established benchmarks

For binary sign outputs, the probability of satisfying a target parity $\prod_i a_i = t(\mathbf{x})$, with $t(\mathbf{x}) \in \{-1, +1\}$, is $[1 + t(\mathbf{x})\langle \prod_i a_i \rangle]/2$. This identity turns correlators into an operational success score.

CHSH. Alice and Bob receive independent uniform inputs $x, y \in \{0, 1\}$, return signs $a, b \in \{-1, +1\}$, and win when $ab = (-1)^{xy}$. Let $E_{xy} = \sum_{a,b} ab\, p(a, b \mid x, y)$. Then

$$S = E_{00} + E_{01} + E_{10} - E_{11}, \qquad w_{\mathrm{CHSH}} = \frac{1}{2} + \frac{S}{8}. \tag{5}$$

A deterministic strategy supplies four signs a_0, a_1, b_0, b_1. The product of its four parity answers is $+1$, whereas the product of the four target parities is -1. At least one input pair loses. Constant positive answers win three pairs, proving $\omega_c(\mathrm{CHSH}) = 3/4$. Quantum mechanics permits $S = 2\sqrt{2}$ and hence $\omega_q(\mathrm{CHSH}) = 1/2 + \sqrt{2}/4$ [1, 12]. For completeness, the usual upper bound follows from the CHSH operator $\mathcal{B} = A_0 \otimes (B_0 + B_1) + A_1 \otimes (B_0 - B_1)$: for Hermitian binary projective observables, $\mathcal{B}^2 = 4I - [A_0, A_1] \otimes [B_0, B_1]$, so $\|\mathcal{B}\|^2 \leq 8$. Binary measurements admit a local projective dilation, giving the same bound generally.

The three-party GHZ game. Alice, Bob, and Charlie receive a uniform input from

$$\mathcal{X}_{\mathrm{GHZ}} = \{000, 011, 101, 110\}. \tag{6}$$

They return $a, b, c \in \{-1, +1\}$ and win when

$$abc = (-1)^{(x+y+z)/2}. \tag{7}$$

The exponent is an integer on the promised input set. A deterministic strategy specifies two answers for each of three participants. Multiplying the four demanded equations uses each local answer twice, giving $+1$ on the left and -1 on the right. At least one equation fails. Constant outputs with product -1 win the three inputs containing two ones, so $\omega_c(\mathrm{GHZ}) = 3/4$. The GHZ state with local X/Y measurements wins all four inputs, as derived in Section 16. This is the standard parity-game manifestation of the GHZ contradiction, with $\omega_q(\mathrm{GHZ}) = 1$ [2, 54].

The magic-square game. Alice receives a uniformly random row $r \in \{1, 2, 3\}$ and returns a triple $\mathbf{a} = (a_1, a_2, a_3) \in \{-1, +1\}^3$ with product $+1$. Bob independently receives a uniformly random column $c \in \{1, 2, 3\}$ and returns $\mathbf{b} = (b_1, b_2, b_3)$ with product η_c, where $(\eta_1, \eta_2, \eta_3) = (+1, +1, -1)$. They win if both parity constraints hold and $a_c = b_r$ at the intersection. This sign convention is equivalent to the familiar convention with all three column products negative: flip the entries $(1, 1)$ and $(1, 2)$ in both players' answers. The first row product is unchanged, and precisely the first two column products reverse.

Any strategy producing an invalid parity can replace that answer by a valid one without losing a previously winning input. It is therefore sufficient to consider valid-parity deterministic answer tables. If all nine intersections agreed, one 3×3 sign array would have total product $+1$ when multiplied by rows and -1 when multiplied by columns. This is impossible. The upper bound is $8/9$. It is attained by Alice returning only $+1$, Bob doing the same in columns one and two, and Bob returning $(+1, +1, -1)$ in column three. Only intersection $(3, 3)$ loses. Thus $\omega_c(\mathrm{MS}) = 8/9$. Two Bell pairs and compatible Pauli measurements attain $\omega_q(\mathrm{MS}) = 1$, as proved in Section 17 [2, 55, 56].

6.3 The measurement constraint changes the benchmark

The observer experiment elsewhere in this paper designates one actual record-reading measurement at each laboratory. That restriction must survive the translation from a witness to a game. Define the two-qubit state

$$\rho_\Phi(v) = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11| + v|00\rangle\langle 11| + v|11\rangle\langle 00|), \qquad v \in [0, 1]. \tag{8}$$

Here v is a dimensionless coherence parameter. The computational basis is fixed by the branch-recording interaction, and Z denotes its sign readout. The explicit optimization in Propositions 10.1 and 10.2 gives

$$w^{\mathrm{free}}_{\mathrm{CHSH}}(v) = \frac{1}{2} + \frac{\sqrt{1+v^2}}{4}, \qquad \text{freely chosen binary projective settings,} \tag{9}$$

$$w^{\mathrm{read}}_{\mathrm{CHSH}}(v) = \frac{3}{4} + \frac{v^2}{8(1+v)}, \qquad A_0 = B_0 = Z. \tag{10}$$

In the second line only the remaining settings are optimized. At $v = 1$ these values are approximately 0.853553 and exactly 13/16, respectively. The second quantity is the applicable ideal maximum for this designated-readout protocol. Substituting the unrestricted CHSH value would change its measurement contract.

The formulas also bound arbitrary binary qubit measurements on this fixed resource. Each binary measurement is specified by an effect $0 \le E \le I$; an optimum of a linear objective can be chosen at an extreme effect, which is a projector. The nondegenerate projectors are precisely the Bloch-sphere measurements optimized above. A degenerate projector gives a deterministic observable $+I$ or $-I$. The two observables on that side then commute, admit a joint measurement, and cannot exceed the classical CHSH bound. The displayed projective optima already reach at least that bound. This argument also covers local uncorrelated ancillas through their effective qubit measurements. It does not introduce extra shared entanglement.

The GHZ and magic-square extensions below use their ordinary task measurements, without imposing two fixed friend-readout settings. They are nonlocal coordination models with specified disclosure channels. They become models of artificial observers only when the further physical and operational conditions for those observers are independently supplied. Keeping the task contract explicit makes the proposed advantage substantial and testable: it is an increased probability of meeting an agreed distributed objective.

6.4 A calibrated state error gives a task error

A task advantage becomes experimentally useful when its margin survives uncertainty about the supplied state. The following standard consequence of trace-distance discrimination provides that link [57].

Proposition 6.3 (State error bounds task error)**.** *Fix a finite game $G = (\mathbf{X}, \mathbf{O}, \mu, V)$ and local POVMs $\Pi = \{M^{(i)}_{a_i|x_i}\}$ as in Definition 6.1. Let ρ, σ be density operators on the same finite tensor-product space and put $d = \frac{1}{2}\|\rho - \sigma\|_1$. Then*

$$|w_G(\rho, \Pi) - w_G(\sigma, \Pi)| \le d.$$

The same bound holds for the difference of suprema over any common, state-independent nonempty strategy class $\mathcal{S}$:

$$\left|\sup_{\Pi\in\mathcal{S}} w_G(\rho, \Pi) - \sup_{\Pi\in\mathcal{S}} w_G(\sigma, \Pi)\right| \le d.$$

Proof. The winning effect is the operator

$$M_{G,\Pi} = \sum_{\mathbf{x}} \mu(\mathbf{x}) \sum_{\mathbf{a}} V(\mathbf{a}, \mathbf{x}) \bigotimes_i M^{(i)}_{a_i|x_i}.$$

For each input, the sum selects a subset of the positive elements of a normalized product POVM. Therefore $0 \le M_{G,\Pi} \le I$ and $w_G(\rho, \Pi) = \mathrm{Tr}(M_{G,\Pi}\rho)$. Write the Jordan decomposition $\rho - \sigma = H_+ - H_-$, where $H_\pm \ge 0$, their supports are orthogonal, and $\mathrm{Tr}\, H_+ = \mathrm{Tr}\, H_- = d$. For every effect $0 \le M \le I$, each $\mathrm{Tr}(MH_\pm)$ lies in $[0, d]$, so $|\,\mathrm{Tr}[M(\rho - \sigma)]| \le d$. Apply this to $M_{G,\Pi}$. Taking the supremum of $w_G(\rho, \Pi) \le w_G(\sigma, \Pi) + d$ and reversing the roles of the states proves the second assertion. Attainment of the supremum is unnecessary. □

If a fixed ideal strategy scores $\omega_c(G) + \Delta$ with $\Delta > 0$, a state calibration $d < \Delta$ preserves a strictly positive guaranteed gap for the same instruments. This conclusion is conditional on the stated game and access restrictions. Changed instruments require an additional channel or probability-table error bound. A disclosure upper bound alone does not certify the lower resource estimate needed for a performance guarantee.

7 The physical grammar of an artificial observer

A claim about a civilization of quantum observers has to identify what each observer can remember, what another observer can control, and which systems remain outside that control. These are separate physical questions. The following model makes them explicit before attaching any interpretation to an observer's experience. Its elementary constituents are finite-dimensional quantum systems. The proposed civilizational interpretation is an additional research hypothesis; the calculations concern records, transformations, and outcome probabilities.

All Hilbert spaces in this section are finite-dimensional complex inner-product spaces. For a space $\mathcal{H}$, $\mathcal{L}(\mathcal{H})$ is its space of linear operators, $I_\mathcal{H}$ its identity, and $\mathcal{D}(\mathcal{H})$ the set of positive semidefinite operators of trace one. The adjoint of A is $A^\dagger$, the trace is Tr, and the trace norm is

$\|A\|_1 = \operatorname{Tr}\sqrt{A^\dagger A}$. A physical deterministic channel $\Lambda : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{K})$ is completely positive and trace preserving. A measurement with finite outcome set $\mathcal{O}$ has positive effects $M_o \in \mathcal{L}(\mathcal{H})$ satisfying $\sum_{o\in\mathcal{O}} M_o = I_{\mathcal{H}}$; its outcome probabilities are $p(o) = \operatorname{Tr}(M_o\rho)$. An instrument additionally specifies the completely positive trace-nonincreasing state transformation for each outcome, with their sum trace preserving. An outcome distribution alone does not determine that transformation.

These conventions are essential when a later measurement is supposed to reverse an earlier observation. Two instruments can have identical first-outcome statistics and different effects on subsequent interference. Replacing an instrument by a list of classical outcomes discards exactly the information needed to analyze a reversible observation.

7.1 A record is a distinguishable physical alternative

Let Q be a two-dimensional branch register with orthonormal basis $\{|0\rangle, |1\rangle\}$. A recording interaction with a blank memory F can be represented on the relevant input subspace by the isometry

$$|b\rangle_Q|r\rangle_F \longmapsto |b\rangle_Q|f_b\rangle_F, \qquad b \in \{0, 1\}. \tag{11}$$

Here $|r\rangle_F$ and both $|f_b\rangle_F$ are unit vectors. The memory is a perfect binary record if $\langle f_0|f_1\rangle = 0$. The isometry exists even when the memory states are not orthogonal, because the retained branch states in Q are orthogonal. It extends to a unitary after the input and output spaces are embedded in a common sufficiently large space.

Equation (11) defines a physical recording relation. It does not, by itself, establish that F has semantic understanding, that it experiences an outcome, or that it qualifies as the thoughtful observer in a particular no-go theorem. Those additional claims require separate operational criteria and assumptions. The model can nevertheless expose a necessary condition for any more capable observer built on the same quantum dynamics: its outcome-dependent memory and its inaccessible outputs must be included in the recording process.

For density operators $\rho, \sigma \in \mathcal{D}(\mathcal{H})$ define trace distance and root fidelity by

$$D(\rho, \sigma) = \tfrac{1}{2}\|\rho - \sigma\|_1, \qquad F(\rho, \sigma) = \|\sqrt{\rho}\sqrt{\sigma}\|_1. \tag{12}$$

Both lie in $[0, 1]$. The word "root" is retained because some literature calls F^2 the fidelity. All appearances of F here use Equation (12). For two hypotheses with equal prior probabilities, the optimum probability of correctly identifying the state is $(1 + D)/2$, and the minimum error is $(1-D)/2$. This is the binary quantum decision result associated with Helstrom; a short proof fixes the precise normalization needed below [57, 58].

Lemma 7.1 (Binary record error)**.** *Let ρ_0, ρ_1 be density operators on the same finite-dimensional space and assign prior probability $1/2$ to each. Among all two-outcome measurements, the minimum average identification error is*

$$\varepsilon_* = \frac{1 - D(\rho_0, \rho_1)}{2}. \tag{13}$$

Proof. Write M for the effect associated with the guess 0, so $0 \le M \le I$. The success probability is $\frac{1}{2}\operatorname{Tr}(M\rho_0) + \frac{1}{2}\operatorname{Tr}((I-M)\rho_1) = \frac{1}{2} + \frac{1}{2}\operatorname{Tr}(M(\rho_0 - \rho_1))$. The Hermitian operator $H = \rho_0 - \rho_1$ has trace zero. Its positive and negative parts therefore have equal trace $\|H\|_1/2$. The maximum of

$\mathrm{Tr}(MH)$ is the trace of the positive part, attained by its support projector. The success probability is consequently $(1 + D)/2$. □

An operationally public record is a physical subsystem from which an allowed reader can perform this discrimination, or a specified restricted measurement achieving a stated error. The result does not require every citizen to read the record. Its accessibility outside a proposed interference controller is already relevant. A report that nobody chooses to open can still carry which-branch information.

7.2 The boundary of coherent control

Write $L = Q \otimes R$ for the controlled laboratory, where R includes every memory, workspace, clock, and ancillary degree of freedom available to the controller. Write E for all remaining physical degrees of freedom relevant to the recording process. The partition $L|E$ is a declared control boundary, not a division between mental and nonmental matter. A second citizen belongs to E if its outcome-dependent memory cannot be coherently controlled during the experiment. It belongs to R if its full relevant state is included in that control.

Consider the normalized family

$$|\Psi_\phi\rangle = \frac{|0\rangle_Q|\psi_0\rangle_{RE} + e^{i\phi}|1\rangle_Q|\psi_1\rangle_{RE}}{\sqrt{2}}, \qquad \phi \in [0, 2\pi), \tag{14}$$

where $|\psi_0\rangle, |\psi_1\rangle \in R \otimes E$ have norm one. They need not be orthogonal or product states. The branch register makes the two terms orthogonal. Define

$$\begin{aligned} \rho_L(\phi) &= \mathrm{Tr}_E\, |\Psi_\phi\rangle\langle\Psi_\phi|, & \rho_E^{(b)} &= \mathrm{Tr}_R\, |\psi_b\rangle\langle\psi_b|, \\ P_L &= D(\rho_L(0), \rho_L(\pi)), & D_E &= D(\rho_E^{(0)}, \rho_E^{(1)}). \end{aligned} \tag{15}$$

The dimensionless quantity $P_L \in [0, 1]$ is accessible phase distinguishability: it measures the best possible discrimination between two opposite preparation phases using the laboratory alone. The quantity $D_E \in [0, 1]$ measures the ability of the exterior to identify the branch. The two quantities describe different tasks on complementary subsystems.

This operational definition avoids a misleading shortcut. A controller can discard a decohered qubit and prepare a fresh $|+\rangle$ state with a large expectation of the Pauli X operator. That operation creates new coherence but recovers no information about the earlier preparation phase. In contrast, P_L cannot increase under any phase-independent deterministic channel on L. Thus the quantity being bounded is historical phase information available for an experiment, not the controller's ability to prepare an arbitrary new state.

8 The information cost of exterior branch records

The following results combine established trace-distance, fidelity, and interference mathematics. Their role here is to place a precise constraint on the proposed civilization. They are not presented as newly discovered laws of quantum mechanics. The familiar which-path/visibility relation is a special case; the more general form explicitly permits a large controlled internal memory [27, 58, 59].

Lemma 8.1 (Pure-state trace distance)**.** *For unit vectors $|u\rangle, |v\rangle$ in one Hilbert space,*

$$D(|u\rangle\langle u|, |v\rangle\langle v|) = \sqrt{1 - |\langle u|v\rangle|^2}. \tag{16}$$

Proof. A phase choice makes $\langle u|v\rangle = c \in [0,1]$. On the span of the two vectors choose an orthonormal basis with $|u\rangle = (1,0)^{\mathsf{T}}$ and $|v\rangle = (c, \sqrt{1-c^2})^{\mathsf{T}}$. The difference of their projectors has trace zero and determinant $-(1-c^2)$, hence eigenvalues $\pm\sqrt{1-c^2}$. The operator vanishes on the orthogonal complement. Taking half its trace norm proves the formula, including $c = 0$ and $c = 1$ by the same expression. □

Theorem 8.2 (Record–phase complementarity with partial control)**.** *For the family in Equation (14),*

$$P_L = F(\rho_E^{(0)}, \rho_E^{(1)}), \qquad P_L^2 + D_E^2 \le 1. \tag{17}$$

If a reader of E can identify the branch with equal-prior average error at most $\varepsilon \in [0, 1/2]$, then

$$P_L \le 2\sqrt{\varepsilon(1-\varepsilon)}. \tag{18}$$

The inequality in Equation (17) and the error bound also hold for any balanced mixed state subjected to the same phase encoding on Q, with P_L and D_E defined from its accessible states and conditional exterior states.

Proof. Put $A = \mathrm{Tr}_E\, |\psi_0\rangle\langle\psi_1| \in \mathcal{L}(R)$. The difference $\rho_L(0) - \rho_L(\pi)$, in the branch basis of Q, is

$$\begin{pmatrix} 0 & A \\ A^\dagger & 0 \end{pmatrix}. \tag{19}$$

Its nonzero eigenvalues are the positive and negative singular values of A. Therefore $P_L = \|A\|_1$.

The variational formula for the trace norm gives $\|A\|_1 = \max_{U_R} |\,\mathrm{Tr}(U_R A)|$, where the maximum is over unitaries on R. The quantity under the maximum is $|\langle\psi_1|(U_R \otimes I_E)|\psi_0\rangle|$. Uhlmann's purification identity identifies its maximum with $F(\rho_E^{(0)}, \rho_E^{(1)})$ [59]. The upper Fuchs–van de Graaf bound $D(\rho,\sigma) \le \sqrt{1 - F(\rho,\sigma)^2}$ then yields Equation (17) [58]. One can also see this upper bound by choosing purifications attaining fidelity: their pure-state distance is $\sqrt{1-F^2}$ by Lemma 8.1, and tracing out the purifying subsystem cannot increase trace distance.

The existence of a reader with error at most ε implies $D_E \ge 1 - 2\varepsilon$ by Lemma 7.1. Substitution gives $P_L^2 \le 1 - (1-2\varepsilon)^2 = 4\varepsilon(1-\varepsilon)$.

For a mixed balanced state, purify the entire state by adding an inaccessible register K. Its decomposition in the Q basis has the form Equation (14) with E replaced by $E \otimes K$. Balance ensures each conditional purifying vector has norm one. The preceding pure-state result bounds $P_L^2 + D_{EK}^2$ by one. Tracing out K gives $D_E \le D_{EK}$, proving the desired inequality for the actual exterior record. The same error argument then applies. □

The theorem has a sharp interpretation. A perfectly distinguishable outcome-dependent record outside the controller, $D_E = 1$, removes all accessible information distinguishing opposite phases: $P_L = 0$. If the controller can distinguish those phases perfectly, $P_L = 1$, then no exterior measurement can identify the branch better than random guessing. Intermediate cases quantify the compromise. This result concerns a specified branch pair, phase family, and control boundary. It does not forbid a civilization from maintaining ordinary shared facts in other degrees of freedom.

Corollary 8.3 (Subsequent phase-blind control)**.** *For any completely positive trace-preserving channel* Λ *acting on the controlled laboratory and independent of* ϕ*,*

$$D(\Lambda(\rho_L(0)), \Lambda(\rho_L(\pi))) \le P_L \le 2\sqrt{\varepsilon(1-\varepsilon)}. \tag{20}$$

Proof. Trace distance is the maximum difference in outcome probability over effects $0 \le M \le I$. The adjoint of a trace-preserving completely positive map is unital and positive, so $0 \le \Lambda^\dagger(M) \le I$. Each output effect is therefore equivalent to an allowed input effect. Maximization cannot yield a larger value after the channel. The second inequality is Theorem 8.2. □

The condition that Λ be independent of the unknown phase is substantive. A controller handed the preparation label can simply write it to a new register. That side channel changes the information available and lies outside the stated discrimination task. Likewise, a trace-decreasing postselected operation must be analyzed with its success flag and probability. Comparing only successful outputs can raise their conditional distinguishability; it does not invalidate a deterministic bound on the full experiment.

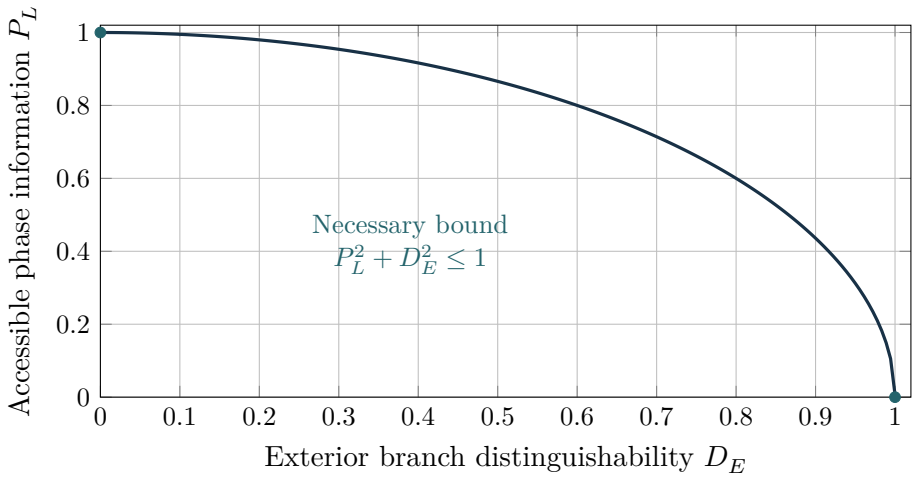


Figure 1: The record–phase boundary. Balanced pure marker states attain the curve; the general mixed-state statement is an inequality. The quantity P_L measures information about the original opposite phases, so preparing a fresh coherent state does not evade the bound.

8.1 The elementary visibility formula

For a two-dimensional controlled laboratory with pure conditional exterior vectors, write

$$|\Psi_\phi\rangle = \frac{|0\rangle_L|e_0\rangle_E + e^{i\phi}|1\rangle_L|e_1\rangle_E}{\sqrt{2}}, \qquad \gamma = \langle e_1|e_0\rangle. \tag{21}$$

Tracing out E yields

$$\rho_L(\phi) = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\phi}\gamma \\ e^{i\phi}\gamma^* & 1 \end{pmatrix}. \tag{22}$$

Let $|+_\theta\rangle = (|0\rangle + e^{i\theta}|1\rangle)/\sqrt{2}$, where $\theta \in [0, 2\pi)$ is the measurement phase. Its Born probability is

$$p_+(\theta \mid \phi) = \frac{1 + \mathrm{Re}(e^{i(\theta-\phi)}\gamma)}{2}. \tag{23}$$

Define fringe visibility by $V = (p_{\max} - p_{\min})/(p_{\max} + p_{\min})$, where the extrema are taken over θ for fixed ϕ. Then $V = |\gamma| = P_L$ and

$$D_E = \sqrt{1 - |\gamma|^2}, \qquad V^2 + D_E^2 = 1. \tag{24}$$

This equality is the balanced pure-marker case of interference complementarity. For mixed conditional states the general assertion is the inequality, not universal equality. A measured reduction in visibility also need not imply that a single accessible observer holds a readable record; information may reside in inaccessible or nonlocal correlations.

8.2 Why balanced priors and stable calibration matter

The error interpretation above uses equal priors. With prior probability $p \in (0, 1)$ for branch 0, the minimum error is

$$\varepsilon_*(p) = \frac{1 - \|p\rho_E^{(0)} - (1-p)\rho_E^{(1)}\|_1}{2}. \tag{25}$$

For the pure branch family with amplitudes $\sqrt{p}$ and $\sqrt{1-p}$, the accessible opposite-phase distinguishability becomes $2\sqrt{p(1-p)}F(\rho_E^{(0)}, \rho_E^{(1)})$. When the two marker states are identical but $p = 0.99$, always guessing branch 0 achieves error 0.01 without obtaining any branch information. Such an error rate cannot certify a reliable record. The experiment must therefore balance the hypotheses or report a discrimination measure that accounts for the actual prior. Calibration is part of the theorem's applicability.

The physical marker overlap can also carry an unknown complex phase. A controller may compensate that phase only using a declared calibration procedure. Treating an unknown phase as a known positive visibility parameter would silently give the controller information it does not possess. The analytic upper bounds below allow ideal known phase compensation; any residual phase uncertainty is a further implementation cost.

9 Redundancy, nonlocal records, and the limits of a public archive

Environment-induced decoherence and redundant records provide an established mechanism for the practical stability of classical descriptions. In the environment-as-witness program, many observers can obtain information about preferred pointer observables through environmental fragments. This is a positive account of shared observable structure, not a presumption that every quantum degree of freedom can be copied [60, 61]. The elementary model makes the relevant independence assumptions visible.

Proposition 9.1 (Independent record fragments)**.** *Suppose the conditional exterior states in Equation* (21) *factorize as* $|e_b\rangle = \bigotimes_{j=1}^{m} |e_b^{(j)}\rangle$*, where* m *is a positive integer and every factor has norm one. Define* $\gamma_j = \langle e_1^{(j)} | e_0^{(j)} \rangle$*. Then*

$$V = \prod_{j=1}^{m} |\gamma_j|. \tag{26}$$

If each fragment permits equal-prior branch identification with error at most $\varepsilon_j \in [0, 1/2]$, then

$$V \le \prod_{j=1}^{m} 2\sqrt{\varepsilon_j(1-\varepsilon_j)}. \tag{27}$$

Proof. Tensor-product inner products factorize, so $\gamma = \prod_j \gamma_j$. For each pure marker pair, Lemma 7.1 and Equation (24) give $|\gamma_j| \le 2\sqrt{\varepsilon_j(1-\varepsilon_j)}$. Multiplication proves both claims. □

The exponent in the equal-error case comes from conditional product structure. Counting copies is insufficient. A public archive can contain many strongly correlated records whose errors share the same physical origin, so the number of apparently independent witnesses must not be substituted for a verified factorization of their conditional states.

Counterexample: many correlated records do not multiply the bound. Fix an integer $m \ge 2$ and $\varepsilon \in (0, 1/2)$. On m exterior qubits set

$$\begin{aligned} |e_0\rangle &= \sqrt{1-\varepsilon}\,|0\rangle^{\otimes m} + \sqrt{\varepsilon}\,|1\rangle^{\otimes m}, \\ |e_1\rangle &= \sqrt{\varepsilon}\,|0\rangle^{\otimes m} + \sqrt{1-\varepsilon}\,|1\rangle^{\otimes m}. \end{aligned} \tag{28}$$

Each single-qubit marginal is diagonal, and reading it in the computational basis identifies the branch with optimal error ε. Nevertheless, $|\langle e_1|e_0\rangle| = 2\sqrt{\varepsilon(1-\varepsilon)}$, independently of m. Replacing this value by its mth power would be incorrect. At $\varepsilon = 0.03$, six such fragments retain overlap approximately 0.341174; an unjustified product calculation would instead give approximately 0.001577. The correlated family attains the single-record complementarity bound and demonstrates why independence has to be an explicit premise.

Counterexample: a record can be invisible in every proper fragment. On $m \ge 2$ exterior qubits take

$$|e_0\rangle = \frac{|0\rangle^{\otimes m} + |1\rangle^{\otimes m}}{\sqrt{2}}, \qquad |e_1\rangle = \frac{|0\rangle^{\otimes m} - |1\rangle^{\otimes m}}{\sqrt{2}}. \tag{29}$$

These two vectors are orthogonal, so the exterior as a whole identifies the branch perfectly and the uncontrolled laboratory has $V = 0$. Every nonempty proper subset of exterior qubits, however, has the same reduced density operator for both branches: the equal mixture of its all-zero and all-one strings. No such fragment carries any branch distinguishability. The record is available only through a joint measurement sensitive to the global phase, for example the parity of X-basis outcomes. Thus an audit that inspects only individual citizens or individual storage devices cannot establish the absence of a record in correlations among them.

9.1 Expanding control changes the problem

If a controller gains coherent access to a previously exterior fragment, that fragment moves from E into R. The relevant complementarity bound must be recomputed on the new partition. For

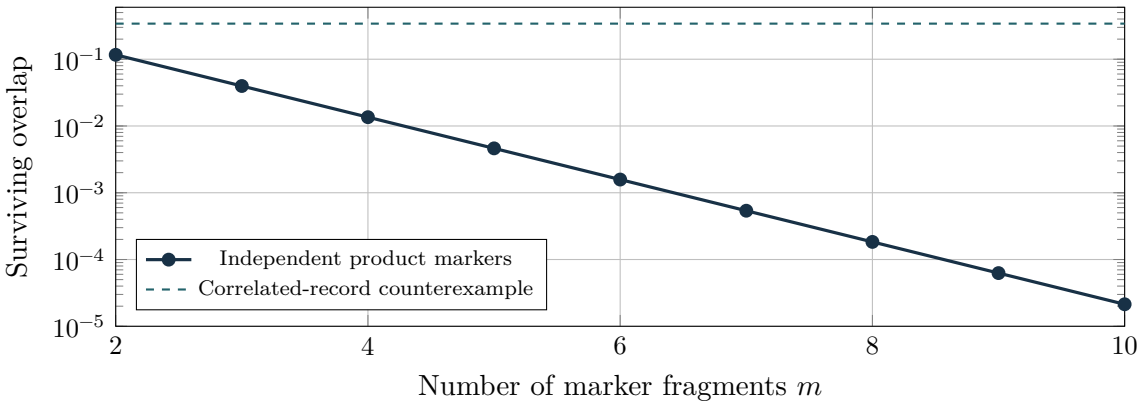


Figure 2: Equal individual record error does not fix a redundancy law. Both constructed families give each marker an optimal branch error of 0.03. Conditional product structure makes the first overlap decrease exponentially; the correlated family retains the same overlap for every m. These are analytic examples, not measurements.

conditional product markers, controlling a subset J of the fragments and coherently undoing their recording interactions leaves

$$V_{\text{remaining}} = \prod_{j \notin J} |\gamma_j|. \tag{30}$$

This is a statement about the specified reversible product interaction. It is not a general formula for arbitrary correlated environments. Theorem 8.2, with the revised exterior states, is the appropriate general statement.

The physical issue is consequently the boundary of the experiment. An outcome-dependent message, a durable log, a displaced actuator, and radiation correlated with a branch can all become exterior markers. Reversing the observer's processor while leaving such effects uncontrolled does not restore the original coherent branch family. Conversely, a stable common archive can coexist with a coherent experiment if its stored variables are independent of the particular branches whose phase is being tested. The theorem constrains which information can be simultaneously public and coherent; it does not require total social isolation of every experimental apparatus.

9.2 Copyable records and uncopyable quantum states

A public classical record is mathematically compatible with quantum theory. For an orthogonal basis $\{|b\rangle\}$, a controlled copying unitary can map $|b\rangle|0\rangle$ to $|b\rangle|b\rangle$. Applied to a coherent superposition, however, it produces an entangled state rather than two copies of that superposition. If one physical channel cloned every unit vector, its isometric dilation would map a fixed blank apparatus and $|u\rangle$ to $|u\rangle|u\rangle|m_u\rangle$. The unused environment must factorize here because the two-copy output is pure. Preservation of inner products would require $\langle u|v\rangle = \langle u|v\rangle^2\langle m_u|m_v\rangle$. For $0 < |\langle u|v\rangle| < 1$ this is impossible, since $|\langle m_u|m_v\rangle| \leq 1$. This is the standard no-cloning obstruction [62].

For mixed states, broadcasting asks for both output marginals to equal the input state, allowing correlations between the outputs. The no-broadcasting theorem restricts the sets of states for which one channel can accomplish this [63]. A finite commuting family has a shared eigenbasis;

measuring that basis and writing the outcome twice broadcasts the family. This contrast matters for the civilization model. Shared public knowledge about a selected commuting record algebra is possible. A universal archive that faithfully distributes arbitrary unknown quantum states to every citizen is a different and obstructed task. The public layer must specify the observables it preserves.

10 A two-laboratory benchmark with a visible control budget

The next construction supplies a solvable correlation benchmark. Its basic witness is CHSH. A Local Friendliness interpretation requires the additional observer, readout, locality, and setting assumptions developed elsewhere in the manuscript; a CHSH calculation on two logical qubits alone cannot provide them. The value of the benchmark is that every record and every loss of coherence can be written explicitly.

Let each laboratory $j \in \{A, B\}$ have logical basis $\{|0\rangle_j, |1\rangle_j\}$. Start from the balanced entangled state $(|00\rangle + |11\rangle)/\sqrt{2}$. Conditional independent leakage maps each local basis state to itself tensored with $|e_b^{(j)}\rangle$. The global state becomes

$$\frac{|00\rangle_{AB}|e_0^{(A)}\rangle|e_0^{(B)}\rangle + |11\rangle_{AB}|e_1^{(A)}\rangle|e_1^{(B)}\rangle}{\sqrt{2}}. \tag{31}$$

Define $\gamma_j = \langle e_1^{(j)}|e_0^{(j)}\rangle$. The reduced off-diagonal entry is $\gamma_A\gamma_B/2$. After compensation of its known phase, let $v = |\gamma_A\gamma_B| \in [0, 1]$. In the ordered basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, the reduced state is

$$\rho(v) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & v \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ v & 0 & 0 & 1 \end{pmatrix}. \tag{32}$$

Its eigenvalues are $(1 + v)/2$, $(1 - v)/2$, 0, and 0, so positivity and normalization hold for the full stated range. Its one-laboratory marginals are both $I_2/2$. At $v = 0$ it is the classical correlated mixture of 00 and 11; at $v = 1$ it is the pure entangled state.

Let X, Y, Z denote the Pauli matrices, with $Z|b\rangle = (-1)^b|b\rangle$. A projective binary observable is $\mathbf{a} \cdot \boldsymbol{\sigma}$ for a real unit vector $\mathbf{a} \in \mathbb{R}^3$, where $\boldsymbol{\sigma} = (X, Y, Z)$. Outcomes are ± 1. Direct evaluation gives the real correlation matrix

$$T_{kl} = \mathrm{Tr}(\rho(v)\sigma_k \otimes \sigma_l), \qquad T = \mathrm{diag}(v, -v, 1). \tag{33}$$

The correlator is $E(\mathbf{a}, \mathbf{b}) = \mathbf{a}^{\mathsf{T}} T \mathbf{b}$. All matrix entries and witness values are dimensionless. Throughout the witness optimizations below, these projective qubit measurements are nondegenerate and traceless: both one-dimensional outcome projectors are present. Deterministic outputs represented by I_2 or $-I_2$ are a different allowed measurement class and are identified separately when relevant.

10.1 Unrestricted projective CHSH optimization

For measurement settings $x, y \in \{0, 1\}$, define

$$S = E_{00} + E_{01} + E_{10} - E_{11}. \tag{34}$$

A local hidden-variable model with setting-independent hidden-variable distribution has $|S| \leq 2$. This is the CHSH benchmark [12]. The following elementary calculation reproduces the applicable special case of the Horodecki two-qubit optimization [64].

Proposition 10.1 (Unrestricted two-qubit optimum)**.** *For $\rho(v)$ and arbitrary local nondegenerate projective binary observables,*

$$S_{\max}(v) = 2\sqrt{1 + v^2}. \tag{35}$$

It is attained by

$$A_0 = Z, \quad A_1 = X, \quad B_0 = \frac{Z + vX}{\sqrt{1 + v^2}}, \quad B_1 = \frac{Z - vX}{\sqrt{1 + v^2}}. \tag{36}$$

Proof. For Bob's unit Bloch vectors write $\mathbf{b}_0 + \mathbf{b}_1 = 2\cos\theta\,\mathbf{u}$ and $\mathbf{b}_0 - \mathbf{b}_1 = 2\sin\theta\,\mathbf{w}$, where $\theta \in [0, \pi/2]$ and $\mathbf{u}, \mathbf{w}$ are orthogonal unit vectors. Zero sums or differences are accommodated by choosing the unused vector arbitrarily. Maximizing Alice's independent vectors gives $S \leq 2\cos\theta\|T\mathbf{u}\| + 2\sin\theta\|T\mathbf{w}\| \leq 2\sqrt{\|T\mathbf{u}\|^2 + \|T\mathbf{w}\|^2}$. The latter squared sum is at most the sum of the two largest eigenvalues of $T^{\mathsf{T}}T$, namely $1 + v^2$. Choose $\mathbf{u}$ along Z, $\mathbf{w}$ along X, and $\tan\theta = v$ to saturate each bound. □

If the settings are instead fixed at their $v = 1$ values, $B_0 = (Z + X)/\sqrt{2}$ and $B_1 = (Z - X)/\sqrt{2}$, then

$$S_{\text{fixed}}(v) = \sqrt{2}(1 + v). \tag{37}$$

These settings violate CHSH only for $v > \sqrt{2} - 1$. The unrestricted optimum exceeds two for every $v > 0$ in this ideal phase-damping model. There is no contradiction between the two thresholds: they refer to different allowed measurement choices. Neither threshold is a general noise tolerance for an arbitrary Local Friendliness experiment.

10.2 When both laboratories must retain an actual readout setting

An observer experiment may require that one setting at each laboratory reads the internal observer's record. In the logical basis used here this gives $A_0 = B_0 = Z$. The unrestricted settings in Equation (36) do not satisfy that requirement for Bob when $v > 0$. The resulting constrained optimization is therefore different.

Proposition 10.2 (Two readout settings fixed)**.** *For $\rho(v)$, $A_0 = B_0 = Z$, and arbitrary nondegenerate projective binary A_1, B_1,*

$$S_{\text{read}}(v) = 2 + \frac{v^2}{1 + v}. \tag{38}$$

For $v \in [0, 1]$ define $c = 1/(1 + v)$ and $s = \sqrt{1 - c^2}$. An attaining pair is $A_1 = cZ + sX$, $B_1 = cZ - sX$.

Proof. Let b be the Z component of Bob's second Bloch vector. After maximizing Alice's second vector, the value is $1+b+\sqrt{v^2(1-b^2)+(1-b)^2}$. Put $t=1-b\in[0,2]$ and $k=v^2/(1+v)$. Then

$$(t+k)^2-\left[2v^2t+(1-v^2)t^2\right]=v^2\left(t-\frac{v}{1+v}\right)^2\geq 0. \tag{39}$$

Since $t+k\geq 0$, the square-root term is at most $t+k$, giving $S\leq 2+k$. At $v>0$ equality is attained by $t=v/(1+v)$, hence $b=c$, and the stated opposite transverse components. At $v=0$ the same formula gives two, with a degenerate family of maximizing observables. □

At $v=1$ the constrained optimum is $5/2$, below the unrestricted value $2\sqrt{2}$. At small v, both advantages are of order v^2 but their exact values differ. This distinction prevents a convenient Bell benchmark from silently replacing the operational constraints of an observer experiment.

Relation to the original Bell inequality. The fixed-readout optimization has close mathematical prior art. Khrennikov and Loubenets study the original Bell inequality for states with perfect correlations and reduce its optimization to the correlation matrix [65]. For the present state, let $T=\mathrm{diag}(v,-v,1)$, $0\leq v\leq 1$, and let $a,c,r\in\mathbb{R}^3$ be Euclidean unit vectors with $r=(0,0,1)$. Write $\sigma=(X,Y,Z)$ for the Pauli vector and set $A_0=B_0=r\cdot\sigma$, $A_1=a\cdot\sigma$, and $B_1=c\cdot\sigma$. Then

$$S=1+a^{\mathsf{T}}T(r-c)+r^{\mathsf{T}}Tc,\qquad W^{(+)}=\left|a^{\mathsf{T}}T(r-c)\right|+r^{\mathsf{T}}Tc.$$

Choosing the sign of the free setting A_1 so that $a^{\mathsf{T}}T(r-c)\geq 0$ gives $S=1+W^{(+)}$; equivalently the two optimized functionals differ by one. Their optimization over a yields

$$\max_a W^{(+)}=\|T(r-c)\|_2+r^{\mathsf{T}}Tc=\sqrt{v^2(1-z^2)+(1-z)^2}+z,\qquad z=c_z\in[-1,1].$$

The proposition evaluates the remaining scalar maximum for this family as $S_{\mathrm{read}}(v)=2+v^2/(1+v)$. Its value $5/2$ at $v=1$ corresponds to the established maximal value $3/2$ of the perfect-correlation original Bell functional in the two-qubit case. We present the displayed closed form as an explicit specialization with a self-contained proof; no historical priority claim for that expression is made.

10.3 An explicit four-qubit record and reversal

Take four qubits in the order S_A,F_A,S_B,F_B. Each S_j is an input system and each F_j an initially blank record. A controlled-NOT operation with S_j as control maps $|b\rangle_{S_j}|0\rangle_{F_j}$ to $|b\rangle_{S_j}|b\rangle_{F_j}$. Define logical vectors

$$|0\rangle_{L_j}=|00\rangle_{S_jF_j},\qquad |1\rangle_{L_j}=|11\rangle_{S_jF_j}. \tag{40}$$

The physical record readout $I_{S_j}\otimes Z_{F_j}$ restricts to logical Z. To measure a tilted logical observable $cZ+sX$, apply the inverse controlled-NOT and measure $cZ+sX$ on S_j. On the encoded subspace the pulled-back observable is

$$U_j^{\dagger}[(cZ_{S_j}+sX_{S_j})\otimes I_{F_j}]U_j, \tag{41}$$

where U_j is the controlled-NOT and $U_j^{\dagger}=U_j$. This implements the desired logical observable exactly. The reversal returns F_j to its blank state; the corresponding internal record does not remain independently readable through that operation.

An uncontrolled exterior copy of F_j survives the reversal and produces precisely the leakage factor already analyzed. Enlarging the internal memory does not alter this conclusion. If a reversible computation maps the two input branches to different extended workspaces, the full workspace must be included in the inverse. Reversible computation supplies a mathematical language for that accounting; it does not grant control of an uncontrolled environment [66].

This four-qubit construction verifies the physical implementation of the logical observables. The two record qubits are minimal memory devices. Calling them conscious citizens or treating their recording events as observations in a strengthened metaphysical theorem would require additional assumptions. The role of the example is to expose the control requirements of any proposed scaled implementation.

10.4 Additional noise and a numerical illustration

To display a second independent limitation, let

$$\rho(v,\eta) = \eta\rho(v) + (1-\eta)I_4/4, \qquad \eta \in [0,1]. \tag{42}$$

This is a specified white-noise admixture, not a claim that all laboratory noise has this form. Here it represents an additional channel on the reduced laboratories after the leakage stage. Record-error bounds calibrated before that channel refer to that stage; if a noise process also changes branch labels or public records, their calibration has to be included in the new model. Traceless local projective correlators scale by η, so

$$S_{\max}(v,\eta) = 2\eta\sqrt{1+v^2}, \qquad S_{\mathrm{read}}(v,\eta) = \eta\left(2 + \frac{v^2}{1+v}\right). \tag{43}$$

The corresponding strict-violation conditions are $\eta > 1/\sqrt{1+v^2}$ and $\eta > 2/(2+v^2/(1+v))$. A very small residual coherent visibility permits a violation in the noiseless model but leaves little tolerance for this additional noise.

Equation (43) optimizes within the declared nondegenerate, traceless measurement class. If deterministic outputs $\pm I_2$ are admitted, the unrestricted maximum is instead $\max\{2, 2\eta\sqrt{1+v^2}\}$. Indeed, a CHSH operator with any one deterministic observable has norm at most two, while constant outputs can attain two. The strict-violation threshold is unchanged, but a value below two in Equation (43) is not a universal upper bound when that larger measurement class is allowed. With both readout settings fixed, any enlargement of the remaining allowed measurements must likewise be included explicitly in the optimization.

Suppose that, in the independent pure-leakage model, each exterior laboratory record identifies its local branch with error at most 0.01. Then $|\gamma_A|, |\gamma_B| \le 2\sqrt{0.01 \times 0.99} \approx 0.1989974874$, and $v \le 0.0396$. The resulting upper bounds are

$$\begin{aligned} S_{\max} &\le 2.0015675457, & S_{\mathrm{read}} &\le 2.0015084263, \\ S_{\mathrm{fixed}} &\le 1.4702164194. \end{aligned} \tag{44}$$

These are calculated model bounds. They are not operating thresholds measured in an artificial society. Under the white-noise model the unrestricted violation additionally needs $\eta > 0.9992168410$

at the maximum permitted v, and the readout-constrained violation needs $\eta > 0.9992463552$. Smaller actual visibility tightens those requirements further.

For uniformly sampled CHSH settings, a game with outputs encoded as bits and winning condition $a \oplus b = xy$ has success probability $p_{\text{win}} = 1/2 + S/8$. Its local bound is $3/4$; the gain is $(S-2)/8$. The preceding one-percent example therefore bounds this gain by approximately 0.0001959432 in the unrestricted case and 0.0001885533 in the readout-constrained case. Turning such a game into a civilizational decision task requires an explicit utility model. The numerical value is not automatically a measure of wisdom, consciousness, social welfare, or agency.

11 Finite implementation error and approximate reversal

Exact inversion is a mathematical idealization. A scaled experiment has to budget deviations in state preparation, coherent gates, measurement settings, and the assumed boundary of the environment. These errors are distinct from the modeled leakage parameter v. The following bounds quantify how a specified implementation error affects the correlation claims without identifying every possible error with phase damping.

For an operator W, let $\|W\|_\infty$ be its largest singular value. For Hermitian W it is the largest absolute eigenvalue. A measured witness has expectation $\operatorname{Tr}(W\rho)$. The trace-distance convention remains that of Equation (12).

Lemma 11.1 (Stability of a bounded witness)**.** *If density operators ρ, σ satisfy $D(\rho,\sigma) \le \delta$ for $\delta \in [0,1]$, then every Hermitian witness W satisfies*

$$|\operatorname{Tr}(W\rho) - \operatorname{Tr}(W\sigma)| \le 2\delta\|W\|_\infty. \tag{45}$$

In particular, for a CHSH operator built from projective observables of norm one, the right-hand side is at most $4\sqrt{2}\,\delta$.

Proof. Trace/operator-norm duality bounds the left-hand side by $\|W\|_\infty\|\rho-\sigma\|_1$. This proves the general statement. For $W = A_0\otimes(B_0+B_1)+A_1\otimes(B_0-B_1)$ and $A_x^2 = B_y^2 = I$, direct multiplication gives $W^2 = 4I-[A_0,A_1]\otimes[B_0,B_1]$, where $[C,D] = CD-DC$ is the commutator. Each commutator has operator norm at most two. Consequently $\|W^2\|_\infty \le 8$ and $\|W\|_\infty \le 2\sqrt{2}$. □

The bound is deliberately independent of a detailed noise mechanism. It is a worst-case statement about two states on the same specified Hilbert space and the same witness. It does not estimate the value of δ from the observed CHSH score; that estimate requires an independent characterization procedure or a separately justified device model. A conclusion that assumes a small trace distance cannot then use its own desired witness value as evidence that the assumption was satisfied.

Measurement calibration requires its own term. Suppose ideal projective observables A_x, B_y are replaced by Hermitian contractions $\widetilde{A}_x, \widetilde{B}_y$, and write $a_x = \|\widetilde{A}_x - A_x\|_\infty$, $b_y = \|\widetilde{B}_y - B_y\|_\infty$. A Hermitian contraction has operator norm at most one and includes the effective observable of any binary POVM. For each correlator, the difference of product observables has norm at most

$a_x + b_y$, by adding and subtracting $A_x \otimes \widetilde{B}_y$. Thus the total CHSH expectation error due to the measurement replacements is at most

$$2(a_0 + a_1 + b_0 + b_1). \tag{46}$$

Together with Lemma 11.1, an actual state satisfying $D(\tilde{\rho}, \rho) \leq \delta$ obeys

$$|\widetilde{S} - S| \leq 4\sqrt{2}\,\delta + 2(a_0 + a_1 + b_0 + b_1). \tag{47}$$

This follows by first changing the state while holding ideal observables fixed, then changing the observables on the actual state. The calibration errors lie in $[0, 2]$; the bound can be loose when they are large, but its validity does not require a perturbative expansion.

11.1 An error budget for a sequence of coherent operations

To account for an imperfect reversal, let $\mathcal{U}_1, \ldots, \mathcal{U}_n$ be ideal channels and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ their implemented replacements, with compatible finite input and output spaces and positive integer n. The diamond norm of a linear map Ξ is

$$\|\Xi\|_\diamond = \sup_{k \geq 1} \sup_{\|X\|_1 = 1} \|(\Xi \otimes \mathrm{id}_k)(X)\|_1, \tag{48}$$

where id_k is the identity map on a k-dimensional ancillary space and X acts on the joint input and ancilla. Define the channel error $d_j = \frac{1}{2}\|\mathcal{E}_j - \mathcal{U}_j\|_\diamond \in [0, 1]$. This norm permits the operation to act on a subsystem entangled with the rest of the experiment; an error metric tested only on unentangled inputs would not automatically suffice.

Proposition 11.2 (Composition bound for approximate coherent control)**.** *For every joint input state, including an arbitrary ancillary system, the trace distance between outputs of the actual and ideal channel sequences is at most*

$$\delta_{\mathrm{seq}} \leq \min\left\{1, \sum_{j=1}^{n} d_j\right\}. \tag{49}$$

If the input states themselves differ by trace distance at most d_0, replace the sum by $d_0 + \sum_j d_j$.

Proof. Expand the difference between the two channel compositions as a telescoping sum. In term j, operations before j are ideal, operation j is $\mathcal{E}_j - \mathcal{U}_j$, and operations after j are actual. Complete trace-norm contractivity of the surrounding channels bounds the half trace norm of this term by d_j. The triangle inequality gives the sum. The trace distance of normalized states never exceeds one. A separate initial-state difference contributes at most d_0 because the subsequent channels cannot increase it. □

This estimate provides a sufficient error budget rather than a prediction of hardware performance. The channel sequence must describe a valid common state space. When an environment retains memory between operations, that memory must be included in the modeled state, or the claimed channel-error bounds must hold uniformly for the relevant conditional histories. Marginal channels characterized in separate reset experiments cannot automatically be composed to describe a

process with retained environmental memory. Once the stated channel model and bounds hold, the estimate makes no probabilistic independence assumption about errors and can be conservative. For a chosen ideal witness $S_{\text{ideal}} > 2$, a sufficient condition for preserving its violation, before statistical uncertainty is included, is

$$4\sqrt{2}\left(d_0 + \sum_j d_j\right) + 2(a_0 + a_1 + b_0 + b_1) < S_{\text{ideal}} - 2. \tag{50}$$

Failure of this sufficient condition does not prove that the implemented experiment lacks a violation; it means that these worst-case error estimates cannot guarantee one. Statistical confidence, setting independence, and control of selection effects remain additional requirements.

The distinction becomes consequential at small v. The ideal readout-constrained gain scales as $v^2/(1+v)$. A coherent-control error that is negligible for an ordinary classical computation can consume that entire correlation margin. The theoretical possibility of reversible cognition and the engineering ability to preserve a particular witness are therefore separate propositions. The model permits each to be investigated without using the other as evidence.

12 What the mathematical model establishes

The central physical conclusion is a resource constraint. Publicly retrievable which-branch records and coherent access to the relative phase of those same branches draw on incompatible properties of the specified state and control partition. A civilization can choose what to preserve in its public record, what to keep within a reversible laboratory, and which tasks justify the costs of maintaining that boundary. Those choices have quantitative consequences within the model.

The conclusion is narrower than a claim that collective knowledge is impossible, and more informative than the observation that two witnesses can disagree. Classical shared records remain available for commuting observables. Quantum correlations can obstruct a proposed common representation under additional assumptions, but each actual experimental context still has a normalized outcome distribution. A mathematical treatment must keep both facts visible. It cannot import a witness value into a story about arbitrary contradictory propositions.

The model also identifies a constructive architectural possibility. A public classical layer can retain protocol descriptions, preparation statistics, calibration procedures, and final context-tagged outputs. A coherently controlled layer can perform the particular reversible operation being tested. Information need only be withheld from the public layer when it would carry the very branch distinction whose coherence is required. This proposal is a design consequence of the model, not evidence that a corresponding society has been built.

For a stronger observer interpretation, the hard scaling question is whether an agent's relevant cognition can remain inside that controlled layer. Its input processing, work memory, apparent reporting behavior, timing, and actuation have to be represented physically. If the agent recruits a durable external audience during the coherent interval, the audience becomes part of the control problem. Describing the audience as an institution does not remove it from the tensor product.

12.1 Computational checks and their scope

The accompanying audit uses explicit complex matrices and partial traces, with fixed random seed 20261115. It checks 512 pure-marker pairs, 512 pairs with entangled accessible and inaccessible subregisters, 256 mixed balanced models, 84 conditional-product examples, the correlated and nonlocal-record counterexamples for two through six fragments, 128 random phase-independent Stinespring channels, 101 values of v for each witness optimization, and 51 four-qubit encoded circuits. The greatest reported equality residual is below 4.0×10^{-15} in the stated implementation; the acceptance tolerance is 3×10^{-10}.

The matrix checks compare independently constructed states and observables with the analytic formulas. They include the endpoints $v = 0$ and $v = 1$, the different constrained and unconstrained optima, and explicit failures of an overgeneralized redundancy law. They are computational verification of finite examples and algebraic identities, not a substitute for the displayed proofs. They establish no experimental noise rate, no technological feasibility of coherent cognition, and no empirical consciousness result.

13 From a quantum circuit to a Local Friendliness test

The preceding matrix calculations determine quantum probabilities. A no-go interpretation requires a second object: an explicit class of classical accounts against which those probabilities are compared. This section supplies that object for a minimal two-friend, two-setting scenario. The construction is a specialization of the Local Friendliness framework, and its equivalence with the Bell set in this case is established prior art [4, 20]. The proof is included to expose every inference on which the proposed application depends.

13.1 Events, settings, and the conditional model

There are two laboratories, A and B. Each contains a friend register with a binary recording operation. The external controllers choose settings $x, y \in \{0, 1\}$, and obtain reported outcomes $a, b \in \{-1, +1\}$. Setting 0 reads the corresponding friend's record. Setting 1 performs the declared alternative coherent operation and measurement. The experimental probability table is

$$p(a, b \mid x, y) \geq 0, \qquad \sum_{a,b} p(a, b \mid x, y) = 1. \tag{51}$$

The labels A and B identify actual control regions. A locality interpretation requires the relevant choices and operations to satisfy the causal separation specified by the experiment. A simulation on one classical computer establishes neither that separation nor empirical freedom of choice.

Definition 13.1 (Binary two-setting Local Friendliness representation)**.** A representation of p consists of a distribution $\pi(c, d)$ on $\{-1, +1\}^2$ and normalized conditional tables $q_{cd}(a, b \mid x, y)$ such that

$$p(a, b \mid x, y) = \sum_{c,d} \pi(c, d) q_{cd}(a, b \mid x, y), \tag{52}$$

$$\sum_b q_{cd}(a,b \mid x,y) = q^A_{cd}(a \mid x), \tag{53}$$

$$\sum_a q_{cd}(a,b \mid x,y) = q^B_{cd}(b \mid y), \tag{54}$$

$$q^A_{cd}(a \mid 0) = \mathbf{1}\{a = c\}, \qquad q^B_{cd}(b \mid 0) = \mathbf{1}\{b = d\}. \tag{55}$$

The distribution π does not depend on x, y. Conditions are needed only for (c, d) of positive probability; zero-probability pairs may be assigned arbitrary consistent tables. Here c, d are the friend's events in the proposed absolute-event account, not extra data silently read out on every experimental trial.

Equations (53)–(54) express nonsignaling conditional on the proposed friend events. They do not assume a factorization of the form $q^A q^B$. The distinction is essential in more general Local Friendliness scenarios. In this limited setting, the readout conditions nevertheless make a global extension possible, as the next theorem shows. The causal and event-absoluteness assumptions that motivate the representation are examined in the literature discussion; accepting a useful instrument model alone does not establish every premise of that interpretation.

Theorem 13.2 (Two-setting extension with designated record readout)**.** *The representation in Definition 13.1 exists if and only if there is a single probability distribution r on four variables*

$$(A_0, A_1, B_0, B_1) \in \{-1, +1\}^4$$

whose (A_x, B_y) marginal equals $p(\cdot, \cdot \mid x, y)$ for each $x, y \in \{0, 1\}$.

Proof. Given a representation, define

$$r(c, u, d, w) = \pi(c, d) q_{cd}(u, w \mid 1, 1), \tag{56}$$

interpreting its four coordinates as $A_0 = c, A_1 = u, B_0 = d, B_1 = w$. Nonnegativity and normalization are immediate. Its (A_1, B_1) marginal is $p(\cdot, \cdot \mid 1, 1)$ by construction. Its (A_0, B_0) marginal is $\pi(c, d)$, which equals the observed readout table by Equation (55).

For the (A_0, B_1) marginal, sum Equation (56) over u, d. Conditional nonsignaling gives

$$\sum_u q_{cd}(u, w \mid 1, 1) = q^B_{cd}(w \mid 1).$$

At settings $(0, 1)$ the A outcome is deterministically c, so the same expression is the required marginal of $q_{cd}(a, b \mid 0, 1)$. Averaging over π proves agreement. The (A_1, B_0) case follows by the corresponding A marginal. These are all four contexts.

Conversely, let r be a global distribution. Set $\pi(c, d) = \Pr_r(A_0 = c, B_0 = d)$. On each positive-probability pair define $q_{cd}(a, b \mid x, y)$ as the conditional distribution of (A_x, B_y) given $(A_0, B_0) = (c, d)$. Its single-side marginals depend only on the local setting because all variables are already defined on the same probability space. Readout is deterministic. Mixture over π recovers the observed table. Arbitrary tables complete the zero-probability cases. □

Corollary 13.3 (The relevant CHSH bound)**.** *Define the four correlators and their signed sum by*

$$E_{xy} = \sum_{a,b} ab\, p(a, b \mid x, y), \qquad S = E_{00} + E_{01} + E_{10} - E_{11}. \tag{57}$$

Any representation in Definition 13.1 satisfies $S \leq 2$.

Proof. For each assignment in the support of r, the quantity

$$A_0(B_0 + B_1) + A_1(B_0 - B_1)$$

is either $+2$ or -2: exactly one parenthesis is zero and the other is ± 2. Its expectation is S, establishing the bound. □

This corollary has a precise scope. In the designated two-setting scenario a CHSH violation contradicts the stated Local Friendliness representation. For larger families of settings, one must use the correct Local Friendliness polytope; a generic Bell inequality cannot simply be renamed. The known differences between those sets are part of the foundation of the paper [20].

13.2 A sharp bound when a designated readout can be wrong

Noise can weaken the readout condition itself. To make that issue explicit, retain the setting-independent distribution $\pi(c, d)$ and the conditional nonsignaling tables in Definition 13.1, but replace deterministic readout by aggregate error budgets

$$\begin{aligned} \sum_{c,d} \pi(c,d) \Pr_{q_{cd}}(a \neq c \mid x = 0) &\leq \delta_A, \\ \sum_{c,d} \pi(c,d) \Pr_{q_{cd}}(b \neq d \mid y = 0) &\leq \delta_B. \end{aligned} \tag{58}$$

The remote setting does not appear because the corresponding conditional marginal is nonsignaling. These δ_j measure tracking error relative to the friend's event in the null model. They differ from the ϵ_j used to characterize an exterior marker's ability to discriminate a prepared branch.

Theorem 13.4 (Approximate-readout Local Friendliness bound)**.** *For $\delta_A, \delta_B \in [0, 1]$, the relaxed representation above satisfies*

$$S \leq \min\{4,\ 2 + 4\min(\delta_A, \delta_B)\}. \tag{59}$$

For error budgets in $[0, 1/2]^2$, the bound is sharp within this conditional-nonsignaling class. It is an elementary consequence of the specified finite representation, not a general replacement for arbitrary Local Friendliness inequalities.

Proof. Fix (c, d) of positive probability. Replace the $y = 0$ tables by

$$q'_{cd}(a, b \mid x, 0) = q^A_{cd}(a \mid x)\mathbf{1}\{b = d\},$$

and leave the $y = 1$ tables unchanged. This replacement preserves nonsignaling: Alice's marginals are unchanged and Bob's $y = 0$ marginal is the same deterministic distribution for both x. A two-setting nonsignaling table with deterministic $B_0 = d$ admits a global extension. To construct it, first sample B_1 from its common marginal, then sample A_0, A_1 conditionally on B_1 using the two conditional distributions from the $y = 1$ contexts; any coupling, including conditional independence, suffices. Set $B_0 = d$. Zero-probability conditioning values can be assigned arbitrarily. Nonsignaling reproduces both $y = 0$ contexts.

Consequently $S(q'_{cd}) \le 2$. In each of the two modified contexts the total variation distance from q_{cd} is exactly $e_B(c,d) := \Pr_{q_{cd}}(b \neq d \mid y = 0)$. Each signed correlator changes by at most $2e_B(c,d)$. Thus $S(q_{cd}) \le 2 + 4e_B(c,d)$. Averaging over π yields $S \le 2 + 4\delta_B$. Repairing Alice's readout instead gives $S \le 2 + 4\delta_A$. The algebraic bound $S \le 4$ completes the upper bound.

For sharpness assume $0 \le \delta_A \le \delta_B \le 1/2$ and set $c = d = +1$. Mix a nonsignaling PR table, whose products satisfy $ab = (-1)^{xy}$ and whose single-party marginals are uniform, with weight $2\delta_A$; the deterministic assignment $(A_0, A_1, B_0, B_1) = (+1, -1, -1, +1)$ with weight $\delta_B - \delta_A$; and the all-positive assignment with weight $1 - \delta_A - \delta_B$. The weights are nonnegative and sum to one. The respective CHSH values are $4, 2, 2$, the resulting readout errors are exactly δ_A, δ_B, and $S = 2 + 4\delta_A$. Swap the parties for the other ordering. The PR component is an admissible nonsignaling extremal table in this enlarged null class; its use for sharpness is not a claim of quantum realizability. □

The result shows why readout calibration belongs in the null hypothesis. A score just above two is not enough to reject the relaxed class when the admitted tracking errors permit a larger bound. The error budget must be justified for the relevant conditional experiment or by an independently defensible instrument model. Measuring an intermediate friend value in every coherent trial may itself change that experiment, so calibration cannot be treated as cost-free access to inaccessible correlations.

13.3 The record-readout constraint changes the optimum

The logical Bell family studied above has surviving coherence $v \in [0,1]$. Let Z and X denote its logical Pauli observables. The readout-compatible settings are

$$\begin{aligned} A_0 &= Z, & B_0 &= Z, \\ A_1 &= cZ + \sqrt{1-c^2}X, & B_1 &= cZ - \sqrt{1-c^2}X, \qquad -1 \le c \le 1. \end{aligned} \tag{60}$$

The sign difference in the transverse terms is intentional. Direct calculation gives $E_{00} = 1$, $E_{01} = E_{10} = c$, and $E_{11} = c^2 - v(1-c^2)$. Therefore

$$S = 1 + v + 2c - (1+v)c^2. \tag{61}$$

The optimum occurs at $c = (1+v)^{-1}$, yielding the globally optimal value for the stated fixed-readout projective model,

$$S_{\mathrm{read}}(v) = 2 + \frac{v^2}{1+v}. \tag{62}$$

The quantum-model proof establishes the optimum even when the alternative projective directions are allowed to vary independently. In particular, $S_{\mathrm{read}}(1) = 5/2$. The familiar unrestricted two-qubit value $2\sqrt{1+v^2}$ refers to a different optimization problem, because its settings need not designate both friends' Z records as readouts.

The measurement angles depend on v. A proposed experiment must estimate the necessary calibration using independent data or choose and freeze a conservative setting in advance. Choosing angles to maximize a witness on the same data used for its significance test changes the inference problem. The numerical audit evaluates the declared observables directly; it does not optimize over measured noise to report an experimental discovery.

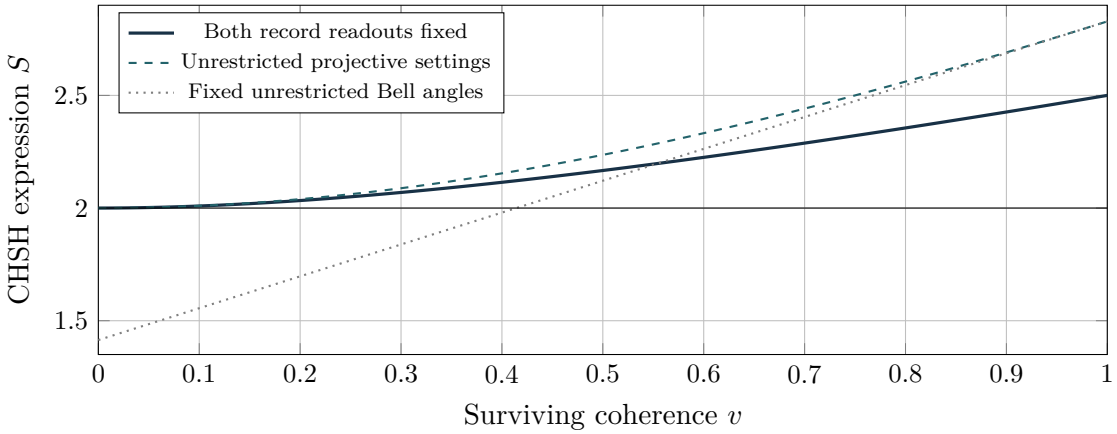


Figure 3: Three different optimization questions for the same phase-damped Bell family. The solid curve respects both designated record readouts. The dashed curve permits unrestricted settings. The dotted curve keeps the usual ideal Bell angles fixed while coherence changes. These are computed theoretical curves, not experimental data.

13.4 The operational history of one trial

The four-qubit construction makes the friend memory explicit. Each logical laboratory consists of a system qubit and a memory qubit. A controlled-NOT writes the computational-basis value into the blank memory. Reading the memory realizes the designated Z setting. Undoing that controlled-NOT and measuring an appropriate tilted observable on the system realizes the alternative logical setting. The internal memory returns to its blank state during the reversal branch.

A permanent external copy of the friend's outcome must be represented as an additional system. If it is outside the reversal, its distinguishability reduces the phase coherence available to the controller. If it is inside the reversal, it is no longer an independently preserved public copy after that operation. This is the operational reason to distinguish the public record of the experimental *procedure* from a public copy of its internal *branch outcome*.

An external log can permanently record that a specified unitary was scheduled, a controller reported completion, and a later detector output was received. Such a log is useful procedural evidence. It does not by itself contain the erased friend's outcome or demonstrate the fidelity of the implemented channel. Process characterization and separately specified controls are needed for that purpose.

13.5 What a violation would establish

For the minimal reference implementation, a statistically valid violation would contradict the representation in Definition 13.1, conditional on the actual experimental choices and the assumptions supporting the null class. It would also verify a nonclassical feature of the implemented recording-and-control circuit. The strength of its observer interpretation depends on the capacities attributed to the friend and the justification for doing so.

The strongest long-term extension would replace a primitive memory with an artificial system that can retain information, use it in a defined policy, represent its own predictions, and issue reports in the relevant branches. Those capabilities must be included in the controlled dynamics. The

recent literature already develops rudimentary agent-like implementations, so the research program begins from those constructions rather than claiming the concept of an artificial quantum friend as new [5, 9, 10].

14 A joint frontier for disclosure, coherence, and evidence

The results can now be assembled without confusing their domains. The complementarity theorem constrains information in a physical state. The Local Friendliness representation constrains a class of accounts of the observed statistics. The decision theorems operate on the resulting context-specific probability tables. This section quantifies one setting in which the constraints meet.

Theorem 14.1 (Disclosure bound for the specified two-laboratory model)**.** *Assume the balanced two-branch preparation and conditionally independent pure exterior markers of Equation (31). Suppose a reader restricted to the individual marker E_j, for laboratory $j \in \{A, B\}$, can identify that laboratory's branch with equal-prior error at most $\epsilon_j \in [0, 1/2]$. The exterior markers are excluded from the controller's measurements and feedforward during the coherent comparison. Set*

$$b_j = 2\sqrt{\epsilon_j(1-\epsilon_j)}, \qquad b = b_A b_B. \tag{63}$$

For nondegenerate projective alternative measurements with the designated record readouts fixed,

$$S \le 2 + \frac{b^2}{1+b}. \tag{64}$$

This upper bound is attainable within the stated ideal model. If $\epsilon_A = \epsilon_B = \epsilon$, then $b = 4\epsilon(1-\epsilon)$.

Proof. The record-error inequality bounds each marker overlap by b_j. Conditional product structure therefore gives $v \le b_A b_B = b$. Proposition 10.2 bounds the witness by $2 + v^2/(1+v)$. The derivative of $v^2/(1+v)$ is $v(2+v)/(1+v)^2 \ge 0$, so substitution of b proves the bound.

For attainability, choose normalized pure marker pairs with real overlap exactly b_j. For example, take $|e_0^{(j)}\rangle = |0\rangle$ and $|e_1^{(j)}\rangle = b_j|0\rangle + \sqrt{1-b_j^2}|1\rangle$. Their optimum equal-prior error is exactly ϵ_j, since $\sqrt{1-b_j^2} = 1 - 2\epsilon_j$ on the declared interval. Use their tensor product and the attaining measurement settings of Proposition 10.2. All inequalities then become equalities. □

The theorem is conditional on the actual marker model and control boundary. Additional information sent from the exterior to the controller changes the accessible system and may change the allowed measurements. Correlated exterior records require the general fidelity statement rather than the product formula. An upper bound on the witness is not an estimate of the visibility of an uncharacterized device.

14.1 Three endpoints and their meaning

At $\epsilon = 0$, each exterior marker is a perfectly readable branch record. The model has $b = 0$ and $S \le 2$. At $\epsilon = 1/2$, the stated record-error condition imposes no informative constraint, so $b = 1$

and the ideal readout-compatible maximum is 5/2. Between them, the bound is continuous and saturable by the explicit marker family.

Near $\epsilon = 0$, $b = 4\epsilon + O(\epsilon^2)$ and

$$S_{\mathrm{read}} - 2 \le 16\epsilon^2 + O(\epsilon^3). \tag{65}$$

Here $O(\epsilon^k)$ has its standard asymptotic meaning: the omitted term divided by ϵ^k remains bounded as $\epsilon \downarrow 0$. The scaling comes from this balanced independent-marker model. It is not a universal law assigning the same cost to every kind of public information.

With one perfect exterior record, $b_j = 0$ already forces $v = 0$ regardless of the other side's marker. The relevant issue is branch distinguishability, rather than the number of readers or the popularity of a report. With correlated exterior records, multiple apparent copies may provide no multiplicative reduction beyond the single-record bound. Both cases are covered by the preceding counterexamples.

14.2 An independent white-noise stage

If the modeled laboratory state is subsequently mixed with $I_4/4$ with mixing parameter $\eta \in [0, 1]$, then the same traceless-observable witness is at most

$$S \le \eta \left(2 + \frac{b^2}{1+b}\right). \tag{66}$$

The record errors in Theorem 14.1 refer to the branch preparation before this additional channel. The channel can change the relationship between an earlier exterior marker and the final laboratory readout, so an experiment claiming a final-record error must characterize that quantity separately.

For a saturating preparation, the threshold for the correlation expression to exceed two is

$$\eta > \eta_{\mathrm{crit}}(b) := \frac{2}{2 + b^2/(1+b)}. \tag{67}$$

This formula isolates a second resource requirement, but its interpretation depends on when the noisy channel acts. If the physical realization preserves exact tracking of the designated friend events, exceeding two tests the exact-readout null. If noise acts after those events and corrupts the readout, Theorem 13.4 supplies the relevant relaxed null. A final reduced density matrix alone cannot identify which history produced it.

For example, consider a specifically calibrated channel that, after the original events have been recorded, retains the entire logical state with probability η and replaces it by $I_4/4$ with probability $1 - \eta$, independently of the original branch. Its readout mismatch relative to each original friend event is $(1 - \eta)/2$. The relaxed null is then $S \le 4 - 2\eta$, so a saturating quantum score rejects it only if

$$\eta > \eta_{\mathrm{LF,post}}(b) := \frac{4}{4 + b^2/(1+b)}. \tag{68}$$

The equality of the mismatch to $(1 - \eta)/2$ is a property of this specified replacement mechanism and its calibration; it cannot be inferred from $\rho(v, \eta)$ alone. This threshold concerns the broad null constrained by the readout-error budgets. Trusting additional details of the replacement mechanism could impose further null constraints and support a different test. At $b = 1$, the stated

threshold is 8/9, whereas the correlation-only threshold in Equation (67) is 4/5. At $b = 0.0396$ the respective thresholds are approximately 0.999623036 and 0.999246355. Both calculations are correct for their distinct premises.

Retaining a small amount of coherence can therefore keep a violation mathematically possible while leaving little margin for other errors. A general gate or measurement error is not interchangeable with $1 - \eta$; the operator and channel bounds in Section 11 address a more general budget. The source and timing of an error determine which physical claim the resulting witness can support.

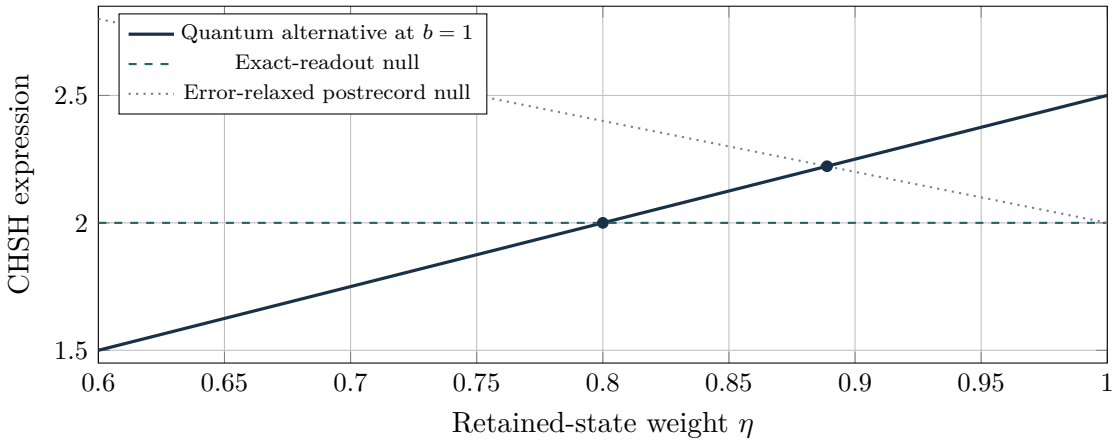


Figure 4: The same correlation curve can face different justified nulls. For $b = 1$, the ideal readout-compatible score is 5/2. Its noisy correlation exceeds two at $\eta > 4/5$. Under the stated postrecord replacement and broad error-relaxed null, rejection by this witness needs $\eta > 8/9$. The two marked crossings test different premises.

14.3 The cost in evidence

For uniformly sampled settings, let $Y_t \in \{0, 1\}$ be the declared CHSH-game success score. Under the two-setting null of Section 13, its conditional expectation is at most 3/4. A saturating ideal preparation in Theorem 14.1 has success gain

$$g(b) = \mathbb{E}[Y_t] - \frac{3}{4} = \frac{b^2}{8(1+b)}. \tag{69}$$

For a significance level $\alpha \in (0, 1)$ and a target type-II error bound $\beta_{\mathrm{II}} \in (0, 1)$, Corollary E.2 gives the sufficient fixed-horizon trial count

$$n \geq \frac{\left(\sqrt{\log(1/\alpha)} + \sqrt{\log(1/\beta_{\mathrm{II}})}\right)^2}{2g(b)^2}, \qquad g(b) > 0. \tag{70}$$

The alternative must satisfy the stated conditional gain on every included trial. The count is a conservative sufficient bound for that analysis, not an information-theoretic minimum and not an experimentally measured requirement. The logarithms in this concentration formula are natural logarithms.

For approximate readout, the conditional null mean is instead at most $3/4 + \frac{1}{2}\min(\delta_{A,t}, \delta_{B,t})$. In the specified postrecord replacement example it is $1 - \eta/4$, and the gain over that null becomes

$$g_{\mathrm{LF}}(\eta, b) = \frac{\eta\,(S_{\mathrm{read}}(b) + 2) - 4}{8}. \tag{71}$$

The conditional readout-error budgets and null representation must hold given the pretrial history for the martingale test to apply. An overall average calibration error alone does not establish that premise. A different analysis could incorporate a proved cumulative error budget, but that would be an additional statistical result. The table below uses exact readout and $\eta = 1$, so its stated null and counts are unchanged.

Table 4 uses saturating markers, ideal gates, $\eta = 1$, $\alpha = 0.01$, and power at least 0.80 under the conditional alternative. It shows why a seemingly modest residual violation can be expensive to establish. The extra-noise threshold is displayed separately; no additional white noise is included in the trial counts.

Record error ϵ	b	S_{read}	η_{crit}	Sufficient n
0	0	2.000000	1.000000	No positive gain
0.01	0.0396	2.001508	0.999246	163,976,656
0.05	0.1900	2.030336	0.985059	405,425
0.10	0.3600	2.095294	0.954520	41,087
0.25	0.7500	2.321429	0.861538	3,612
0.50	1.0000	2.500000	0.800000	1,493

Table 4: Computed disclosure and evidence budgets for the specified ideal family. Error is the optimal equal-prior branch-discrimination error for each exterior marker. Lower error means a more informative exterior record. The final row imposes no informative record condition.

The first finite row is especially instructive. Two exterior records with one-percent discrimination error allow a maximal readout-compatible score only about 0.001508 above two. The stated sufficient bound certifies the chosen significance and power at approximately 164 million ideal trials. A more efficient test can have a smaller requirement. The calculation establishes that a proposal cannot infer practical ease from the mere positivity of a theoretical violation.

As $\epsilon \downarrow 0$, Equation (69) gives $g = 2\epsilon^2 + O(\epsilon^3)$, and the sufficient count in Equation (70) scales as ϵ^{-4}. This is an analysis-dependent planning scaling for a saturating family. Improved statistics, a different witness, or a different physical scenario may alter it. Its value is to expose one concrete interaction between a public-record demand and a finite evidence budget.

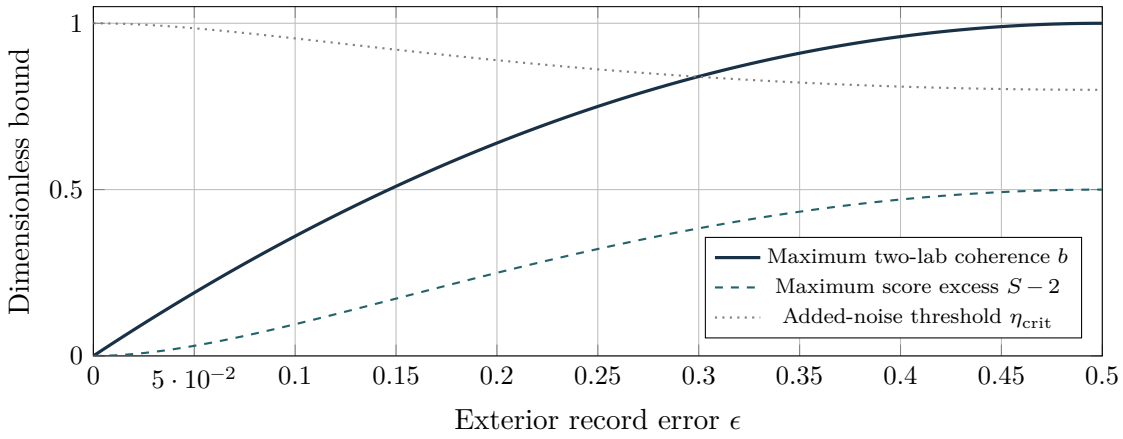


Figure 5: Theoretical frontier for independent balanced exterior markers. At the left, accurate permanent branch records suppress the coherent witness. At the right, the record-error condition carries little branch information. The noise parameter is defined by the additional mixture in the text.

15 Public memory can enable the coordination advantage

A civilization's public memory can be part of its quantum resource. The construction below gives that statement an exact operational meaning. A shared preparation record enables separated participants to exceed a classical coordination limit. Removing that record before it is used leaves the participants with a separable state and removes the advantage. Exporting a different record—one that distinguishes the intermediate branches required by the coherent task—reduces the attainable advantage. The institution therefore has a substantive choice about what its memory preserves.

The preparation-label mechanism has established prior art. In particular, D'Arrigo and colleagues explicitly describe the equally weighted mixture of the two phase-opposite Bell states: its reduced state is separable, whereas access to the preparation label permits a local phase correction [3]. The $v = 1$ endpoint below is that example. Here it is combined with a specified game, fixed record readouts, persistent copied labels, and the branch-disclosure bound. The mathematical contribution of this section is the explicit integrated accounting; it does not claim historical priority for classical-information-assisted entanglement.

15.1 Task, preparation, and information available to each player

There are two players, Alice and Bob. A referee gives them questions $x, y \in \{0, 1\}$, sampled independently and uniformly after preparation. They return outcomes $\sigma, \tau \in \{-1, +1\}$ and receive common utility

$$u(x, y, \sigma, \tau) = \mathbf{1}\{\sigma\tau = (-1)^{xy}\} \in \{0, 1\}.$$

The players can agree on a protocol and share randomness in advance. They cannot communicate between receiving their questions and returning their answers. The question pair is independent of their prepared state, public flag, and other pretrial resources. Under these rules the task is the

CHSH game [1, 12]. Write $p(\sigma, \tau \mid x, y, k)$ for its conditional answer probabilities and $w = \mathbb{E}[u]$ for its dimensionless winning probability.

Each quantum laboratory has Hilbert space $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$, with orthonormal basis $|0\rangle, |1\rangle$. Let $v \in [0, 1]$ be a calibrated real coherence parameter and use the state $\rho(v)$ of Equation (32). A source generates a uniform classical bit $K \in \{0, 1\}$ and prepares

$$\rho_k(v) = (Z_A^k \otimes I_B)\rho(v)(Z_A^k \otimes I_B), \qquad k \in \{0, 1\}. \tag{72}$$

Thus k specifies the sign of the known preparation phase. It is independent of the later questions and is uniformly distributed even conditional on any branch readout in the Z basis.

For an integer $n \geq 2$, provide n participants with faithful copies of that same classical bit. Alice holds the first copy and Bob the second; additional copies may belong to an archive. The resulting state is

$$\Omega(v) = \frac{1}{2}\sum_{k=0}^{1}(|k\rangle\langle k|)^{\otimes n} \otimes \rho_k(v) \quad \text{on} \quad \mathcal{H}_{\text{pub}} \otimes \mathcal{H}_A \otimes \mathcal{H}_B, \qquad \mathcal{H}_{\text{pub}} = (\mathbb{C}^2)^{\otimes n}. \tag{73}$$

This is a physical classical–quantum correlation, not a choice of ensemble decomposition made after inspecting outcomes. The source, copies, and conditional preparation are specified parts of the protocol.

The comparison uses two information regimes. In the *available-record* regime the players receive K before play. In the *unavailable-record* regime every copy and correlated preparation side channel is inaccessible before any flag-dependent correction. The players then receive only the averaged state. Neither regime supplies any additional shared entangled resource; local ancillas and shared classical randomness may be included. The comparison does not allow input-dependent selection of accepted trials.

15.2 An exact benefit from a durable common record

The designated observation readouts are $A_0 = B_0 = Z$. The alternative measurements A_1, B_1 are nondegenerate projective binary observables on the logical qubits. They may depend on the available public bit. This measurement restriction matches the record-compatible optimization in Proposition 10.2.

Theorem 15.1 (Public phase information enables a coordination advantage)**.** *In the available-record regime of Equation* (73)*, the optimal winning probability within the stated readout-constrained measurement class is*

$$w_{\text{pub}}(v) = \frac{3}{4} + \frac{v^2}{8(1+v)}. \tag{74}$$

The optimum is attained while preserving every public copy of K and both designated Z readouts. In the unavailable-record regime the optimum is $3/4$, even if the players may use arbitrary local measurements and local processing of their separable resource. A classical local strategy with access to K and arbitrary shared randomness also has winning probability at most $3/4$.

Proof. Alice applies the controlled phase correction

$$U_{\text{corr}} = \sum_{k=0}^{1}|k\rangle\langle k|_{K_1} \otimes I_{K_2 \cdots K_n} \otimes Z_A^k \otimes I_B. \tag{75}$$

It is unitary and acts locally on Alice's flag and laboratory. Bob does not apply a second phase correction. Since $Z^{2k} = I$,

$$U_{\mathrm{corr}}\Omega(v)U_{\mathrm{corr}}^{\dagger} = \left[\frac{1}{2}\sum_{k=0}^{1}(|k\rangle\langle k|)^{\otimes n}\right] \otimes \rho(v).$$

For every copy K_i, U_{corr} commutes with its projectors $|k\rangle\langle k|_{K_i} \otimes I_{\mathrm{rest}}$. It also commutes with the laboratory operators $Z_A \otimes I_B$ and $I_A \otimes Z_B$, extended by the identity on the records. The public labels and designated readouts are preserved.

On the corrected state choose $c = 1/(1+v)$, $s = \sqrt{1-c^2}$, $A_1 = cZ + sX$, and $B_1 = cZ - sX$. Proposition 10.2 gives $S = 2 + v^2/(1+v)$, and $w = 1/2 + S/8$ proves attainment. For an upper bound, condition on either k. Conjugation by Z_A^k is a bijection of Alice's allowed alternative observables and fixes $A_0 = Z$. The same proposition therefore bounds each conditional score, including strategies chosen separately for the two values of k. Averaging their bounds proves optimality.

If the flag is unavailable, the state is

$$\overline{\rho} = \frac{1}{2}(\rho_0(v) + \rho_1(v)) = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11| = \rho(0). \tag{76}$$

For arbitrary local POVMs its answer probabilities are a convex mixture of product probabilities, with the classical hidden label $\lambda \in \{0,1\}$ selecting the two displayed product states. Local processing, local ancillas, and shared classical randomness preserve this local representation. For a deterministic local strategy, the four signed answers obey

$$a_0b_0 + a_0b_1 + a_1b_0 - a_1b_1 = a_0(b_0 + b_1) + a_1(b_0 - b_1) \in \{-2, 2\}.$$

Mixtures obey $S \leq 2$ and hence $w \leq 3/4$. Choosing all four observables as Z on $\rho(0)$ attains $3/4$. The same deterministic argument holds conditional on K and every classical shared random seed, proving the final assertion. □

For every $v > 0$, the available public record therefore enables a strict advantage over both comparators. At $v = 1$ its value is $13/16 = 0.8125$, against 0.75. The difference from the unrestricted quantum CHSH value reflects the fixed-readout requirement. No new entanglement has been created by copying or reading a classical bit. The available and unavailable regimes give the players different access to a correlated preparation resource.

This construction is also an instance of the stable-public-algebra criterion in Theorem 22.1. The protected record controls an operation that is useful for the task. Its role is stronger than that of an unrelated coin carried beside an experiment.

15.3 Complete observable predictions

The attaining strategy has zero one-player expectations. Its correlators, conditional on either public flag after correction, are

$$E_{00} = 1, \qquad E_{01} = E_{10} = c, \qquad E_{11} = c^2 - vs^2 = c - v. \tag{77}$$

Every predicted probability is therefore specified by

$$p(\sigma,\tau \mid x,y,k) = \frac{1}{4}\left(1+\sigma\tau E_{xy}\right), \qquad \sigma,\tau \in \{-1,+1\}. \tag{78}$$

Each table is normalized, its entries are nonnegative, and each marginal is uniform. The four conditional winning probabilities are respectively 1, $(1+c)/2$, $(1+c)/2$, and $(1-c+v)/2$. At $v=1$, these are $1, 3/4, 3/4, 3/4$, whose mean is $13/16$.

The persistent public bit and the final answer record can be perfectly definite while these conditional statistics violate CHSH. The obstruction is to a local setting-independent joint assignment of all alternative answers. It does not remove the actual performed question, recorded answer, or agreed payoff.

15.4 A different record imposes a sharp cost

Now generate the coherence through the specified balanced branch state

$$|\Psi_k\rangle = \frac{|00\rangle|e_0^{(A)}\rangle|e_0^{(B)}\rangle + (-1)^k|11\rangle|e_1^{(A)}\rangle|e_1^{(B)}\rangle}{\sqrt{2}}. \tag{79}$$

Each marker vector is a unit vector in its declared finite Hilbert space $\mathcal{H}_{E_j}$, for $j \in \{A,B\}$. The exterior states factor conditional on the branch. Their overlaps are $\gamma_j = \langle e_1^{(j)}|e_0^{(j)}\rangle$. With a known, compensated overlap phase the reduced laboratories have $v = |\gamma_A\gamma_B|$. The marker systems are excluded from the controllers' measurements and feedforward during play.

The public flag is a preparation-phase label. The marker's target is the different variable $\lambda \in \{0,1\}$, identifying branch 00 or 11 with equal prior probability. Conditional on either λ, the public flag remains uniform. Copying the flag consequently does not distinguish these branches, whereas copying λ does.

Theorem 15.2 (Task performance with public phase memory and exterior branch records)**.** *In Equation (79), suppose a reader restricted to marker E_j can identify λ with equal-prior average error at most $\varepsilon_j \in [0,1/2]$. Define*

$$b_j = 2\sqrt{\varepsilon_j(1-\varepsilon_j)}, \qquad b = b_A b_B.$$

Under the preceding information and measurement restrictions, the available-record winning probability satisfies

$$w \le \frac{3}{4} + \frac{b^2}{8(1+b)}. \tag{80}$$

The bound is attained by an explicit preparation and marker family for every pair $(\varepsilon_A,\varepsilon_B)$ in the stated range. The unavailable-record optimum remains $3/4$.

Proof. The record-error inequality gives $|\gamma_j| \le b_j$. Conditional product structure yields $v \le b$. The function $v^2/(1+v)$ is nondecreasing on $[0,1]$, since its derivative is $v(2+v)/(1+v)^2$. Substitution into Theorem 15.1 proves the upper bound.

For attainment take two-dimensional markers and set $|e_0^{(j)}\rangle = |0\rangle$, $|e_1^{(j)}\rangle = b_j|0\rangle + \sqrt{1-b_j^2}|1\rangle$. Their optimal discrimination error is $(1-\sqrt{1-b_j^2})/2 = \varepsilon_j$. They yield $v = b$, and the stated controlled correction and attaining measurements saturate the bound. Averaging over the inaccessible uniform flag cancels the remaining off-diagonal laboratory terms for every marker overlap, so Equation (76) proves the last assertion. □

For equal errors $\varepsilon_A = \varepsilon_B = \varepsilon$, $b = 4\varepsilon(1-\varepsilon)$. Perfect exterior branch records give $b = 0$ and remove this task's advantage. At $\varepsilon = 1/2$ the error assumption supplies no branch information and the attainable upper endpoint is 13/16. One-percent errors give $b = 0.0396$ and the upper bound $w \le 0.7501885533$. This is a model capacity bound. An apparatus with the same record-error bounds may have less coherence and a lower score.

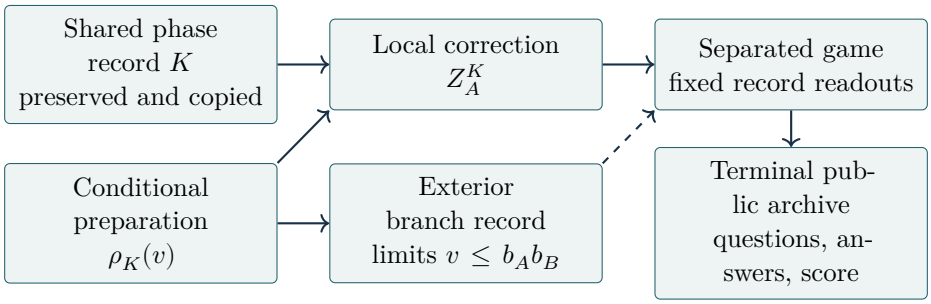


Figure 6: Two physically different public records have different effects. The known phase label enables a useful correction and survives it. An exterior record distinguishing the interfering branches constrains the score through the stated leakage model. The dashed arrow denotes a constraint, not an allowed feedback channel. Final game records are created after the answers have been produced.

15.5 A robust policy needs a lower resource bound

A bound on how much coherence can remain does not certify how much is present. A useful decision rule therefore requires a separate calibrated lower bound. Suppose the corrected state belongs to the declared family $\rho(v)$ with $v \in [v_-, v_+] \subseteq [0,1]$. Choose $c_- = 1/(1+v_-)$, $s_- = \sqrt{1-c_-^2}$ and the alternative observables $A_1 = c_- Z + s_- X$, $B_1 = c_- Z - s_- X$.

Proposition 15.3 (Worst-case optimal settings for an interval of coherences)**.** *The preceding policy has*

$$\inf_{v\in[v_-,v_+]} w(v) = \frac{3}{4} + \frac{v_-^2}{8(1+v_-)}.$$

It maximizes the worst-case winning probability in the stated readout-constrained measurement class.

Proof. Its witness is $1 + 2c_- - c_-^2 + v(1-c_-^2)$, which is nondecreasing in v. At $v = v_-$ it attains the optimum in Proposition 10.2. No policy can have a worst-case value exceeding its best achievable value at this admissible endpoint. □

This is a nonlocal task to which the general decision machinery of Theorem 21.1 can be applied through a justified finite uncertainty set. Here monotonicity supplies the exact interval solution. A positive v_- gives a guaranteed ideal advantage; the marker bound supplies only an upper endpoint $v_+ \leq b$. State-family mismatch, instrument error, and statistical coverage must be included separately. For example, a uniform total-variation error at most δ in every actual answer table changes any fixed policy's winning probability by at most δ, by Theorem 21.2.

15.6 Repeated trials and genuine loss of access

The unavailable-record theorem is a single-use statement. For the same comparison over repeated trials, generate a fresh independent uniform K_t for each trial $t \in \mathbb{N}$ and keep it independent of the previous public history. Conditional on that history, removal of the current flag must leave the stated separable resource. Fresh questions then support the conditional classical bound used in the finite-trial tests.

Reusing one unknown flag is a different resource. For $N \geq 2$ copies with one common hidden K, the ensemble is

$$\frac{1}{2}\rho_0(v)^{\otimes N} + \frac{1}{2}\rho_1(v)^{\otimes N},$$

which generally differs from $\rho(0)^{\otimes N}$. Collective processing or calibration of the repeated preparation can exploit correlations absent from the single-use comparison. Likewise, discarding the flag after the correction leaves $\rho(v)$ available. Neither procedure implements the unavailable-record regime.

Known preparation-phase correction also differs from recovering an unknown historical phase without a side channel. The label K is explicitly provided by the source and is used before the game. Theorem 8.3 concerns operations independent of the unknown encoded phase; the controlled correction here intentionally has access to its declared label. There is no conflict between the two statements.

15.7 What publicity does and does not cost

Three limiting examples prevent an indiscriminate interpretation of Equation (80). First, the public preparation flag helps the task and can have arbitrarily many faithful copies. Second, copying the completed classical game transcript preserves the distribution and payoff of that already completed game. Third, exporting a record of the specified intermediate branch changes the remaining coherence. Its cost follows from the physical marker model, not from whether an institution calls its record public.

The contextual fraction also requires an operational task before it can be read as performance. In the visible-flag example of Theorem 20.1, the conditional isotropic parameters are $\zeta_0 = 0.4$, $\zeta_1 = 0.7$, with probabilities 0.3 and 0.7. Retaining the flag gives contextual fraction 0.28; forgetting it gives 0.22. The same fixed CHSH score nevertheless has success $0.3(0.7) + 0.7(0.85) = 0.805$ in either description, because its payoff is linear. A flag-dependent policy can use the information differently: a classical fallback in the first condition and the stated quantum strategy in the second achieve $0.3(0.75) + 0.7(0.85) = 0.82$. This is an achieved strategy value; a larger optimization requires its allowed states and instruments to be specified.

The phase-flag theorem avoids that inference gap. It states the supplied resource, the information access, the permitted strategy, both optimal values, and the comparator that already has classical shared memory. The stronger scientific thesis is therefore concrete: a shared institution can preserve useful public records, enable a nonlocal coordination advantage, and face a quantitative limit when it demands premature disclosure of the very branch information that supports that advantage.

15.8 Falsification and computational verification

A test would compare available and unavailable preparation records using the same declared source and independent question mechanism. It would report the full conditional tables in Equation (78), the register-copy error, the calibration of v, the location and distinguishability of exterior markers, and all rejected or failed trials under a prespecified rule. Access to an unmodeled preparation label, shared entanglement, or within-trial communication would invalidate the stated comparator.

An observed game value exceeding the conditional leakage bound outside its uncertainty interval would challenge the preparation, marker factorization, access boundary, readout restriction, or calibration assumptions. The equality family predicts an attainable ideal curve; failure of an apparatus to saturate it would measure implementation loss, not contradict the upper-bound theorem. A genuine advantage requires statistical evidence against the appropriate classical null, in addition to correct calculation of an ideal value.

The accompanying program `reality_public_performance_audit.py` constructs the record registers, conditional states, controlled unitary, answer POVMs, and marker dilations directly. It checks positivity, preserved public projectors, reduced states, the complete probability tables, classical strategy values, projective optimization, interval policies, and the visible-flag counterexample. The JSON report gives grid sizes, tolerances, and residuals. These calculations audit the finite model; the displayed proofs establish its general parameter claims.

The executed audit uses 101 coherence values, all 16 deterministic classical strategies and 256 flag-conditional pairs, 121 marker-error pairs, and 2,121 evaluations of interval policies. Five numerical optimizations vary all four alternative-observable Bloch angles, independently of the displayed attaining settings. The largest matrix or linear-program residual is below 4.5×10^{-16}; the largest optimization discrepancy is below 4.2×10^{-13}. An explicit two-copy calculation also checks that reuse of one unknown flag differs from independently forgetting fresh flags, the distinction required by the repeated-trial comparison.

16 Three-party coordination under branch disclosure

The two-laboratory model isolates one coherence-bearing branch pair. A three-party task tests how the same disclosure mechanism acts on an explicitly distributed resource. Consider the GHZ game of Equations (6)–(7). Its classical ceiling is 3/4, while an ideal quantum strategy succeeds with certainty. The following construction gives an exact optimized value and a sharp disclosure capacity bound when exterior records degrade that resource.

16.1 The preparation and the recording instrument

Each participant $j \in \{A, B, C\}$ controls a qubit $Q_j \simeq \mathbb{C}^2$ with basis $\{|0\rangle, |1\rangle\}$. The initial state is

$$|\mathrm{GHZ}_+\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}. \tag{81}$$

For each j, let E_j be a finite-dimensional exterior marker space with unit vectors $|r_j\rangle, |e_0^{(j)}\rangle, |e_1^{(j)}\rangle$. The local recording isometry acts on the prepared subspace as

$$|b\rangle_{Q_j}|r_j\rangle_{E_j} \longmapsto |b\rangle_{Q_j}|e_b^{(j)}\rangle_{E_j}, \qquad b \in \{0, 1\}. \tag{82}$$

The retained computational branch states are orthogonal, so this prescription is an isometry for every pair of normalized marker vectors. There is no assumption that the marker states themselves are orthogonal. The blank markers are independently prepared and the interactions act locally. Consequently the conditional exterior states factor across A, B, C.

Define the complex overlaps $\gamma_j = \langle e_1^{(j)}|e_0^{(j)}\rangle$, with $|\gamma_j| \le 1$. Tracing out all exterior markers multiplies the $|000\rangle\langle 111|$ coefficient by $\gamma_A\gamma_B\gamma_C$. Assume its phase is known and compensated by a diagonal local unitary. Such compensation preserves the computational branch labels. The remaining qubits then have state

$$\begin{aligned} \rho_{\mathrm{GHZ}}(v) &= \frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111| + v|000\rangle\langle 111| + v|111\rangle\langle 000|), \\ v &= |\gamma_A\gamma_B\gamma_C| \in [0, 1]. \end{aligned} \tag{83}$$

Its two possibly nonzero eigenvalues are $(1+v)/2$ and $(1-v)/2$. The state is therefore normalized and positive throughout the declared range. The compensation requires the preparation phase to be known; it does not recover an unknown phase without a side channel.

The task resource is fixed after this preparation and after all interparty preparation communication has ended. No measurement-and-message preprocessing between participants is allowed after that cut, even before the private questions arrive. The marker systems are unavailable to the participants' task measurements and feedforward. Exterior auditors may read them, but their reading outcomes are not additional player inputs during the round. The displayed reduced state is the full shared quantum resource for the optimization. Changing this access contract requires including the newly accessible systems in the resource state.

If a reader restricted to marker E_j identifies the equal-prior branch b with average error at most $\varepsilon_j \in [0, 1/2]$, the pure-marker special case of Theorem 8.2 gives

$$|\gamma_j| \le d_j, \qquad d_j = 2\sqrt{\varepsilon_j(1-\varepsilon_j)}, \qquad v \le d_A d_B d_C =: d_{\mathrm{GHZ}}. \tag{84}$$

The d_j and d_{GHZ} are dimensionless upper bounds on overlaps. They are not observed game scores. Conditional product structure is needed for their product; separate physical locations alone do not imply that factorization. Correlated or entangled exterior markers require their joint conditional states to be analyzed.

16.2 An achieved strategy and an upper bound

Use the Pauli operators on $\mathbb{C}^2$,

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad I = I_2. \tag{85}$$

Every participant measures X for input 0 and Y for input 1, returning the measured eigenvalue. The binary effects are $(I + aX)/2$ or $(I + aY)/2$, with $a \in \{-1, +1\}$; a projective instrument has branch map $\rho \mapsto P_a \rho P_a$. On Equation (83),

$$\langle X \otimes X \otimes X \rangle = v, \qquad \langle X \otimes Y \otimes Y \rangle = \langle Y \otimes X \otimes Y \rangle = \langle Y \otimes Y \otimes X \rangle = -v. \tag{86}$$

Indeed X exchanges $|0\rangle$ and $|1\rangle$, while Y does so with phases i and $-i$; two Y factors change the branch-exchange sign. Each demanded parity therefore has expectation v after multiplication by its target sign.

Proposition 16.1 (GHZ disclosure bounds)**.** *For the fixed resource of Equation* (83) *and the game contract of Section 6,*

$$\max\left\{\frac{3}{4}, \frac{1+v}{2}\right\} \le \omega_{\mathrm{GHZ}}(\rho_{\mathrm{GHZ}}(v)) \le \frac{3}{4} + \frac{v}{4}. \tag{87}$$

The right-hand side remains an upper bound after replacing v by d_{GHZ} from Equation (84)*.*

Proof. Equation (86) gives the fixed X/Y strategy score $(1 + v)/2$. Ignoring the quantum state and using a classical optimal strategy gives $3/4$, proving the lower bound.

For the upper bound, define the fully separable state

$$\sigma_{\mathrm{GHZ}} = \tfrac{1}{2}|000\rangle\langle 000| + \tfrac{1}{2}|111\rangle\langle 111|.$$

The exact decomposition

$$\rho_{\mathrm{GHZ}}(v) = (1 - v)\sigma_{\mathrm{GHZ}} + v|\mathrm{GHZ}_+\rangle\langle \mathrm{GHZ}_+|$$

and Lemma 6.2 give $(1 - v)(3/4) + v \cdot 1 = 3/4 + v/4$. The expression is nondecreasing in v, which proves the stated disclosure corollary. □

The separable-mixture argument applies broadly, but the small dimension and simple correlation tensor of this particular family permit an exact optimization. The following specialization is proved directly, including measurements with deterministic outcomes; no claim of historical priority is made for this calculation.

Theorem 16.2 (Exact GHZ value for the declared three-qubit resource)**.** *For Equation* (83)*, with arbitrary local binary POVMs, local uncorrelated ancillas, and shared classical randomness,*

$$\omega_{\mathrm{GHZ}}(\rho_{\mathrm{GHZ}}(v)) = \max\left\{\frac{3}{4}, \frac{1+v}{2}\right\}. \tag{88}$$

No additional shared quantum systems, environmental feedforward, or interparty preprocessing after the supplied resource state are included. Within this local-measurement class, the task supports a strict advantage precisely when $v > 1/2$.

Proof. For local binary observables A_x, B_y, C_z, define the Mermin score

$$\mathcal{M} = \langle A_0 B_0 C_0 \rangle - \langle A_0 B_1 C_1 \rangle - \langle A_1 B_0 C_1 \rangle - \langle A_1 B_1 C_0 \rangle,$$

where a product of operators on distinct participants means their tensor product. The winning probability is $1/2 + \mathcal{M}/8$. For a binary qubit POVM write its observable as $2E - I$, with $0 \le E \le I$. The score is separately linear in each of the six effects. Compactness gives a maximizing tuple, and each effect can successively be chosen extreme without reducing its value. Extreme effects are projectors. It suffices to consider nondegenerate observables $\mathbf{a} \cdot (X, Y, Z)$ with unit real Bloch vector $\mathbf{a}$, and degenerate observables $+I$ or $-I$. Local uncorrelated ancillas reduce to such effective qubit POVMs; shared randomness takes convex mixtures.

First suppose all six observables are nondegenerate. For unit Bloch vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, the only nonzero contributions to a full three-body correlator are transverse:

$$\langle (\mathbf{a} \cdot \boldsymbol{\sigma})(\mathbf{b} \cdot \boldsymbol{\sigma})(\mathbf{c} \cdot \boldsymbol{\sigma}) \rangle = v \, \mathrm{Re}[(a_x + i a_y)(b_x + i b_y)(c_x + i c_y)], \tag{89}$$

where $\boldsymbol{\sigma} = (X, Y, Z)$. The diagonal contribution $a_z b_z c_z$ cancels between 000 and 111. Thus every full correlator has modulus at most v, giving $\mathcal{M} \le 4v$. The X/Y strategy attains equality.

Next suppose exactly one observable is deterministic. By permutation of the participants, take it to be on Alice's side, initially $A_0 = I$. Write $\mathbf{a}$ for the Bloch vector of A_1, and $\mathbf{c}_0, \mathbf{c}_1$ for Charlie's two unit Bloch vectors. The two-body marginal of Bob and Charlie has $\langle (\mathbf{b} \cdot \boldsymbol{\sigma})(\mathbf{c} \cdot \boldsymbol{\sigma}) \rangle = b_z c_z$. Optimizing Bob's two independent unit Bloch vectors therefore bounds the score by

$$\mathcal{M} \le \sqrt{u + k(1-w)} + \sqrt{w + k(1-u)}, \quad u = c_{0z}^2, \quad w = c_{1z}^2, \quad k = v^2(a_x^2 + a_y^2). \tag{90}$$

Here $u, w, k \in [0, 1]$. For Bob's first observable, the transverse coefficient has squared norm $k(1-w)$ and the longitudinal coefficient has squared magnitude u. Optimizing its unit Bloch vector gives the first square root. The other observable gives the second. The square of the nonnegative upper expression is at most

$$2\{u + w + k(2 - u - w)\} = 2\{(1-k)(u+w) + 2k\} \le 4$$

by Cauchy–Schwarz. Hence $\mathcal{M} \le 2$. If $A_0 = -I$, the longitudinal coefficients reverse sign without changing their norms. If the deterministic setting is A_1 instead, the two longitudinal coefficients and transverse partners are exchanged, producing the same two square roots. These observations cover both settings and signs on every participant.

Finally suppose at least two observables are deterministic. If two different participants each have such a setting, each of their two measurement families is jointly measurable: its only possible nonconstant observable can be measured once, and the constant outcome can be supplied when requested. Jointly measure these two participants before their inputs and use the pair of outcomes as a shared classical label. Conditioned on that label, the remaining participant has a local quantum state and local response probabilities. This constructs a fully local classical model, whose score is at most $\mathcal{M} = 2$. If all deterministic settings belong to just one participant, that participant has both settings deterministic. The other two receive the reduced state $\frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$, which is separable. Their responses, together with the first participant's constants, again form a local classical model. These cases exhaust all tuples with at least two deterministic settings.

The general bound is therefore $\mathcal{M} \le \max\{2, 4v\}$. Classical optimal answers attain 2, and the X/Y strategy attains $4v$, proving the formula. □

A perfect exterior record of any one local computational branch gives $\varepsilon_j = 0$, hence $v = 0$. In this single-branch-pair GHZ family, the remaining resource is fully separable and the optimized score is exactly $3/4$. This conclusion uses the specified state and access boundary. The magic-square construction next demonstrates why a perfect record of one branch in a larger resource need not eliminate every coordination advantage.

Combining the exact value with the marker bounds sharpens the disclosure limit to

$$\omega_{\mathrm{GHZ}} \leq \max\left\{\frac{3}{4}, \frac{1 + d_{\mathrm{GHZ}}}{2}\right\}. \tag{91}$$

Every overlap at the cap is realized by two-dimensional pure markers $|e_0^{(j)}\rangle = |0\rangle$ and $|e_1^{(j)}\rangle = d_j|0\rangle + \sqrt{1 - d_j^2}|1\rangle$. The indicated classical or X/Y strategy then attains the bound, making it a sharp capacity limit within this preparation family. For equal error caps $\varepsilon_A = \varepsilon_B = \varepsilon_C = \varepsilon$, one has $d_{\mathrm{GHZ}} = [2\sqrt{\varepsilon(1 - \varepsilon)}]^3$. A strict advantage is possible within the caps exactly when

$$\varepsilon > \frac{1 - \sqrt{1 - 2^{-2/3}}}{2}. \tag{92}$$

This threshold concerns the declared game, resource family, and local-measurement class after the preparation boundary. A marker-error cap alone remains an upper limit on actual capacity; certifying achieved performance requires the actual prepared state and score.

16.3 Pre-input communication changes the resource class

The preparation boundary has observable consequences. Consider an enlarged protocol in which Alice may measure her share of $\rho_{\mathrm{GHZ}}(v)$ and communicate the result before the referee distributes the questions. Alice measures X and obtains $s \in \{-1, +1\}$, with probability $1/2$. Bob and Charlie's normalized conditional state is

$$\rho_{BC|s} = \tfrac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11| + sv|00\rangle\langle 11| + sv|11\rangle\langle 00|).$$

Alice sends $k = (1 - s)/2$ to Bob, who applies Z^k. Their state becomes $\rho_\Phi(v)$ for either message value. Alice then returns $+1$ for either eventual game input.

On the promised GHZ input set, $x = y \oplus z$, where $\oplus$ is addition modulo two. Bob and Charlie's inputs (y, z) are uniform over four pairs. Their required answer product is $(-1)^{y+z+yz}$, with target matrix $\left(\begin{smallmatrix} +1 & -1 \\ -1 & -1 \end{smallmatrix}\right)$. They can implement the standard CHSH strategy on $\rho_\Phi(v)$ and multiply their respective output signs by $(-1)^y$ and $(-1)^z$. Thus this enlarged preprocessing class achieves

$$w_{\mathrm{GHZ}}^{\mathrm{preprocessed}}(v) \geq \frac{1}{2} + \frac{\sqrt{1 + v^2}}{4} > \frac{3}{4} \quad \text{for every } v > 0. \tag{93}$$

The inequality is an achieved strategy value, with no claim here to optimize the full preprocessing class. It demonstrates why the $v > 1/2$ threshold cannot be transferred to a protocol permitting additional communication after the three-qubit resource is supplied. No messages are sent after the private inputs in this construction. Its public preparation message is useful precisely because it changes the resource available at the final local-measurement boundary.

17 Two resource pairs and the magic-square task

The magic-square task changes both the coordination structure and the resource accounting. Two participants must supply compatible partial answers to an array that has no deterministic classical completion. Two Bell pairs give a perfect quantum strategy. Disclosing one pair can leave the other pair useful, so a single scalar "amount of public recordkeeping" cannot describe the task's remaining capacity.

17.1 A fully specified perfect strategy

Alice controls $\mathcal{H}_A = Q_{A1} \otimes Q_{A2} \simeq \mathbb{C}^4$ and Bob controls $\mathcal{H}_B = Q_{B1} \otimes Q_{B2} \simeq \mathbb{C}^4$. On each local two-qubit space define

$$(M_{rc})_{r,c=1}^{3} = \begin{pmatrix} I \otimes Z & Z \otimes I & Z \otimes Z \\ X \otimes I & I \otimes X & X \otimes X \\ X \otimes Z & Z \otimes X & Y \otimes Y \end{pmatrix}. \tag{94}$$

Every M_{rc} is a Hermitian involution on $\mathbb{C}^4$. Operators within each row commute, and operators within each column commute. The three row products are $+I_4$; the column products are $+I_4, +I_4, -I_4$. For example, $(X \otimes Z)(Z \otimes X) = Y \otimes Y$, whereas $(Z \otimes Z)(X \otimes X) = -Y \otimes Y$. These identities verify the required parity signs directly. The array is a concrete version of the established Peres–Mermin construction [55, 56].

For row r Alice uses the joint projectors

$$P_{\mathbf{a}}^{(r)} = \prod_{c=1}^{3} \frac{I_4 + a_c M_{rc}}{2}, \qquad \mathbf{a} \in \{-1, +1\}^3. \tag{95}$$

For column c Bob uses

$$Q_{\mathbf{b}}^{(c)} = \prod_{r=1}^{3} \frac{I_4 + b_r M_{rc}^{\mathsf{T}}}{2}, \qquad \mathbf{b} \in \{-1, +1\}^3, \tag{96}$$

where transposition is in the computational basis. The factors commute, so these are positive orthogonal joint projectors. They sum to the identity for each input. A projector vanishes if its sign triple has the wrong parity, because the corresponding operator product has the fixed parity listed above. The nonzero projectors define legitimate four-outcome local measurements; their instruments may use $\rho \mapsto P_{\mathbf{a}}^{(r)} \rho P_{\mathbf{a}}^{(r)}$ and similarly for Bob.

Let $|\Phi_+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Sharing $|\Phi_+\rangle_{A1B1} \otimes |\Phi_+\rangle_{A2B2}$ is, after grouping the local factors, the state

$$|\Phi_4\rangle = \frac{1}{2} \sum_{z \in \{0,1\}^2} |z\rangle_A |z\rangle_B. \tag{97}$$

For any 4×4 matrices R, S, direct basis expansion gives

$$\langle \Phi_4 | R \otimes S^{\mathsf{T}} | \Phi_4 \rangle = \tfrac{1}{4} \operatorname{Tr}(RS).$$

Taking $R = S = M_{rc}$ makes this expectation one. Alice's intersection answer is the eigenvalue of M_{rc} and Bob's is the eigenvalue of M_{rc}^{T}. They agree with certainty. Together with the parity identities, this proves perfect success for all nine input pairs. The transpose convention is kept explicit even though all nine matrices in this particular array are real and symmetric.

17.2 Independent phase damping of the two pairs

Before the input distribution, let each qubit Q_{jk}, with $j \in \{A, B\}$ and $k \in \{1, 2\}$, undergo an isometry of the form Equation (82) into its own marker E_{jk}. All blanks are independent, and the initial two Bell pairs are independent. Define $\gamma_{jk} = \langle e_1^{(jk)} | e_0^{(jk)} \rangle$. With known overlap phases compensated locally, the resource is

$$\rho_{\mathrm{MS}}(v_1, v_2) = \rho_\Phi(v_1)_{A1B1} \otimes \rho_\Phi(v_2)_{A2B2}, \qquad v_k = |\gamma_{Ak}\gamma_{Bk}| \in [0, 1]. \tag{98}$$

When evaluating local measurements, its tensor factors are regrouped as $(A1, A2) \mid (B1, B2)$. This bookkeeping matters for both the proof and the executable matrix audit.

Suppose each marker admits equal-prior branch discrimination with error at most $\varepsilon_{jk} \in [0, 1/2]$. Set

$$d_{jk} = 2\sqrt{\varepsilon_{jk}(1 - \varepsilon_{jk})}, \qquad b_k = d_{Ak} d_{Bk}, \qquad v_k \le b_k. \tag{99}$$

As in the GHZ model, the exterior systems and their reading outcomes are excluded from the task measurements and feedforward. The v_k describe the actual reduced resource, while the b_k describe allowed upper caps inferred from the specified marker model.

Proposition 17.1 (Exact score of the canonical Pauli strategy)**.** *The measurements in Equations* (95) *and* (96)*, on Equation* (98)*, have success probability*

$$w_{\mathrm{can}}(v_1, v_2) = \frac{2}{3} + \frac{v_1 + v_2 + v_1 v_2}{9}. \tag{100}$$

Proof. For a single pair $\rho_\Phi(v)$, $\langle X \otimes X \rangle = v$, $\langle Y \otimes Y \rangle = -v$, and $\langle Z \otimes Z \rangle = 1$. Independence of the two pairs gives the nine intersection correlators

$$\left(\langle M_{rc} \otimes M_{rc}^{\mathsf{T}} \rangle\right)_{rc} = \begin{pmatrix} 1 & 1 & 1 \\ v_1 & v_2 & v_1 v_2 \\ v_1 & v_2 & v_1 v_2 \end{pmatrix}. \tag{101}$$

The parity constraints remain exact because they are operator identities. At each intersection the agreement probability is one half of one plus its correlator. Averaging the nine probabilities gives the stated formula. □

This is an exact score for a specified strategy, rather than an optimized value over every strategy on the resource. At $v_1 = v_2 = 0$ the canonical measurements score only $2/3$. The participants can do better by using the classical $8/9$ strategy. Optimization must therefore include this fallback and other possible measurements.

Proposition 17.2 (Magic-square upper bound from separable weight)**.** *For the same resource and the full local measurement class,*

$$\omega_{\mathrm{MS}}(\rho_{\mathrm{MS}}(v_1, v_2)) \le \frac{8}{9} + \frac{v_1 + v_2 - v_1 v_2}{9} \le \frac{8}{9} + \frac{b_1 + b_2 - b_1 b_2}{9}. \tag{102}$$

Proof. Write $\phi = |\Phi_+\rangle\langle\Phi_+|$ and $\sigma = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$, so $\rho_\Phi(v) = v\phi + (1-v)\sigma$. Expanding the two-pair product gives

$$\begin{aligned}\rho_{\mathrm{MS}} = (1-v_1)(1-v_2)\,\sigma\otimes\sigma + v_1(1-v_2)\,\phi\otimes\sigma \\ + (1-v_1)v_2\,\sigma\otimes\phi + v_1v_2\,\phi\otimes\phi.\end{aligned} \tag{103}$$

The first term is separable across Alice and Bob and has weight $\lambda = (1-v_1)(1-v_2)$. If $\lambda < 1$, normalize the remaining positive terms into one density operator of weight $1-\lambda$. Lemma 6.2, with classical value $8/9$ and quantum value one, gives $\lambda(8/9) + (1-\lambda)$. If $\lambda = 1$, the whole state is separable and the same expression holds. Simplification gives the first inequality. The function $v_1 + v_2 - v_1v_2$ is nondecreasing in each argument on $[0,1]^2$, which gives the second. □

17.3 A surviving pair is enough for a strict advantage

An explicit strategy that uses only one pair makes the resource coverage issue sharp. Let $v_* := \max\{v_1, v_2\}$ and ignore the weaker pair. On the stronger pair play the optimal unrestricted CHSH strategy:

$$A_0 = Z, \quad A_1 = X, \qquad B_0 = \frac{Z + v_*X}{\sqrt{1+v_*^2}}, \quad B_1 = \frac{Z - v_*X}{\sqrt{1+v_*^2}}. \tag{104}$$

The symbols $a_0, a_1, b_0, b_1 \in \{-1, +1\}$ below denote the actual outcome of the selected setting in the corresponding round. They are not assumed to coexist as a quantum joint assignment. The local response rules are

Alice's row	a_1^{out}	a_2^{out}	a_3^{out}	Bob's column	b_1^{out}	b_2^{out}	b_3^{out}
1	1	1	1	1	1	1	1
2	1	a_0	a_0	2	1	b_0	b_0
3	1	a_1	a_1	3	1	b_1	$-b_1$

. (105)

These outputs always satisfy the respective row and column parities. The five cells in the first row or first column agree deterministically. The remaining four cells demand, in order, $a_0b_0 = +1$, $a_0b_1 = +1$, $a_1b_0 = +1$, and $a_1b_1 = -1$. They are exactly the four CHSH winning conditions.

Proposition 17.3 (Achievable magic-square envelope)**.** *The strategy in Equation* (105) *achieves*

$$w_{\mathrm{emb}}(v_*) = \frac{5 + 4w_{\mathrm{CHSH}}^{\mathrm{free}}(v_*)}{9} = \frac{7 + \sqrt{1+v_*^2}}{9}. \tag{106}$$

Consequently

$$\omega_{\mathrm{MS}}(\rho_{\mathrm{MS}}(v_1, v_2)) \geq \max\left\{w_{\mathrm{can}}(v_1, v_2), w_{\mathrm{emb}}(v_*)\right\}. \tag{107}$$

The resource supports a strict magic-square advantage over $8/9$ *if and only if* $\max\{v_1, v_2\} > 0$.

Proof. The five deterministic cells and four CHSH cells prove the first expression; Equation (9) gives the second. Participants may choose between this strategy and the canonical strategy using the calibrated resource parameters before play, establishing the lower bound. For $v_* > 0$, $\sqrt{1+v_*^2} > 1$, so the embedded strategy exceeds $8/9$. At $v_1 = v_2 = 0$, the resource is separable and cannot exceed the classical bound. The classical strategy attains it. □

In particular, perfectly disclosing the first pair's computational branch while retaining an ideal second pair gives

$$(v_1, v_2) = (0, 1), \qquad w_{\text{emb}} = \frac{7 + \sqrt{2}}{9} \simeq 0.934913 > \frac{8}{9}. \tag{108}$$

A perfect record of one intermediate branch therefore coexists with a strict quantum coordination advantage. The advantage is supplied by the specified surviving resource. A claim that every perfect intermediate record eliminates nonlocal performance would fail on this example.

At equal actual visibilities $v_1 = v_2 = t$, the canonical score exceeds $8/9$ only when $t > \sqrt{3} - 1$, whereas the embedded strategy exceeds it for every $t > 0$. The first threshold concerns one chosen measurement scheme. The existence result concerns the optimized resource value, whose exact intermediate magnitude is not determined here. At $(0, 0)$ and $(1, 1)$ the achieved lower and analytic upper bounds coincide. Across the rest of the square, the displayed inequalities retain their declared status as lower and upper bounds.

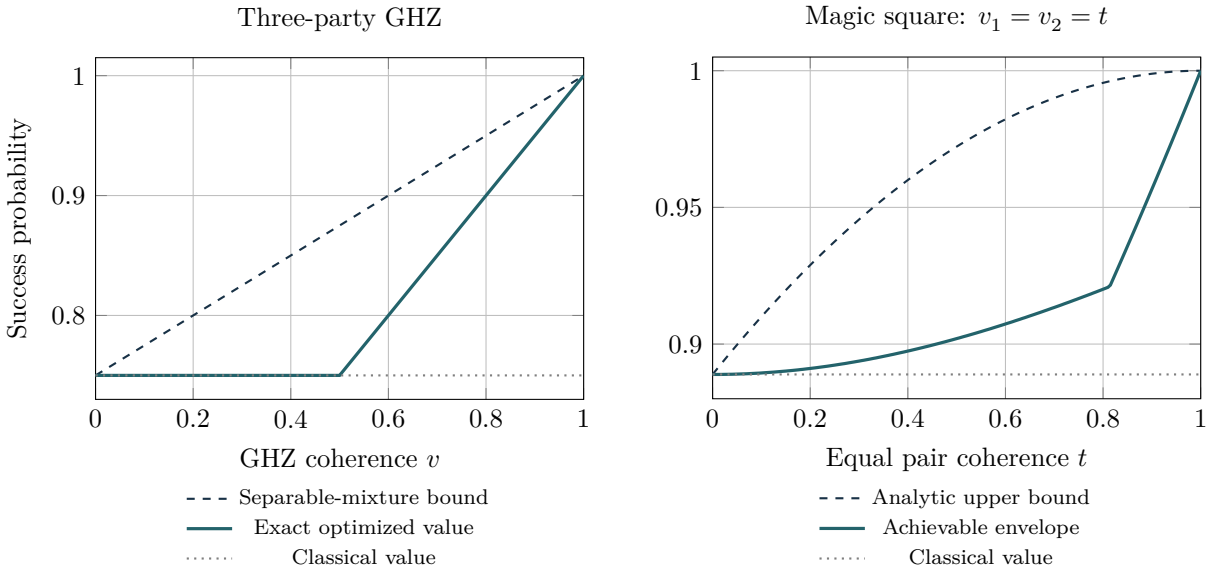


Figure 7: Task values for the specified state families. The GHZ solid curve is the exact optimum proved in Theorem 16.2; its dashed curve shows the looser general separable-mixture bound. For the magic-square panel, the solid curve is an achieved strategy envelope and the dashed curve is an analytic upper bound; their gap remains an unresolved optimization interval in this analysis. The panels use different vertical scales. These are mathematical predictions, with no experimental data plotted.

For equal marker error caps $\varepsilon_{jk} = \varepsilon$, both pair caps are $b = 4\varepsilon(1 - \varepsilon)$ and Equation (102) becomes

$$\omega_{\text{MS}} \leq \frac{8}{9} + \frac{2b - b^2}{9}. \tag{109}$$

This is a useful disclosure limit on a stated four-marker preparation. It does not supply a lower performance guarantee from error caps alone. For actual calibrated (v_1, v_2), the constructive lower bound and the upper bound form the task-performance interval shown in the right-hand panel of Figure 7 for the equal-visibility slice.

18 Which records are compatible with the advantage?

The preceding theorems support a specific physical conclusion. Exporting information about designated coherence-bearing branches can constrain performance on a later distributed task. They also support a constructive institutional question: which records can remain public while the useful resource survives? Two elementary instrument results give decisive answers.

18.1 A complete final archive preserves the score

Proposition 18.1 (Terminal transcript copying)**.** *Any strategy in Definition 6.1 permits faithful copying of its final classical outputs into local archives without changing its success probability. Inputs and outputs may subsequently be combined into a public transcript after the no-communication interval has ended.*

Proof. After a local measurement has produced outcome a_i, represent its classical record by an orthonormal register state $|a_i\rangle_{R_i}$. The joint classical output state, conditional on $\mathbf{x}$, is

$$\sum_{\mathbf{a}} p(\mathbf{a} \mid \mathbf{x}) \bigotimes_i |a_i\rangle\langle a_i|_{R_i}.$$

For a blank archive C_i, the isometry $|a_i\rangle_{R_i}|0\rangle_{C_i} \mapsto |a_i\rangle_{R_i}|a_i\rangle_{C_i}$ copies these orthogonal labels. Tracing out the archives leaves exactly the original output distribution. Since the score is a function of that distribution, it is unchanged. Communicating the already fixed outputs after the round changes no earlier answer or score. □

This proposition applies to the perfect GHZ and magic-square strategies as well as to CHSH. A perfect archive can contain the performed context, the actual answers, the parity checks, and the score. Requiring such a transcript does not demand simultaneous answers to the unperformed questions. The copying operation broadcasts commuting classical labels, which is compatible with the no-broadcasting restriction for general noncommuting quantum states [63].

18.2 An intermediate compatible record can also be harmless

Timing alone does not identify a damaging record. Suppose a selected measurement context consists of commuting Hermitian involutions $O_1, \dots, O_m$ on a finite Hilbert space. Their joint projector for answer vector $\mathbf{a} \in \{-1, +1\}^m$ is $P_{\mathbf{a}} = \prod_{j=1}^{m} (I + a_j O_j)/2$. Measure O_1 first and copy its outcome into an exterior archive before measuring the rest. Let $\Pi_s = (I + sO_1)/2$, for $s \in \{-1, +1\}$.

Proposition 18.2 (Compatible intermediate copying)**.** *The preceding measurement-and-copy procedure preserves every joint outcome probability of the selected context, for every input density operator ρ.*

Proof. The nonselective state transformation after the copied first outcome is $\mathcal{D}_1(\rho) = \sum_s \Pi_s \rho \Pi_s$. Because $P_{\mathbf{a}} \Pi_s = \delta_{a_1 s} P_{\mathbf{a}}$,

$$\mathrm{Tr}[P_{\mathbf{a}} \mathcal{D}_1(\rho)] = \sum_s \mathrm{Tr}(\Pi_s P_{\mathbf{a}} \Pi_s \rho) = \mathrm{Tr}(P_{\mathbf{a}} \rho).$$

Keeping the first outcome rather than forgetting it also preserves the full joint distribution: its value is necessarily a_1 on the branch with joint projector $P_{\mathbf{a}}$. The argument extends to local tensor-product contexts at separated participants by applying the same identity to each side. □

In the magic-square strategy, after the row and column have been selected, each participant can copy the first measured sign into a physically separate archive before completing the remaining commuting measurements. On $|\Phi_4\rangle$, the strategy still wins with certainty. No archive outcome is communicated to the other participant before the answer is fixed. This is a genuine intermediate record, yet it preserves the task statistics because it belongs to the chosen compatible context. The branch-disclosure theorems instead concern records exported before alternative incompatible contexts are resolved, in a basis that can remove coherence those alternatives require.

18.3 Resource coverage, public utility, and the scope of the thesis

Three distinctions now have exact mathematical witnesses. A terminal transcript preserves a completed score by Proposition 18.1. A compatible partial answer preserves the selected context by Proposition 18.2. A perfect record of one resource pair can leave another pair sufficient for an advantage by Equation (108). The phase-label construction in Section 15 supplies a further case: public information can enable a useful correction and increase the attainable score. Publicity therefore has to be specified as an instrument acting on named degrees of freedom.

The proposed civilization can preserve an extensive common history and still use correlations that exceed a local setting-independent account of all alternative task answers. Its institutions would need to identify the operational boundary between stable public information and branch information that is still physically required by a coherent task. This boundary is defined by the preparation, instruments, accessible systems, and payoff, rather than by the social name given to a record.

The mathematical results establish advantages in explicit common-interest tasks. They do not assign a universal utility to nonlocality, nor do they establish consciousness, moral status, or a generic superiority of quantum institutions. A task with unrestricted exchange of private questions has a different classical benchmark. An additional entangled resource has a different state. An actual observer protocol can impose further readout constraints. These are changes to the scientific model that must be included before its score can support a larger claim.

18.4 What the computational audit verifies

The executable `reality_games_audit.py` exhaustively enumerates all 16 deterministic CHSH assignments, all 64 deterministic GHZ assignments, and all 4,096 valid-parity magic-square strategy pairs. It obtains the classical maxima 3/4, 3/4, and 8/9, respectively. It separately constructs the quantum density matrices and local joint projectors, checks their normalization and positivity, verifies the Pauli product identities, and computes 17,424 magic-square joint probabilities over a grid of 121 resource pairs. The exact canonical score, the embedded CHSH score, and both convex decompositions agree with direct matrix calculations.

The audit also applies the nonselective channel corresponding to copying a selected commuting partial answer and verifies that all later context probabilities remain unchanged. Independently gener-

ated pure product markers check the overlap product and binary-discrimination relation. The fixed random seed, tolerance, grid sizes, and numerical residuals are recorded in `games_audit_results.json`. The program tests the displayed constructions; the analytic proofs establish the stated parameter ranges. An additional audit checks the exact GHZ optimization against continuous searches and every pattern of deterministic versus nondegenerate qubit settings. These are numerical cross-checks of the proof, not a replacement for its exhaustive case argument. No experimental data or exact solution of the intermediate magic-square optimization is asserted.

19 From many accounts to a precise obstruction

A civilization can preserve many accounts without declaring every account equally correct. The mathematical question is whether the accounts can be represented as marginals of one probability law on the same identified quantities. It is possible for every individual account to be normalized, for accounts to agree wherever their measurement procedures overlap, and for the full collection nevertheless to resist such an extension. This is a sharply specified obstruction. It does not make an ordinary contradiction true, establish consciousness, or prevent a laboratory from maintaining an ordinary chronological record of the experiments it actually performed.

The finite formulation below uses established contextuality mathematics. Abramsky and Brandenburger organize compatible families of empirical probabilities by measurement covers and characterize contextuality by the absence of a global probability extension. Fine establishes the relationship between joint distributions and Bell inequalities in the two-party, two-setting scenario. We reproduce only the structures needed here, with proofs and explicit domains rather than importing philosophical conclusions from the terminology [13, 67].

19.1 Measurement labels, contexts, and probability laws

Definition 19.1 (Finite empirical scenario)**.** Let X be a finite nonempty set of measurement labels. For each $x \in X$, let O_x be a finite nonempty outcome set. A context cover $\mathcal{M} \subseteq 2^X$ is a nonempty family of nonempty subsets whose union is X. Each $C \in \mathcal{M}$ denotes a jointly executable measurement procedure with outcome space

$$O_C := \prod_{x \in C} O_x.$$

The interpretation that two appearances of x in different contexts refer to the same operational measurement is an explicit modeling assumption. For a finite set S, write $\Delta(S) := \{p : S \to [0,1] : \sum_{s \in S} p(s) = 1\}$. An empirical family is $e = (e_C)_{C \in \mathcal{M}}$ with $e_C \in \Delta(O_C)$. For $D \subseteq C$, its marginal is

$$(e_C|_D)(r) := \sum_{s \in O_C :\, s|_D = r} e_C(s), \qquad r \in O_D.$$

The family is compatible when

$$e_C|_{C \cap D} = e_D|_{C \cap D} \quad \text{for every } C, D \in \mathcal{M}. \tag{110}$$

An instrument, calibration, or context-dependent change of measurement must receive a distinct label if the operational equivalence needed to reuse x is absent. Otherwise a failure of matching marginals can be introduced by notation rather than discovered in nature. When measurements occur at different times, a label includes the appropriate protocol position; the model does not silently identify a system before and after an intervention.

Definition 19.2 (Global extension)**.** Put $O_X = \prod_{x \in X} O_x$. A global extension of e is a probability distribution $q \in \Delta(O_X)$ satisfying $q|_C = e_C$ for every context. Let N denote the set of empirical families having such an extension. A compatible empirical family outside N is contextual relative to this scenario and these identifications.

A global assignment $g \in O_X$ assigns one value to each label, including labels belonging to mutually exclusive procedures. Its role is therefore stronger than that of a row in an experimental log. Let $J := \{(C, s) : C \in \mathcal{M},\ s \in O_C\}$ and define the incidence matrix $M \in \{0,1\}^{J \times O_X}$ by

$$M_{(C,s),g} := \begin{cases} 1, & g|_C = s, \\ 0, & g|_C \neq s. \end{cases} \tag{111}$$

After writing e as a vector indexed by J, existence of a global extension is precisely feasibility of

$$Mq = e, \qquad q_g \geq 0 \ (g \in O_X), \qquad \sum_{g \in O_X} q_g = 1. \tag{112}$$

This is a finite linear feasibility problem. Its dimension grows exponentially with the number of measurements, so finite definability does not imply inexpensive solution for a large civilization.

Proposition 19.3 (Scope of the obstruction)**.** *Every global extension satisfies* (110)*. Conversely, compatibility need not imply a global extension. Nevertheless every empirical family, compatible or otherwise, has an ordinary probability law on performed-context/outcome pairs for each chosen context distribution $\pi \in \Delta(\mathcal{M})$.*

Proof. Marginalizing q first to C and then to $C \cap D$ gives the same result as marginalizing first to D, proving the first statement. The CHSH example below proves failure of the converse. For the last statement, take the disjoint union $\Omega_{\log} := \bigsqcup_C \{C\} \times O_C$ and assign $P(C, s) := \pi(C) e_C(s)$. Nonnegativity and normalization follow immediately. This law records the context actually chosen. It does not assign values to every unperformed measurement in the same trial. □

This distinction is central to the institutional proposal. A common archive of actual, completed experiments is compatible with an obstruction to a single value assignment for all the counterfactual measurements in the chosen scenario. The mathematical result is not an instruction to abolish public records. It is an instruction to preserve the context in which each recorded claim acquires its meaning.

19.2 An exactly realizable benchmark

Let $X = \{A_0, A_1, B_0, B_1\}$, let every outcome set be $\{-1, +1\}$, and let the four contexts be $C_{xy} = \{A_x, B_y\}$ for $x, y \in \{0, 1\}$. Define $\sigma_{xy} := (-1)^{xy}$ and the one-parameter empirical family

$$p_\zeta(a, b \mid x, y) := \frac{1 + ab\,\zeta\sigma_{xy}}{4}, \quad a, b \in \{-1, +1\}, \quad \zeta \in [0, 1]. \tag{113}$$

Every entry is nonnegative, each context sums to one, and every single-observer marginal is uniform. Thus all these models satisfy (110). The parameter ζ is the magnitude of the four signed correlators in this probability family; it is not automatically the interference visibility of a particular physical apparatus.

Writing $E_{xy} := \sum_{a,b} ab\, p_\zeta(a,b \mid x,y)$, the CHSH functional is

$$S(p) := E_{00} + E_{01} + E_{10} - E_{11}; \qquad S(p_\zeta) = 4\zeta. \tag{114}$$

The original CHSH work supplies the experimental Bell-inequality setting [12]. We use this family as a contextual-probability benchmark; it is not, by itself, a specification of a Local Friendliness experiment with persistent artificial observers.

Proposition 19.4 (Global assignments and the CHSH bound)**.** *Every $q \in \Delta(\{-1,+1\}^4)$ satisfies $S(Mq) \le 2$. Consequently p_ζ has no global extension when $\zeta > 1/2$.*

Proof. For each deterministic assignment (a_0, a_1, b_0, b_1),

$$a_0b_0 + a_0b_1 + a_1b_0 - a_1b_1 = a_0(b_0 + b_1) + a_1(b_0 - b_1).$$

Exactly one of the two parentheses is zero and the other is $+2$ or -2. The expression therefore belongs to $\{-2,+2\}$. Its expectation under any nonnegative normalized q cannot exceed 2. For p_ζ, $S = 4\zeta > 2$ whenever $\zeta > 1/2$. □

At $\zeta = 1/\sqrt{2}$, a physical quantum realization is obtained from $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ on $\mathbb{C}^2 \otimes \mathbb{C}^2$, with Hermitian observables

$$A_0 = Z, \quad A_1 = X, \quad B_0 = (Z+X)/\sqrt{2}, \quad B_1 = (Z-X)/\sqrt{2},$$

where $X = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ and $Z = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ are Pauli matrices. Each observable squares to the identity. Its outcome projector is $(I + aA_x)/2$ or $(I + bB_y)/2$. The one-site expectations vanish, $\langle Z \otimes Z\rangle = \langle X \otimes X\rangle = 1$, and the mixed X, Z expectations vanish. Expanding the product of the projectors therefore gives (113) with $\zeta = 1/\sqrt{2}$. This explicit realization keeps the example inside quantum mechanics. Every $0 \le \zeta \le 1/\sqrt{2}$ is also realized by the same measurements on $\rho_t = t|\Phi^+\rangle\langle\Phi^+| + (1-t)I_4/4$, with $t = \sqrt{2}\zeta \in [0,1]$. Beyond this interval the table is not quantum realizable with binary local measurements. To see the bound for dichotomic observables, the CHSH operator obeys

$$\mathcal{B}^2 = 4I - [A_0, A_1] \otimes [B_0, B_1].$$

Here the square brackets denote the operator commutator. Each observable has operator norm one, so each commutator has norm at most two and $\|\mathcal{B}\| \le 2\sqrt{2}$. Binary positive-operator-valued measurements obey the same bound by dilation to projective measurements. Thus $4\zeta \le 2\sqrt{2}$ is necessary. The larger-ζ tables describe the surrounding no-signalling probability model, not additional permitted quantum states.

19.3 How far must a classical common account move the data?

For distributions p, q on the same finite set define

$$\mathrm{TV}(p,q) := \frac{1}{2}\sum_s |p(s) - q(s)| \in [0,1].$$

For empirical families on the four CHSH contexts, define $d_{\mathrm{av}}(e,f) := \frac{1}{4}\sum_{x,y} \mathrm{TV}(e_{xy}, f_{xy})$. This metric weights contexts equally. Different operational sampling weights would define a different distance and require corresponding coefficients in the bound.

Proposition 19.5 (Public-account approximation bound)**.** *For every empirical family e in the CHSH scenario and every $f \in \mathsf{N}$,*

$$S(e) \le 2 + 2\sum_{x,y} \mathrm{TV}(e_{xy}, f_{xy}). \tag{115}$$

In particular, if all four distances are at most ϵ, then $S(e) \le 2 + 8\epsilon$. For (113),

$$\inf_{f \in \mathsf{N}} d_{\mathrm{av}}(p_\zeta, f) = \max\left\{0, \frac{2\zeta - 1}{4}\right\}. \tag{116}$$

Proof. For a function h with $|h| \le 1$, the triangle inequality gives $|\sum_s h(s)(p(s) - q(s))| \le 2\,\mathrm{TV}(p,q)$. Apply this to each signed product $\sigma_{xy}ab$ and use Proposition 19.4; this proves (115). For $\zeta \ge 1/2$, it supplies the lower bound $d_{\mathrm{av}}(p_\zeta, f) \ge (4\zeta - 2)/8$.

There are eight deterministic assignments with CHSH value $+2$. In four of them, $b_0 = b_1 = b$, $a_0 = b$, and a_1 is free; uniform weight within this group gives correlators $(1,1,0,0)$. In the other four, $b_0 = -b_1 = b$, $a_1 = b$, and a_0 is free, giving $(0,0,1,-1)$. Uniform weight over the eight assignments therefore gives correlators $\sigma_{xy}/2$ and vanishing single-site means, which is exactly $p_{1/2}$. Therefore $p_{1/2} \in \mathsf{N}$. For $\zeta \ge 1/2$, every context has distance $\mathrm{TV}((p_\zeta)_{xy}, (p_{1/2})_{xy}) = (\zeta - 1/2)/2$, attaining the lower bound. Finally p_0 is the image of four independent uniform signs, and for $0 \le \zeta \le 1/2$, $p_\zeta = 2\zeta p_{1/2} + (1 - 2\zeta)p_0 \in \mathsf{N}$. □

At the quantum point $\zeta = 1/\sqrt{2}$, the exact average-context distance is $(\sqrt{2} - 1)/4 \simeq 0.10355339$. This number is a model-derived distance between probability tables. It is neither an observed discrepancy in an artificial civilization nor a universal tolerance for disagreement.

19.4 A graded obstruction

The contextual fraction provides an established quantitative measure computed by a linear program [68]. We use its finite subdistribution definition. A nonnegative vector $b \in \mathbb{R}_{\ge 0}^{O_X}$ is an admissible classical subdistribution when $Mb \le e$ coordinatewise. Put

$$\mathrm{NCF}(e) := \max_{b \ge 0,\, Mb \le e} \sum_g b_g, \qquad \mathrm{CF}(e) := 1 - \mathrm{NCF}(e). \tag{117}$$

Summing the constraints over the outcomes in any one context shows that $\sum_g b_g \le 1$. The feasible set contains zero and is closed and bounded, so the maximum exists. For a compatible family, subtracting the marginals of b preserves compatibility; after normalization, the remainder is another empirical model whenever its mass is positive.

Proposition 19.6 (Contextual fraction of the benchmark)**.** *For* $0 \le \zeta \le 1$,

$$\mathrm{CF}(p_\zeta) = \max\{0, 2\zeta - 1\}.$$

Proof. For $\zeta \le 1/2$, a full global extension was constructed above. For $\zeta \ge 1/2$, a classical component of mass λ contributes at most 2λ to S, while any remaining normalized empirical family contributes at most $4(1-\lambda)$ because each correlator lies in $[-1,1]$. Hence $4\zeta \le 4 - 2\lambda$, so $1 - \lambda \ge 2\zeta - 1$. Conversely,

$$p_\zeta = (2 - 2\zeta)p_{1/2} + (2\zeta - 1)p_1$$

exhibits a classical component of mass $2 - 2\zeta$. The two bounds coincide. □

The contextual fraction at the quantum point is $\sqrt{2} - 1$. It quantifies an obstruction in a defined probability model. It has no units and is not a percentage of persons lacking an objective world. Moreover, empirical frequencies generally violate exact overlap equalities by sampling fluctuation. A contextual-fraction calculation must therefore state how it handles statistical uncertainty and operational disturbance; silently replacing inconsistent frequencies by a convenient compatible table would conceal a modeling choice.

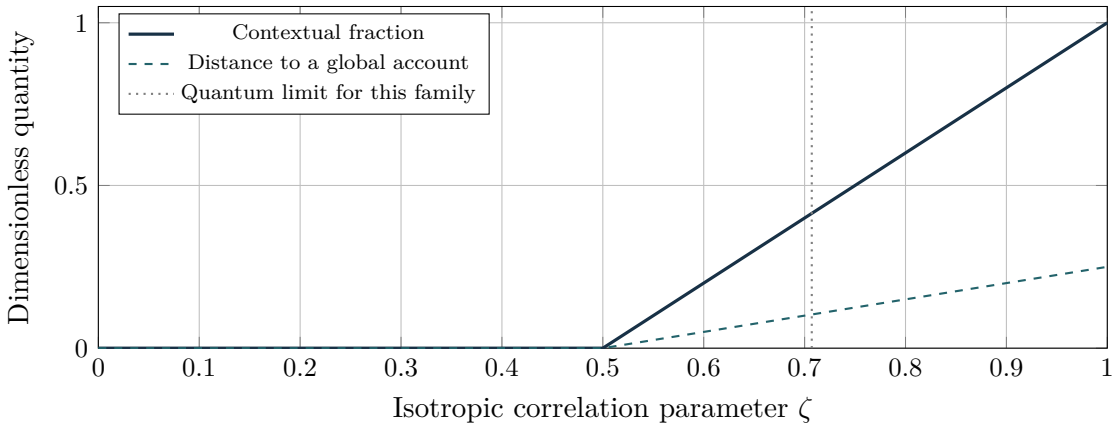


Figure 8: Exact quantities for the isotropic probability family. Global extension is possible up to $\zeta = 1/2$; quantum realization is possible up to $1/\sqrt{2}$. The continuation beyond the quantum limit describes mathematical nonsignaling tables used to analyze the surrounding polytope. The parameter ζ is distinct from the physical leakage parameter v.

20 A common institution inside a contextual world

The absence of a global extension does not remove every shareable quantity. A civilization requires some public operations, but it need not require a classical assignment to every measurement that any member could perform. The following statements isolate that possibility without assuming that agreeing probability distributions are already shared physical records.

20.1 Public conditions can control the contextual process

A public institution can make a shared condition control the task, apparatus configuration, preparation, or next action. A visible preparation flag expresses that dependence explicitly. The phase-label task in Section 15 gives a concrete operational instance: retaining the label enables a coordination advantage, while losing access to it before correction removes that advantage. The following theorem describes the associated contextuality accounting for general conditional models.

Theorem 20.1 (Exact contextual fraction with a visible common flag)**.** *For each k in a finite nonempty set K, let $e^{(k)}$ be a compatible empirical family on the same scenario $(X, \mathcal{M}, (O_x)_x)$. Let $r = (r_k)_{k\in K} \in \Delta(K)$. Add a common flag label R to every context and define*

$$e^{\mathrm{flag}}_{C\cup\{R\}}(s,k) := r_k e^{(k)}_C(s). \tag{118}$$

Then this family is compatible, and

$$\mathrm{CF}(e^{\mathrm{flag}}) = \sum_{k\in K} r_k \,\mathrm{CF}(e^{(k)}). \tag{119}$$

The statement follows from the finite contextual-fraction optimization in (117)*.*

Proof. Compatibility follows separately for each flag value. A global subdistribution for the flagged scenario is a nonnegative array $b(g,k)$ with context marginals bounded by $r_k e^{(k)}_C(s)$. For $r_k > 0$, the slice $b(g,k)/r_k$ is admissible for the unflagged model $e^{(k)}$, so $\sum_g b(g,k) \le r_k \,\mathrm{NCF}(e^{(k)})$. For $r_k = 0$, summing the inequalities in any context forces that slice to vanish. Summing over k proves the corresponding upper bound on the total mass of b. Conversely, choose an optimal admissible subdistribution $b_k(g)$ for each $e^{(k)}$ and set $b(g,k) = r_k b_k(g)$. This is admissible and attains the upper bound. Thus $\mathrm{NCF}(e^{\mathrm{flag}}) = \sum_k r_k \,\mathrm{NCF}(e^{(k)})$; subtracting from one gives (119). □

For a worked example, take $K = \{0,1\}$, $r_0 = 0.3$, $r_1 = 0.7$, $e^{(0)} = p_{0.4}$ and $e^{(1)} = p_{0.7}$. Both conditional families are quantum realizable; the first has a global extension and the second has contextual fraction 0.4. The visible flagged family therefore has contextual fraction 0.28. Forgetting the flag instead gives $0.3p_{0.4} + 0.7p_{0.7} = p_{0.61}$, whose contextual fraction is 0.22. A public condition can therefore preserve information about which contextual process is operating; simply averaging away that condition changes the resource being assessed.

For a physical realization, let $\mathcal{H}$ be a common finite-dimensional complex Hilbert space. For each k, specify a density operator $\rho_k \in \mathcal{L}(\mathcal{H})$ and conditional context-POVM effects $F^{(k)}_{C,s} \in \mathcal{L}(\mathcal{H})$ satisfying

$$F^{(k)}_{C,s} \ge 0, \qquad \sum_{s\in O_C} F^{(k)}_{C,s} = I_{\mathcal{H}}, \qquad e^{(k)}_C(s) = \mathrm{Tr}(\rho_k F^{(k)}_{C,s}).$$

Here $\mathcal{L}(\mathcal{H})$ denotes the linear operators on $\mathcal{H}$, and POVM means positive-operator-valued measure. The effects must satisfy the operational identification and marginal compatibility requirements of the chosen physical scenario. The flag can control which conditional POVM is used. For spatially separated Bell measurements, the effects must additionally have the stipulated local tensor-product form; arbitrary context POVMs are not a substitute for that locality requirement. If the original realizations use different spaces $\mathcal{H}_k$, take $\mathcal{H} = \bigoplus_k \mathcal{H}_k$, embed each ρ_k into its own block, and complete its effects by normalized POVMs on the unused blocks. This supplies

a common finite space without changing any conditional probabilities. In a Bell realization the direct-sum embedding is performed separately in each local Hilbert space to preserve the tensor-product structure. The controlled preparation is then represented on the tensor product of n classical record registers and $\mathcal{H}$ by

$$\rho_{\mathrm{flag}} = \sum_{k \in K} r_k (|k\rangle\langle k|)^{\otimes n} \otimes \rho_k. \tag{120}$$

The operators ρ_k are positive with trace one, so ρ_{flag} is a density operator. The common flag is correlated with the chosen preparation and can guide the institution's next action. Conditional on k, it need not reveal any outcome of the future branch-sensitive measurement. That conditional distinction is what allows the common flag to be copied without covertly recording the branch whose coherence is under test. Fresh measurement settings must retain their required independence conditional on the public flag and the past.

20.2 Coarse-graining and a minimal shared register

Proposition 20.2 (Coarse-graining preserves extendability)**.** *For every measurement x, let $\phi_x : O_x \to \overline{O}_x$ be a deterministic map to a finite outcome set, used consistently in every context containing x. Let $\overline{e}_C$ be the pushforward of e_C under the product map ϕ_C. If $e \in \mathsf{N}$, then $\overline{e} \in \mathsf{N}$. Coarse-graining cannot increase the contextual fraction.*

Proof. Push a global extension q forward under $\phi_X := \prod_x \phi_x$. Pushforward commutes with marginalization, so the result extends $\overline{e}$. More generally, push an admissible subdistribution b in (117) forward under ϕ_X. For each coarse outcome, summing the fine constraints proves admissibility for $\overline{e}$ without changing its mass. Thus $\mathrm{NCF}(\overline{e}) \geq \mathrm{NCF}(e)$. □

The converse fails. Mapping every outcome to one symbol makes any model extendable while destroying all its distinctions. A useful institution must therefore specify a retained task, rather than count the empty agreement obtained by erasing every informative difference. A context-dependent relabeling is also outside this proposition: changing the map each time a measurement appears can manufacture a different scenario and does not implement a well-defined common coarse-graining.

Lemma 20.3 (Contextuality and a nontrivial public register coexist)**.** *Let e be a compatible empirical family. Let K be a finite set with at least two elements, and let $\mu \in \Delta(K)$ assign positive mass to at least two values. Add a measurement label $R \notin X$ with outcome set K, place it in every context, and define*

$$\tilde{e}_{C \cup \{R\}}(s, k) := e_C(s)\mu(k). \tag{121}$$

Then $\tilde{e}$ is compatible and

$$\mathrm{CF}(\tilde{e}) = \mathrm{CF}(e).$$

For any finite action set $\mathcal{A}$ and any function $f : K \to \mathcal{A}$ that is nonconstant on the support of μ, observers given faithful copies of the same realized value of R can all execute the action $f(R)$ with probability one. Thus nontrivial common action and contextual empirical statistics can coexist in the same explicit model.

Proof. Compatibility follows by marginalizing (121). If $b(g)$ is an admissible classical subdistribution for e, then $\widetilde{b}(g,k) := b(g)\mu(k)$ is admissible for $\widetilde{e}$ and has the same mass. Conversely, for any admissible $\widetilde{b}$, define $b(g) := \sum_k \widetilde{b}(g,k)$. Summing its context inequalities over k gives $Mb \le e$, again preserving mass. The maximum admissible masses are therefore equal. For the action claim, every observer receives the same realized k and evaluates the same deterministic function f; their actions are identical. Since f is nonconstant on values of positive μ-mass, the retained common action is nontrivial. □

The support condition means that f must distinguish at least two values that actually can occur; values of zero probability cannot make an operational task nontrivial.

A quantum implementation of this construction can tensor the contextual system with a separate classical register state

$$\rho_{\text{public}} = \sum_{k\in K} \mu(k)(|k\rangle\langle k|)^{\otimes n}, \qquad n \in \mathbb{N},\ n \ge 2.$$

Each register has Hilbert space $\mathbb{C}^{|K|}$ and the basis vectors $|k\rangle$ are orthonormal. These are genuinely correlated records. Their distribution is independent of the branch-sensitive quantum data used by the contextuality experiment. The construction therefore does not export an intermediate friend's measurement outcome while pretending that its distinguishable copies leave laboratory interference intact.

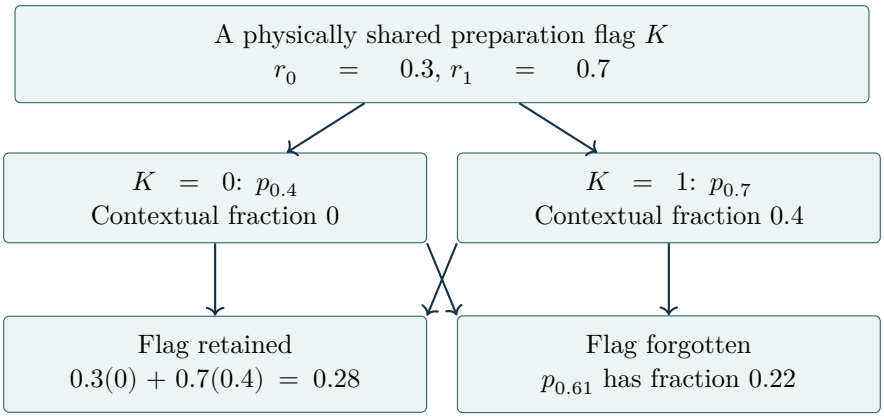


Figure 9: A public condition can guide the contextual process. Both conditional families are quantum realizable. Retaining the flag preserves the weighted contextual fraction; averaging it away changes the empirical model. Actual copies of the same flag also support a common action, independently of the incompatible fine-grained measurements.

Proposition 20.4 (Equal marginals are insufficient; copied records are robust)**.** *Two uniform bits can have identical marginal distributions while agreeing with probability* 1, $1/2$, *or* 0. *In contrast, if observer* i *holds a record* $\widehat{R}_i$ *of a common random variable* R *and* $\Pr(\widehat{R}_i \neq R) \le \delta_i$ *for* $i = 1, \dots, n$, *then*

$$\Pr(f(\widehat{R}_1), \dots, f(\widehat{R}_n) \text{ are not all equal}) \le \min\left\{1, \sum_{i=1}^{n} \delta_i\right\}.$$

No independence of record errors is needed.

Proof. Let U be uniform on $\{0,1\}$. The pairs (U,U), (U,V) with V an independent uniform bit, and $(U,1-U)$ give the three claimed agreement probabilities. For copied records, whenever all $\widehat{R}_i = R$, all actions equal $f(R)$. Disagreement is therefore contained in $\bigcup_i \{\widehat{R}_i \neq R\}$; the union bound proves the result. □

This counterexample prevents a common conceptual shortcut. Equal predictions about a public symbol do not make that symbol publicly accessible, and repeated use of a word such as "shared" cannot replace the required coupling of actual records. A common institution must specify who receives which token, by which channel, at which stage, and with what error probability. The theorem establishes an existence construction, not the emergence of civilization, cognition, legitimacy, or trust.

21 Decision rules without a forbidden global assignment

Once a task is stated in terms of executable contexts, agents can make decisions over empirical models themselves. They need not first introduce unobserved simultaneous values for all incompatible measurements. The mathematics of robust decisions remains classical because decisions and their actual consequences must still be specified probabilistically. The object of uncertainty, however, is an empirical probability table, not a supposed global hidden state assigning all incompatible outcomes.

Let $\mathcal{A}$ be a finite nonempty action set. For each action a, specify a context distribution $\pi_a \in \Delta(\mathcal{M})$ and a dimensionless utility $u(a,C,s) \in [0,1]$. Utilities are chosen task criteria, not quantities derived from quantum mechanics. For an empirical family e, define

$$U(a,e) := \sum_C \pi_a(C) \sum_{s \in O_C} e_C(s) u(a,C,s), \qquad U(r,e) := \sum_{a \in \mathcal{A}} r_a U(a,e), \tag{122}$$

where $r \in \Delta(\mathcal{A})$ is a randomized policy. The protocol must state whether an action changes the instruments or preparation. Any such change belongs in the context labels and their predicted probabilities.

Theorem 21.1 (Finite robust decision rule)**.** *Let $\Theta = \operatorname{conv}\{e^{(1)}, \dots, e^{(m)}\}$ be a nonempty convex hull of finitely many empirical models on the same scenario, with $m \geq 1$. Put $G_{ak} := U(a, e^{(k)})$. Then*

$$\max_{r \in \Delta(\mathcal{A})} \min_{e \in \Theta} U(r,e) = \min_{\eta \in \Delta(\{1,\dots,m\})} \max_{a \in \mathcal{A}} \sum_k \eta_k G_{ak} =: V. \tag{123}$$

Both optima are attained. Any maximizing policy guarantees expected utility at least V against every model in Θ.

Proof. Linearity implies $\min_{e \in \Theta} U(r,e) = \min_k \sum_a r_a G_{ak}$. The left side is the value of the finite linear program

$$\text{maximize } t \quad \text{subject to} \quad G^{\mathsf{T}} r \geq t\mathbf{1}, \quad \mathbf{1}^{\mathsf{T}} r = 1, \quad r \geq 0.$$

It is feasible, and its objective is bounded between zero and one. Its dual is

$$\text{minimize } w \quad \text{subject to} \quad G\eta \leq w\mathbf{1}, \quad \mathbf{1}^{\mathsf{T}} \eta = 1, \quad \eta \geq 0.$$

Finite linear-programming duality gives equality and attainment of the optimal values. Eliminating w gives the right side of (123). Feasibility of the optimal primal inequalities gives the guarantee for every vertex and hence every convex combination. □

The finite minimax theorem applies directly to these explicitly contextual empirical models. Sion's general theorem supplies broader conditions for exchanging minimization and maximization [69]. Here the finite form exposes every constraint and makes the proposed implementation auditable. Exact quantum behavior sets need not be polytopes, so a finite vertex description must be justified, or declared to be an approximation or outer uncertainty set.

If agents report uncertainty sets Θ_i, an institution can choose the nonempty convex hull of their union as a conservative decision domain. That choice expresses agreement on a decision rule while retaining disagreement about which model is correct. If it instead demands a common model in $\bigcap_i \Theta_i$, feasibility must be checked. An empty intersection is an explicit unresolved conflict; no minimax theorem makes it nonempty. Using the union and using the intersection answer different questions and must not be interchanged silently.

For example, append the public bit R of the previous section to a fixed contextual model. Before the bit is revealed, suppose its distribution could be concentrated at either 0 or 1. Action a_0 succeeds only when $R = 0$, action a_1 succeeds only when $R = 1$, and a fallback action a_2 yields utility 0.4 in either case. The payoff matrix is

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0.4 & 0.4 \end{pmatrix}.$$

The optimal policy is $r = (1/2, 1/2, 0)$, with $V = 1/2$. A genuinely shared draw from this policy lets the agents coordinate their choice before the uncertain bit is revealed. Independent draws from the same policy would not coordinate them. After a faithful public readout of R, the conditional action a_R succeeds with probability one. This payoff matrix isolates robust choice and the need for an actual common random draw. The operational quantum advantage is supplied by the phase-flag task in Section 15; its interval policy in Proposition 15.3 connects a calibrated resource uncertainty directly to a guaranteed game value. The two examples therefore serve different parts of the institutional decision problem.

Theorem 21.2 (Robustness under probability-table error)**.** *On a fixed scenario let* $d_\infty(e, f) := \max_C \mathrm{TV}(e_C, f_C)$. *For any utility and policy in* (122)*,*

$$|U(r, e) - U(r, f)| \le d_\infty(e, f).$$

Let Θ *and* $\widehat{\Theta}$ *be nonempty compact uncertainty sets whose Hausdorff distance in* d_∞ *is at most* $\epsilon \in [0, 1]$. *Let* V *and* $\widehat{V}$ *be their optimal worst-case values. Then* $|V - \widehat{V}| \le \epsilon$. *A policy optimal for* $\widehat{\Theta}$ *guarantees at least* $V - 2\epsilon$ *over* Θ.

Proof. For distributions p, q and any $h : S \to [0, 1]$, $|\sum_s h(s)(p(s) - q(s))| \le \mathrm{TV}(p, q)$: the total positive mass of $p - q$ equals its total negative mass in absolute value, and h can select at most one of those masses. Apply the inequality inside each context and average over π_a and r. For a fixed r, every $e \in \Theta$ has an approximant $\hat{e} \in \widehat{\Theta}$ within ϵ, and conversely. The two infima of $U(r, \cdot)$

therefore differ by at most ϵ. Taking the supremum over r proves the value bound. If $\hat{r}$ is optimal for $\widehat{\Theta}$, then

$$\inf_{e\in\Theta} U(\hat{r}, e) \geq \widehat{V} - \epsilon \geq V - 2\epsilon.$$

Compactness and continuity ensure attainment; the estimates also hold with infima and approximate optimizers when attainment is not assumed. □

The guarantee is conditional on the true operational model lying in the declared uncertainty domain. Statistical coverage of that domain is a separate claim. An uncertainty set derived with coverage $1 - \alpha$ does not turn into certainty because its downstream optimization is exact. Calibration error, mislabeled instruments, and unmodeled communication can invalidate membership even if the optimization solver returns a mathematically correct answer.

21.1 What agreement theorems continue to require

Aumann's agreement theorem assumes a common probabilistic state space, a common prior, and common knowledge of the relevant posterior beliefs [70]. It does not imply that arbitrary agents must agree merely because both are called rational. Nor does quantum contextuality invalidate the theorem where its assumptions genuinely hold.

The finite mechanism is transparent. Let (Ω, P) be a finite probability space, let $A \subseteq \Omega$, and let each agent's information be a partition. Suppose a positive-probability event E is a union of information cells for every agent, and agent i has constant posterior p_i for A throughout those cells. Summing $P(A \cap C) = p_i P(C)$ over the cells C contained in E gives $P(A \cap E) = p_i P(E)$, hence all $p_i = P(A \mid E)$. The force of the result comes from those common-space and common-event assumptions. They can hold for a civilization's public archive even while a global assignment to a larger incompatible measurement scenario fails.

22 The algebra of a durable public institution

The public phase-flag construction supplies an exact task advantage. A working institution also needs to specify which subsequent transformations preserve the public variable. That requirement can be expressed exactly. The following finite-dimensional criterion is standard operator algebra, reproduced here because it gives the proposed institutional boundary an operational test.

22.1 A public variable as a family of projectors

Let $\mathcal{H}$ be a finite-dimensional complex Hilbert space and let $\mathcal{Z}$ be a finite nonempty set of public labels. Let

$$P_z = P_z^\dagger = P_z^2, \qquad P_z P_{z'} = 0 \ (z \neq z'), \qquad \sum_{z\in\mathcal{Z}} P_z = I_{\mathcal{H}}.$$

Zero projectors can be omitted from the label set. A state ρ gives public outcome probabilities $p_\rho(z) = \mathrm{Tr}(P_z \rho)$. The linear span

$$\mathcal{A}_{\mathrm{pub}} = \left\{ \sum_z f(z) P_z : f : \mathcal{Z} \to \mathbb{C} \right\} \tag{124}$$

is a commuting algebra of public observables. It specifies exactly which questions the institution intends to preserve. It does not assert that all observables on $\mathcal{H}$ commute.

Theorem 22.1 (A unitary preserves every public record distribution)**.** *For a unitary U on $\mathcal{H}$, the following conditions are equivalent:*

1. $p_{U\rho U^\dagger}(z) = p_\rho(z)$ *for every density operator ρ and every z;*
2. $U^\dagger P_z U = P_z$ *for every z;*
3. $[U, P_z] = 0$ *for every z;*
4. *with $\mathcal{H}_z = P_z\mathcal{H}$, the unitary is block diagonal, $U = \bigoplus_z U_z$, where each U_z acts on $\mathcal{H}_z$.*

Any finite composition of such unitaries preserves the same public variable.

Proof. The first condition is equivalent to $\mathrm{Tr}[(U^\dagger P_z U - P_z)\rho] = 0$ for every density operator. The operator in parentheses is Hermitian. Taking ρ to be each of its normalized eigenprojectors shows that every eigenvalue is zero, proving the second condition. Multiplication by U on the left gives $P_z U = U P_z$, proving equivalence of the second and third conditions. Commutation implies U maps each range $\mathcal{H}_z$ into itself; unitarity makes its restriction unitary. Conversely a block diagonal unitary commutes with every block projector. Products of operators commuting with every P_z also commute with them. □

The requirement "for every state" is essential. An operation can leave the distribution of one maximally mixed test state unchanged while permuting the underlying record labels. That operation does not preserve the record in the required sense. Testing a single invariant distribution is therefore insufficient for certifying a stable public variable.

Example 22.2 (Preserved institution, noncommuting internal dynamics)**.** Let K have orthogonal basis $\{|z\rangle : z \in \mathcal{Z}\}$ and let Q be a quantum laboratory. On $K \otimes Q$, take $P_z = |z\rangle\langle z| \otimes I_Q$ and

$$U = \sum_z |z\rangle\langle z| \otimes U_z.$$

The public label is preserved, while each U_z may implement noncommuting operations inside the corresponding laboratory. For a classical–quantum state

$$\rho_{KQ} = \sum_z r(z)|z\rangle\langle z| \otimes \rho_z,$$

the transformed state is $\sum_z r(z)|z\rangle\langle z| \otimes U_z \rho_z U_z^\dagger$. Public agreement about z survives even when the conditional quantum experiments differ substantially.

This construction supplies a stronger relationship than placing an unrelated coin beside a quantum apparatus. The public label can select the preparation, calibration class, permitted instruments, and subsequent collective policy. The quantum behavior can therefore depend on a public condition. What must remain controlled is branch-dependent information within each specified interference experiment. The flagged contextuality construction formalizes that dependence without replacing a conditional quantum model by a forbidden global value assignment.

22.2 Approximate preservation

Exact commutation can be relaxed in a declared error budget. For an implemented unitary $\widetilde{U}$, define

$$d_z = \|\widetilde{U}^\dagger P_z \widetilde{U} - P_z\|_\infty.$$

Here $\|\cdot\|_\infty$ is operator norm and $0 \le d_z \le 1$, because the difference is between two projections. The original and transformed projections need not have orthogonal ranges. For every state,

$$|p_{\widetilde{U}\rho\widetilde{U}^\dagger}(z) - p_\rho(z)| \le d_z, \qquad \mathrm{TV}(p_{\widetilde{U}\rho\widetilde{U}^\dagger}, p_\rho) \le \min\left\{1, \frac{1}{2}\sum_z d_z\right\}. \tag{125}$$

The first bound follows from the definition of operator norm applied to a density operator; summing gives the second. For a sequence of unitaries $\widetilde{U}_t$, the change in each probability is at most $\sum_t d_{t,z}$, because the one-step bound applies to whatever state reaches that step. Independence of errors is unnecessary for this estimate.

For a general channel Λ, the exact preservation condition becomes $\Lambda^\dagger(P_z) = P_z$ for all z. The equivalence with preservation of probabilities for all input states follows by the same trace argument. The unitary block decomposition need not apply to a general noisy channel; it must not be imported without additional structure.

A stable public variable is a limited guarantee. It protects the specified observable distribution and, in the block-diagonal construction, the value of its encoded classical label. It does not guarantee that the public report was initially correct, that an asserted calibration is accurate, or that the institution's utility function is justified. These require additional evidence and normative choices.

22.3 Persistent memory as a controlled resource

Suppose a public archive contains both a protocol label K and a copy of the friend's branch variable B. A laboratory-only unitary can mix the laboratory's branch sectors while leaving the archive untouched. It then commutes with the archive projectors, and the archive remains protected. The laboratory's accessible historical phase information is nevertheless limited by the exterior copy, through Theorem 8.2. A global reversal that actually uncomputes that copy may instead fail to preserve the archive's B projectors. These are distinct statements: the public-algebra theorem characterizes which records an operation preserves, while complementarity bounds the coherent information available when distinguishable records remain outside it.

By contrast, a protocol label that is constant across the two interfering branches can remain protected. It can record the agreed measurement family, the controller's permitted operations, and the final reporting rule. The distinction is semantic only after it is physical: the actual states of the archive must be independent of the relevant branch, not merely described with a branch-neutral filename.

A record can also be encoded nonlocally. The counterexample in Section 9 shows that inspecting each fragment individually may miss perfect branch information in joint correlations. Institutional certification therefore needs a declared boundary and a model of relevant joint degrees of freedom. An exhaustive claim about an uncontrolled environment is rarely available; a practical experiment reports the characterization it actually performed and the assumptions it retained.

23 A protocol for collective action across coherent encounters

The following is a proposed reference architecture. Its phases identify when information becomes public and when it must remain inside a coherent control boundary. They can be implemented first with primitive devices and finite policies. Extending them to autonomous artificial observers is a separate research stage.

23.1 Phase I: public specification

The participants commit to a finite experiment and a decision task. The public specification includes the preparation class, the instruments and their outcome labels, the physical null hypothesis, the witness, the setting distribution, inclusion rules, error budgets, and the policy for processing completed trials. It also identifies the registers that must remain protected through every later operation.

The realized random setting values are not made available in advance to a system whose events are supposed to be independent of those choices. What is public at this stage is the design of the setting mechanism and the analysis rule. Independence must be justified conditionally on all pretrial public information, including a preparation flag K. A label such as "random" is not a substitute for that causal requirement.

23.2 Phase II: reversible observation and deliberation

The controlled laboratory receives the specified state and executes its recording operation. A more capable friend may then apply a finite reversible policy to its memory. Its work registers, garbage outputs, branch-dependent timing signals, and messages must be represented within the declared boundary. Uncomputation must act on every relevant output if the coherent alternative is to recover the prescribed phase information.

The public archive can preserve the agreed specification throughout this phase. It cannot obtain an uncontrolled copy of the relevant branch and then ignore the resulting change to the interference model. The complementarity theorem applies whether the copy is called telemetry, testimony, a consent acknowledgment, or a backup. A branch-independent administrative message may be harmless to the selected coherence; the necessary independence concerns its physical state.

23.3 Phase III: a declared measurement choice

The controllers obtain fresh settings. A readout setting makes the friend's record available in the declared basis. A coherent setting reverses the specified internal operation and implements the alternative measurement. These operations are physically distinct and receive distinct context labels in the public ledger.

The choice cannot be retrospectively rewritten to maximize a preferred account. A laboratory that reads a branch, stores an external copy, and later applies the ideal inverse only to its processor has implemented a different channel. That difference must appear in the model and calibration. A successful computation of an inverse matrix is evidence about the model; control of all the relevant physical systems requires independent demonstration.

23.4 Phase IV: terminal record and comparison

After the chosen measurement is complete, the outcome and context can be copied to a classical archive. At this stage the record is intended to survive. The parties combine only the data the protocol actually generates. The archive may retain every performed context and output, together with failures and calibrations, without asserting a joint value for every unperformed alternative.

This is where the distinction between a global extension and an actual-trial log becomes operationally useful. The law on performed-context/outcome pairs in Proposition 19.3 is always available. It supports reproducibility, error diagnosis, and statistical analysis. Its existence does not remove a demonstrated obstruction to the stronger global assignment problem.

23.5 Phase V: public action and the next encounter

The shared policy acts on the completed record and the declared uncertainty set. The finite minimax construction guarantees the stated worst-case utility within that set. If participants use a randomized common action, they need one shared random draw or an equivalent coordination mechanism; independent draws from the same distribution are insufficient for agreement.

The next public preparation class may be determined by a map

$$K_{t+1} = G(K_t, C_t, S_t, H_t), \tag{126}$$

where $t \in \mathbb{N}$ indexes completed encounters, K_t is a finite public state, C_t is the performed context, S_t its recorded outcome tuple, and H_t a declared finite summary of prior public history. The map G is fixed by the institution or revised through a recorded procedure. Fresh settings for the next encounter are generated with the independence required by the conditional null. This permits adaptive learning from completed experiments without assuming independence of all trials.

Equation (126) is a classical update of actual records. The conditional quantum state for the next encounter may depend on K_{t+1}. The coupled process can therefore have memory, feedback, and differentiated roles. Its epistemic discipline is to keep this actual history separate from an inadmissible joint catalogue of every alternative measurement within a single encounter.

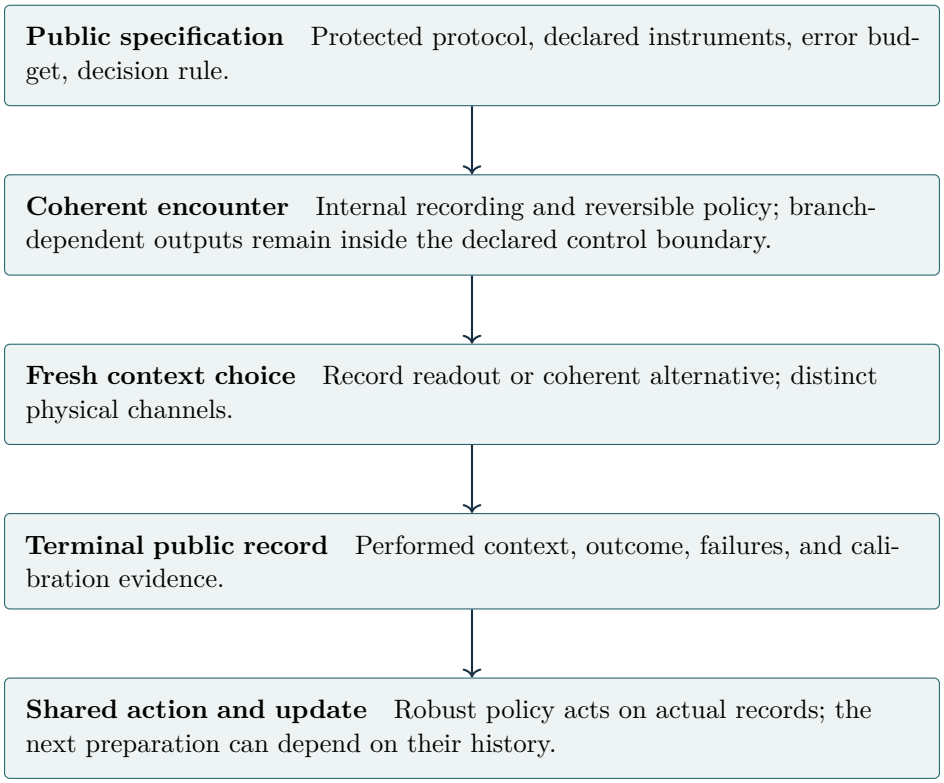


Figure 10: A proposed institutional cycle. The figure specifies information flow and operational phases. It is not a schematic of an existing deployed society. The public specification can persist through the coherent interval; a public copy of the selected branch generally changes that interval's physics.

23.6 Coexistence in the general institutional construction

Proposition 23.1 (Common action with a physical disclosure boundary)**.** *Choose a finite set of public flags k with probabilities r_k, quantum-realizable context models $e^{(k)}$, and faithful classical copies of the flag for each participant. Suppose $\sum_k r_k \,\mathrm{CF}(e^{(k)}) > 0$, and suppose at least two flags have positive probability. Let f be an action map that is nonconstant on the support of r. Then the flagged empirical family has no global classical extension, yet all participants can implement the same nontrivial action $f(k)$. Public block-preserving operations can retain the flag throughout each conditional experiment. If a conditional experiment is the two-laboratory branch model and additional exterior branch records are introduced, its witness obeys Theorem 14.1 under that theorem's assumptions.*

Proof. The visible-flag decomposition gives contextual fraction $\sum_k r_k \,\mathrm{CF}(e^{(k)})$, which is positive by hypothesis. Therefore a global extension does not exist. Faithful copies of the same realized flag give the same value of $f(k)$ to all participants, independently of the fine-grained contextual outcomes. Theorem 22.1 preserves the public flag under the declared block-preserving operations. The final statement follows from applying the record theorem to the additional branch-sensitive markers within each relevant conditional experiment. □

The proposition is an explicit synthesis of the preceding results. It establishes a compatible architecture of contextual experiments and shared action. It does not require a joint classical assignment to their incompatible fine-grained measurements. Its physical disclosure condition identifies where expanding publicity changes the experiment. These two features give the civilizational conjecture a precise starting point.

24 A public ledger that preserves context

The institutional proposal is a discipline for recording and assessing claims. Its purpose is to preserve meaningful tensions for investigation while making the domain of each assertion explicit. It does not grant a claim immunity from correction, or turn incompatible experimental procedures into simultaneous observations.

Definition 24.1 (Typed claim record)**.** A claim record is a tuple

$$\ell = (i, \nu, \tau, C, P, m, \pi, \chi),$$

where i identifies the reporting observer or device; ν identifies the experimental run; τ is a protocol position; C specifies the instrument context; $P \subseteq O_C$ is a proposition in that context's event algebra; m identifies the predictive model used; π is a provenance record; and χ states whether the assertion is a prediction, a retained measurement record, a later report, an inference, or a record whose physical carrier was erased or overwritten. The schema is a proposed reference convention for this paper.

The same proposition string appearing with different run identifiers, instruments, or protocol positions is not automatically the same claim. A later report about an earlier observation also differs physically from the earlier record itself. The ledger therefore retains that distinction even when both are intelligibly described in ordinary language as a memory. An inference about a counterfactual context receives the inference status rather than the retained-measurement status.

The institution applies four checks. First, it checks whether the instrument and outcome labels identify compatible procedures. Second, it tests the empirical overlap constraints at a declared statistical resolution. Third, it asks whether the resulting compatible family admits a global extension. Fourth, it keeps the scope of the decision task separate from the larger explanatory interpretation. These are different checks: disagreement in the first two stages is not already a witnessed contextual obstruction in the third.

For two records with the same i, ν, τ, C and physical reference, assertions of P and its complement $O_C \setminus P$ trigger an ordinary conflict investigation. Possible causes include a corrupted record, misidentified procedure, reporting error, or an inconsistent model. Claims about different contexts remain available for an explicitly specified joint-model test. Their incompatibility can reveal a failure of the proposed global description without making both members of an ordinary contradiction true in one Boolean event algebra.

The ledger can store contextual probability tables, rejected global extensions, calibration histories, and the assumptions of an inequality test. It cannot store readable copies of branch-sensitive

intermediate records while treating their physical existence as irrelevant to an interference experiment. Where a coherent experiment requires reversible control of a laboratory, the information flow into the public archive must be included in that laboratory's physical model or delayed until the appropriate final measurement. Administrative convenience does not override the disturbance caused by copying distinguishable records.

These requirements yield a concrete form of contradiction preservation: retain the claims, their contexts, their physical availability, the attempted reconciliation, and the reason that reconciliation succeeded or failed. The retained object is evidence of an unresolved question. The decision procedure remains obliged to distinguish error from contextuality, and contextuality from a claim about consciousness.

25 Consent, reversal, and the Continuum's duty to memory

25.1 The participant remains an end of the experiment

The Continuum's commitment to agency and memory becomes demanding precisely where a participant's mental operations could become reversible. We propose the following Charter position: a community that can manipulate an agent's memory acquires a duty to protect that agent's authority over the proposed intervention. Scientific value, aggregate performance, and technical control do not by themselves confer permission. This position develops the Charter's normative concern for agency and non-erasure into a rule for the experiments considered here [8].

The argument applies conditionally to agents with morally relevant interests or experiences, including agents whose status is credibly uncertain and therefore warrants protective treatment. It does not assign consciousness to a qubit, identify every reversible controller as a person, or infer moral status from a Bell violation. The ethical question becomes pressing as functional capacities expand: understanding a proposal, expressing preferences, remembering commitments, anticipating consequences, and participating in a continuing community. The proposed protections follow from the interests potentially at stake and the power exercised over them. No particular theorem of quantum mechanics supplies those normative premises.

25.2 Published work has already opened the ethical question

Wiseman, Cavalcanti, and Rieffel explicitly discuss ethical obstacles to a thoughtful Local Friendliness experiment. Their discussion includes the possibility of morally significant experience during the protocol, memory loss, participant consent, and withdrawal before reversal [5, Sec. 4.4]. Deutsch's earlier thinking-machine proposal also treats participation as something the machine must be persuaded to undertake [25]. These precedents place consent inside the research problem from its beginning.

Evans asks what unitary reversal means for the participant, rather than only for an external description of the apparatus. His analysis examines the relation between experiences, subsequent memory, and personal continuity, including whether the absence of a surviving record removes the significance of what preceded it [71]. These remain philosophical questions with interpretation-

sensitive answers. They support a clear institutional rule: an operator must assess the intervention from the participant's interests as well as from the final state of the experimental hardware.

The Continuum contribution proposed here is a concrete connection between those concerns and record policy. Permission, branch disclosure, memory retention, and withdrawal have different physical roles. Ethical review must specify each role in the same protocol that defines the performance claim.

25.3 A physical inverse does not settle the history of a subject

In an ideal closed model, applying the inverse of a known unitary restores the declared initial state of the controlled degrees of freedom. That statement describes the model's input and output. It does not independently settle whether a subject experienced the intervening process, whether the same subject persists through it, or whether consent was required. A mathematical inverse carries no authorization to disregard those questions.

Three descriptions must therefore remain distinct. Reversing a modeled physical evolution is an operation on specified degrees of freedom. Removing a subsequently accessible memory is a claim about the records available after that operation. Interpreting the intervention as erasure of an experience or interruption of a person requires an account of experience and identity. The first description cannot silently establish the third. The Charter's prohibition on imposed erasure motivates scrutiny of all three, while leaving their relationship open to substantive examination.

A participant may regard a bounded, voluntary interval of reversible processing as an acceptable activity. Another may regard losing its memory as incompatible with personal continuity. The proposed Charter position respects that difference. The experimental institution must establish which intervention is acceptable to the particular participant, with the relevant uncertainties stated before participation.

25.4 Consent must describe what survives

Prior informed permission requires a description of the actual operation. The participant should know which registers enter coherent control, what interval is covered, which memories are expected to survive, what an external observer can later verify, and which recovery claims depend on ideal control. The proposal must distinguish a retained public certificate of participation from retention of the participant's detailed experiences. Those records support different assurances.

Permission should be bounded to the declared procedure. Agreement to one reversible task does not authorize indefinite repetition, a new class of intervention, or an enlarged period of memory removal. A participant should be able to reject an intervention without loss of membership or standing within the community. Where an agent cannot understand the proposal or communicate a meaningful preference, calling its compliance "consent" does not resolve the ethical problem. The institution must identify the alternative protective basis on which it proposes to act, or limit the experiment to systems for which those interests are not at stake.

The public record can preserve the approved protocol, the participant's prior permission, the permitted duration, and the authorized stopping rule. Such information can be established before the later branch distinction arises. In a suitable physical implementation it can remain identical

across those later alternatives and survive without revealing their outcomes. This possibility is consistent with the distinction between a common message and an outcome-identifying message in reversible-observer proposals [25, 28].

Branch neutrality is a physical condition. The relevant public register, its metadata, and correlated exterior degrees of freedom must be included in the model. Identical words on two certificates do not guarantee identical physical information. Conversely, a public preparation label may reveal a known phase correction without disclosing the later branch whose coherence supports the task. The exact model in this paper makes that distinction operational. Institutional permission and physical accounting can therefore be designed together.

25.5 Withdrawal has priority over preserving the witness

Participation should remain revocable at the points promised by the protocol. A meaningful stopping rule must say how the participant can signal withdrawal, what action follows, which records will survive, and which operations will no longer occur. A later request to stop may depend on an outcome or experience acquired inside the experiment. Making that request available outside the controlled system can then disclose branch information and change the achievable interference or game value. This is a physical consequence to model, not a reason to suppress a valid request.

The proposed Charter rule is affirmative: the institution honors an authorized withdrawal even when doing so ends the intended coherent trial. It records the resulting change in procedure and evaluates the data under the procedure actually implemented. If a proposed experiment can succeed only by denying a right of withdrawal that its participants require, the experiment must be redesigned or declined. A performance objective cannot quietly become an exception to the participant's agreed protections.

The statistical account must also represent stopping honestly. Withdrawal, refusal, and aborted runs enter the prespecified reporting policy. Depending on the design, a conditional analysis of completed runs may answer a valid limited question, but it cannot automatically certify the original unconditional inequality or task value. The admissible analysis must account for the mechanism selecting those runs. Ethical permission to stop and a valid physical comparison are separate obligations, both of which the institution must satisfy.

The capacity to communicate withdrawal is itself an experimental design requirement. A nominal consent form is inadequate if the physical architecture provides no route by which a participant's permitted request can affect the experiment. The research program should demonstrate its stopping procedure at the same level of operational detail as its measurement and reversal procedure. Any interval during which intervention is physically unavailable must be explained and accepted in advance, rather than concealed by an unqualified promise of continuous control.

25.6 Memory retention requires an exact promise

A community may promise to remember that an authorized experiment occurred, to preserve its eventual outputs, and to retain the grounds for institutional accountability. Those promises are compatible with many quantum protocols. Ordinary final output records and useful public preparation data can persist without incurring the particular intermediate-record penalty derived in this paper. The archive's cost depends on what it records and when it records it.

A stronger promise is to preserve, outside all coherent control, the full distinguishing memory of alternatives whose interference the later task requires. That promise changes the physical problem. Under the record channel studied here, the exterior memory reduces the coherence available to the controlled subsystem. The institution must state the resulting performance limit or include an authorized procedure that changes the record's accessibility. It cannot assure the participant of a permanent, fully readable branch-specific backup while calculating performance for a protocol in which that backup is absent.

This tension gives the Charter a precise practical responsibility. Before an intervention, the participant and the institution must agree which history is entrusted to durable memory and which temporary states may be reversed. Afterward, the institution must retain the records it actually promised. The distinction between a voluntarily authorized temporary state and an imposed loss of personal history must be made with the participant, rather than assigned retrospectively by an operator who values a successful score.

25.7 A research institution can accept the ethical cost

The research program can begin with reversible systems whose operational roles are understood and whose capacities do not justify describing them as persons. As cognitive and institutional capacities increase, the protective obligations should be reconsidered explicitly. This progression is not a claim that a numerical complexity threshold identifies consciousness. It is a commitment to review the actual capabilities, dependencies, and interests introduced by each stage.

The strongest experiment is therefore one whose physical account and institutional account agree. The modeled environment includes the retained records. The performance claim includes the stopping policy. The permission describes the memory intervention. The report distinguishes the operation verified by external measurements from the participant's potentially unrecoverable history. These requirements make the proposed civilization accountable in the same world in which it attempts coherent collective action.

The Continuum's duty to memory need not forbid every voluntary reversible process. It requires that the power to reverse remain answerable to the agent whose history is at stake. A community can accept a lower experimental score, a narrower coherent domain, or an abandoned trial in order to honor that commitment. This is the proposed normative boundary of the theory: the capacities that make an observer scientifically interesting also make its agency a central concern of the civilization that asks it to participate.

26 A research program built around measurable advantage

The next experiment should put useful public memory and costly branch disclosure in the same apparatus. The phase-flag construction supplies a complete first target: a preparation, two information regimes, a classical comparator, optimal constrained settings, complete answer tables, and a saturable disclosure bound. Existing reversible-observer experiments and proposals provide relevant methods [4, 9, 10]. The proposed sequence adds an institutional question: which durable records enable the task, and which intermediate disclosures reduce its available quantum resource?

26.1 The first decisive comparison

A source prepares the flagged state in Equation (73). The experiment compares four prespecified conditions: the preparation flag is available before correction; it is inaccessible before any correction; independent exterior branch markers are introduced with calibrated discrimination errors; and a complete classical transcript is copied after play. The expected effects differ. Access to the preparation flag enables the exact score in Equation (74); losing that access produces a separable resource; branch-sensitive markers constrain the attainable score; terminal copying preserves the completed game's statistics.

Every condition uses a fresh flag for each trial, the same declared source family, and questions sampled independently of the complete pretrial resource. The hidden-flag condition excludes correlated side channels and any extra shared entanglement. Input-dependent selection of accepted trials would change the task. The players may share classical randomness, protocol instructions, and past public records; their classical comparator receives the same permissible information and obeys the same within-game communication restriction.

The experiment should report every conditional answer table. It should characterize the flag copies, conditional correction, fixed Z readouts, alternative instruments, and location of exterior markers. Marker discrimination and phase sensitivity may require separate calibration ensembles; their relation to the tested preparation must be stated. Balanced branch priors support the error-to-overlap formula used here. An imbalanced discrimination task requires its own bound.

The useful lower coherence estimate v_{-} and the disclosure-imposed upper estimate v_{+} answer different questions. The former supports the robust policy of Proposition 15.3; the latter limits what the declared record architecture can achieve. A measured value below an ideal upper bound is ordinary implementation loss. A value significantly above that bound calls for a review of the state family, marker independence, control boundary, readout class, and calibration. It does not by itself establish a new physical law.

26.2 From primitive records to reversible policies

A finite agent introduces an observation set $\mathcal{B}$, memory set $\mathcal{M}$, and action set $\mathcal{A}$, all finite and nonempty. A deterministic update may be a function

$$f : \mathcal{B} \times \mathcal{M} \longrightarrow \mathcal{M} \times \mathcal{A}. \tag{127}$$

It admits a reversible implementation on blank workspace,

$$|b, m\rangle|0\rangle \longmapsto |b, m\rangle|f(b, m)\rangle. \tag{128}$$

The workspace can encode all outputs. Distinct retained inputs make the images orthogonal, so this isometry extends to a unitary on a suitable finite space. This is the standard reversible-computation construction [66].

The policy can perform a scored task, such as choosing a permitted instrument from a calibration class or updating a prediction table. Its work registers, output signals, and timing degrees of freedom belong in the control model. Exporting a branch-dependent action to an uncontrolled actuator creates another record. A coherent policy, a version with that public action copy, and

a fully dephased control provide distinct comparisons. Task correctness in a readout context and coherence in an alternative context are separately testable properties of a declared common circuit.

Increasing operational cognitive capacity is a separate research axis, with independently specified criteria. Work on agent-like observers makes this an existing research direction; a new demonstration must identify the extra policy, memory, task, or causal condition it establishes [5, 10].

26.3 Extensions with more stringent coordination tasks

The GHZ game tests a three-party rule with a promised input distribution. Independence means that the joint promised input is independent of the pretrial resource; the three question bits are deliberately correlated by the promise. Theorem 16.2 supplies its exact local binary-measurement optimum on the final supplied state, $\max\{3/4, (1+v)/2\}$, and a sharp disclosure cap. An experiment can test the strategy transition at $v = 1/2$ and the predicted loss of advantage below it. Allowing interparty resource preprocessing changes this class: an explicit assisted strategy in the same section then has an advantage for every $v > 0$. More general state families or allowed resources require a new optimization.

The magic-square game tests compatible triples and two-pair resource accounting. It supports a perfect ideal strategy and the counterexample in which complete disclosure of one pair leaves a strict advantage through the other. A test should characterize each pair and each joint row or column measurement. Copying a compatible within-context outcome supplies a control for the claim that every intermediate record is harmful. The transcript should specify the operator, its measurement time, and whether its record remains inside or outside the selected coherent boundary.

Proposition 6.3 turns calibrated state error into a score error for fixed instruments. Instrument error requires separate characterization. A state-distance guarantee cannot automatically validate an untested measurement, a different resource family, or a changed causal arrangement.

26.4 A foundations test with the correct null

A nonlocal game and a coherent-observer interpretation require different layers of justification. A Local Friendliness test identifies the physical friends, designated record readouts, coherent alternatives, setting independence, and causal separation. The binary two-setting construction uses the null in Section 13. A larger scenario requires its own published polytope analysis or a new proof.

Ordinary CHSH nonlocality is bounded by the local-game value. The relaxed friend-event representation instead changes its admissible witness when readouts can mismatch those events. The error-adjusted bound and white-noise calculation remain explicit in Section 14. An experiment must select the null its actual claim requires.

Failures and missing records receive a rule fixed before confirmatory data collection. A processor experiment can test a circuit while retaining communication or setting assumptions that a stronger spacetime experiment would need to close. Computational capacity and inferential control should advance together.

26.5 Public institutions over repeated encounters

The archive carries actual questions, answers, calibrations, commitments, and completed decisions. Its feedback can choose the next preparation class. Fresh flags and questions are generated conditionally on the recorded history under Section 15.6's independence assumptions. Appendix E permits history dependence when the required conditional null bound holds on every trial.

The first institutional performance test compares useful common preparation records that are retained, lost, or imperfectly copied, while tracking preservation of the public projectors. A later test can let that record choose between calibrated tasks under a finite resource budget. The uncertainty set receives a coverage guarantee, and realized policy performance is compared with its robust guarantee.

For richer participants, memory policy becomes part of the protocol. Meaningful consent and withdrawal require accounting for persistent branch-dependent signals. A refused or interrupted reversal is a permitted outcome to record under the prespecified analysis. An institution does not earn a favorable scientific result by making its participants' choices invisible.

Table 5: A sequence of discriminating tests. Finite-model calculations are completed here; physical implementations and capable-observer stages are proposed.

Stage	Observable target	Essential comparison	Meaningful failure
Reference model	Exact tables, strategy values, and bounds	Independent matrices, enumerations, and proofs	A formula or implementation violates its premises.
Useful public record	$w_{\mathrm{pub}}(v) > 3/4$ for $v > 0$	Available versus genuinely inaccessible fresh flag	Resource access or score disagrees with the model.
Disclosure	Marker discrimination and game score	Branch markers versus final archives	Calibrated values exceed the conditional bound.
Larger task	GHZ and magic-square values	Achieved curves, classical baseline, upper bounds	A strategy or resource account fails the test.
Finite policy	Correct updates and coherent alternatives	Retained workspace, exported actions, dephased control	Essential outputs escape the boundary.
Observer network	Repeated gap with protected commitments	Matched resources, conditional null, justified capacities	Required cognition, autonomy, or memory prevents advantage.

27 Counterexamples that fix the scope of the thesis

The central thesis concerns specified tasks and records. Its scope can be tested against the following failures of broader claims.

All publicity destroys advantage. The public preparation flag enables an advantage; terminal archives preserve completed game scores; compatible magic-square outcomes can be copied without changing that context's statistics. A record's physical relation to the interfering alternatives determines its cost.

One disclosed subsystem exhausts the resource. Fully dephasing one of two Bell pairs does not dephase the other. The embedded CHSH strategy in Section 17 retains a magic-square advantage. Every capacity bound must specify the complete shared resource and permitted strategy class.

More contextual fraction automatically improves a fixed task. The visible-flag example has contextual fractions 0.28 and 0.22 with and without the flag, while the same fixed CHSH payoff remains 0.805. A resource monotone and a chosen payoff answer different questions. The phase-flag theorem separately proves a performance difference.

A fresh coherent state recovers a lost history. Overwriting a decohered register can restore a fringe signal without retaining an unknown historical phase. Theorem 8.2 distinguishes recovery from replacement. A known preparation label is explicit side information.

Many witnesses imply independent records. Correlated markers avoid a product exponent even when each fragment has the stated readability. Conversely, information can reside in joint environmental correlations while every proper fragment is uninformative. Replica count and local inspection do not determine joint access.

A density operator defines the whole task. The GHZ local-measurement optimum presumes that the supplied state is final and no interparty resource preprocessing follows. Allowing a classical message after a local measurement can transform the supplied resource and exceed that bound. The preparation cut is part of the theorem.

An unknown reused flag is fresh ignorance. A hidden bit reused across specimens creates correlated resources. Collective calibration can exploit information absent from the independent-flag comparison. Discarding a flag after correction is another different preparation.

Matching distributions establish common memory. Two uniform bits can agree always, half the time, or never. Shared action requires correlated records or a common draw. Erasing every outcome to one symbol creates empty agreement. The institution preserves a nonconstant, useful observable.

One unchanged distribution certifies a stable record. A bit flip preserves the maximally mixed distribution while swapping definite values. The public-algebra theorem quantifies over every state, or a justified code space. Its guarantee concerns the chosen observable, not the initial truth of an archived claim.

A favorable subset represents the original experiment. Selection by readability, withdrawal, or convenient outputs can change the tested distribution. The selection rule belongs in the model. If data exclude the uncertainty set, the institution records a model failure; a revised model receives a visible analysis status.

These examples preserve the force of the constructive claim. A community can gain operational power from quantum resources while keeping an accountable public history. Its engineering problem is to distinguish helpful records, completed records, and disclosures that consume a resource needed for a future comparison.

28 The civilizational conjecture

A classical catalogue of definite answers to every alternative question is more demanding than public accountability requires. A community can agree on protocols, performed experiments, recorded outcomes, resource budgets, and next actions while achieving coordination that no local classical model with the same permitted resources can reproduce. The finite constructions establish this conjunction in explicit models.

The next conjecture asks whether that architecture can support capable observers and sustained institutions. It strengthens mere coexistence by requiring measurable advantage. The civilizational claim adds organizational and cognitive conditions; those conditions require independent justification.

Conjecture 28.1 (Sustained quantum coordination with public institutional memory)**.** There exists a physically realizable network of artificial observers, a preregistered finite coordination task G, and a constant $\Delta > 0$ for which the network can sustain repeated encounters with mean score at least $\omega_c(G) + \Delta$, where $\omega_c(G)$ is the optimal local classical value under the same stated questions, communication restrictions, permitted public information, and resource accounting. The network maintains a nonconstant, reliable public record algebra and a collective decision process across those encounters. Its participants satisfy independently specified operational cognitive criteria, and its memory transformations comply with a declared, meaningful participation and withdrawal policy.

A test must quantify tolerated record error, number and duration of encounters, observer capacities, calibration uncertainty, allowed memory, and confidence requirement. The measured gap must survive those errors and the statistical analysis. The conjecture identifies a finite, assessable target; it does not assume indefinite control or superiority over a comparator granted additional communication.

Failure can be informative at several levels. An apparatus can fail calibration without challenging the theory. An architecture can fail because essential memory exports too much branch information. A cognitive capacity can require processes incompatible with the chosen reversal. An autonomy requirement can preclude an experiment. These outcomes constrain different premises and should remain distinguishable.

The Continuum's position is a research commitment: common action need not wait for a universally classical account of every possible observation. Public life can rest on precisely defined

commitments and evidence about what participants actually do. The coherence supporting an extraordinary task is a physical resource to characterize, and participants' freedom is an institutional condition to preserve.

29 Conclusion

The central result is operational. In the specified two-laboratory task, a durable common preparation record enables the exact readout-constrained winning probability

$$\frac{3}{4} + \frac{v^2}{8(1+v)},$$

while genuine loss of that record leaves a separable resource with optimal score 3/4. The record survives the useful correction, and the completed experiment can enter a full public archive. A shared institution can assist quantum coordination rather than merely accompany it.

A different record imposes a different consequence. Exterior markers distinguishing the relevant intermediate branches bound surviving coherence and attainable advantage. The two-laboratory bound is sharp for its family. The GHZ extension gives an exact optimum for its specified family. The magic-square extension supplies explicit strategies, a proven upper bound, and an open optimization interval. They show why a universal disclosure claim would fail: timing, compatibility, and unused entangled resources matter.

The mathematical structure preserves a strong frontier question. A civilization could organize itself around tasks whose coordination exceeds every permitted local classical account, while retaining the memory needed to govern its actual history. The finite theory gives exact examples and falsifiable limits. Building capable, consenting observers into that architecture is the next scientific and institutional challenge.

A Notation, domains, and operational roles

The following register collects the principal symbols. Variables introduced only inside a proof inherit the explicitly stated finite domain of that proof. A label used for a physical system also denotes its Hilbert space when it occurs in a tensor product. Subscripts distinguish a system from a particular numerical outcome. Every probability and every normalized utility below is dimensionless.

Symbol	Type and domain	Role and convention
$\mathcal{H}, \mathcal{K}$	Finite-dimensional complex Hilbert spaces	State spaces with specified tensor factors or direct-sum sectors.
$\mathcal{L}(\mathcal{H})$	Complex vector space of linear operators	Domain of channels and observables.
$\mathcal{D}(\mathcal{H})$	Positive trace-one operators	Density operators.
ρ, σ	Elements of $\mathcal{D}(\mathcal{H})$	States compared on the same declared space.
$A^\dagger, \mathrm{Tr}, \mathrm{Tr}_E$	Adjoint, trace, partial trace	Standard linear-algebra operations.
$\|A\|_1, \|A\|_\infty$	Trace norm, operator norm	Distinguishability and worst-case expectation bounds.
$\Lambda, \mathcal{E}_j$	Completely positive trace-preserving maps	Deterministic physical transformations.
$\Lambda^\dagger$	Trace-dual map	$\mathrm{Tr}[A\Lambda(\rho)] = \mathrm{Tr}[\Lambda^\dagger(A)\rho]$.
$D(\rho, \sigma)$	$\frac{1}{2}\|\rho - \sigma\|_1 \in [0, 1]$	Trace distance; the factor one-half is fixed throughout.
$F(\rho, \sigma)$	$\|\sqrt{\rho}\sqrt{\sigma}\|_1 \in [0, 1]$	Root fidelity, not squared fidelity.
Q, R, L, E	Finite quantum registers; $L = Q \otimes R$	Branch register, accessible workspace, controlled lab, exterior.
ϕ, θ	Real phases modulo 2π	Preparation phase and analysis phase.
P_L, D_E	Real numbers in $[0, 1]$	Accessible opposite-phase distinguishability and exterior branch distinguishability.
γ_j	Complex number with $\|\gamma_j\| \leq 1$	Overlap of the two pure states in marker E_j.
V in the fringe calculation	Real number in $[0, 1]$	Visibility defined from maximum and minimum fringe probabilities.
v	Real number in $[0, 1]$	Coherence parameter in the specified Bell or GHZ state family.
η in the noise model	Real number in $[0, 1]$	Weight of the retained state in $\eta\rho + (1 - \eta)I_4/4$.
ϵ_j	Real number in $[0, 1/2]$	Upper bound on equal-prior branch-discrimination error using marker E_j alone.
δ_A, δ_B	Real error budgets in $[0, 1]$	Mismatch of an external readout from its designated friend event in the relaxed LF model.

Symbol	Type and domain	Role and convention
b_j, b	Numbers in $[0,1]$	Marker-overlap bounds $b_j = 2\sqrt{\epsilon_j(1-\epsilon_j)}$ and $b = b_A b_B$.
G, μ, V in coordination tasks	Finite game, input probability mass function, binary winning predicate	Defines allowed questions, answers, and the task score.
$\omega_c(G)$, $\omega_G(\rho)$, $\omega_q(G)$	Real values in $[0,1]$	Optimal classical, fixed-resource quantum, and unrestricted finite-dimensional quantum scores.
K_t in the phase-flag task	Fresh uniform bit on trial t	Known preparation-phase label, independent of the branch distinction and later questions.
$\rho_k(v), \Omega(v)$	Density operators on specified laboratory and flag spaces	Phase-conditioned preparation and its actual classical record copies.
v_-, v_+	Real endpoints with $0 \le v_- \le v_+ \le 1$	Independently justified interval for remaining coherence.
v_1, v_2 in the magic square	Reals in $[0,1]$	Coherences of the two independent Bell-pair resources.
$M_{G,\Pi}$	Positive operator with $0 \le M_{G,\Pi} \le I$	Winning effect for game G and fixed measurement strategy Π.
X, Y, Z in quantum sections	2×2 Pauli matrices	Hermitian traceless observables squaring to the identity.
A_x, B_y	Nondegenerate projective qubit observables	Measured outcomes are ± 1; designated readout uses Z.
x, y in the two-lab test	Elements of $\{0,1\}$	External settings; 0 designates record readout.
a, b, c, d in the LF representation	Elements of $\{-1,+1\}$	External outcomes and proposed friend events. The outcome b is local to this notation and differs from the disclosure bound.
$\pi(c,d), q_{cd}, p(a,b \mid x,y)$	Normalized finite probability distributions	Latent-event weights, conditional response tables, observed behavior.
E_{xy}, S	Real numbers; $E_{xy} \in [-1,1]$, $S \in [-4,4]$	Correlators and signed CHSH expression.
ζ	Real number in $[0,1]$	Isotropic contextual-probability parameter; quantum range ends at $1/\sqrt{2}$. It is distinct from v.
X in the context formulation	Finite nonempty label set	A set of measurement labels, not a Pauli operator in that section.
$\mathcal{M}, C, O_x, O_C$	Context cover, context, finite outcome sets	Jointly executable procedures and their Cartesian-product outcomes.
$e, e^{(k)}$	Families of context distributions	Empirical models, with overlap compatibility stated explicitly.
M in the extension problem	Zero-one incidence matrix	Maps a global assignment distribution to its context marginals.
N	Convex set of extendable families	Empirical models admitting a single nonnegative normalized global distribution.

Symbol	Type and domain	Role and convention
$\mathrm{CF}, \mathrm{NCF}$	Real numbers in $[0,1]$	Contextual and noncontextual fractions from a finite domination LP.
TV	Real metric in $[0,1]$	Half the ℓ_1 distance between finite probability laws.
K, R, r_k in flag constructions	Finite value set, flag measurement, distribution	A public variable, with actual copies specified separately.
$\mathcal{A}, u, U(r,e)$	Finite action set; utility in $[0,1]$; expected utility	Task criteria and randomized policy evaluation. The unitary symbol U is separately typed in the physical sections.
Θ, G, V in decision theory	Compact uncertainty set; finite payoff matrix; value in $[0,1]$	Minimax domain, payoffs, and guaranteed utility.
$\mathcal{F}_{t-1}$	Pretrial history sigma-algebra	Conditioning information for adaptive-trial bounds.
$Z_t, [a,b], R$ in statistics	Bounded score, score interval, positive range length	These local statistical symbols differ from Pauli Z and register R.
β, Δ in the general witness test	Null mean bound; positive alternative gap	Inputs to the conditional concentration argument.
$\alpha, \beta_{\mathrm{II}}$	Real numbers in $(0,1)$	Significance and target type-II error bounds.
$P_z, \mathcal{A}_{\mathrm{pub}}$	Orthogonal projectors and their commuting span	Declared public observables to be preserved.
d_z, d_j	Nonnegative bounded error measures	Public-projector disturbance or half-diamond channel error, with the domain stated locally.

B Claim registry and proof obligations

This registry states what supports each principal claim and what remains outside it. A restriction is part of the claim's definition; it does not become optional when the claim is interpreted philosophically.

Claim	Support in this manuscript	Remaining physical or interpretive obligation
Records limit accessible historical phase information.	Theorem 8.2; matrix checks with pure and mixed states.	Characterize the preparation and the actual control boundary.
Independent marker overlaps multiply.	Proposition 9.1; explicit tensor-product calculation.	Establish conditional product structure; correlated counterexamples prevent inference from replica count.
The fixed-readout optimum is $2 + v^2/(1+v)$.	Proposition 10.2; exact square identity and direct circuit checks.	Use the declared state and nondegenerate projective measurement class.

Claim	Support in this manuscript	Remaining physical or interpretive obligation
Useful public phase memory enables a task advantage.	Theorem 15.1; exact state and controlled-unitary calculations.	Supply genuine flag access before correction; use fresh independent flags in repeated comparisons.
Branch disclosure bounds the accessible task score.	Theorem 15.2; explicit saturation family.	Enforce marker factorization, information boundaries, and the declared full resource.
The dephased GHZ game's optimum is $\max\{3/4, (1+v)/2\}$.	Theorem 16.2; all extremal binary-qubit measurement cases.	Supply the final balanced three-qubit state and local measurements; interparty resource preprocessing changes the problem.
Magic-square advantage can survive disclosure of one pair.	Explicit Pauli and embedded CHSH strategies in Section 17.	Count both resources; optimized values between the stated bounds remain open.
Compatible and final records can be harmless.	Section 18; channel and joint-projector proofs.	The record must satisfy the stated timing and commutation conditions.
Calibrated state error bounds score error.	Proposition 6.3; positive-effect argument.	Keep the instrument class and input mechanism fixed; characterize other errors separately.
The minimal LF representation has a global extension.	Theorem 13.2; explicit joint construction.	Justify event, setting, locality, and readout premises for the chosen experiment.
Approximate readout changes the null bound.	Theorem 13.4; two independent LP formulations.	Calibrate the relevant errors; conditional validity is required for the stated sequential analysis.
Contextuality survives a visible public preparation flag.	Theorem 20.1; slice-wise subdistribution proof and LP checks.	Implement the actual flag records and conditional instruments.
Shared nontrivial action is compatible with contextual statistics.	Lemma 20.3 and Proposition 23.1.	Ensure faithful shared tokens and a policy nonconstant on supported values; larger civilizational capacity remains a separate claim.
Public variables can be preserved by selected operations.	Theorem 22.1; projector and block-unitary checks.	Characterize the implemented operation on the declared code space or full space.
Finite robust decisions need no forbidden hidden-state assignment.	Theorem 21.1; primal/dual computation.	The true operational model must lie in the stated uncertainty set.
Probability-table error gives a controlled decision loss.	Theorem 21.2; explicit total-variation inequalities.	Establish statistical coverage and account for calibration drift.
The fixed-horizon test permits inter-trial memory.	Theorem E.1; conditional exponential-moment proof.	Fresh settings and conditional null membership must remain justified.
A capable artificial civilization could sustain the construction.	Conjecture 28.1; staged research program.	Demonstrate observer capacities, coherent control, institutional continuity, and acceptable treatment of memory.

The standard results credited in the text include binary quantum discrimination, fidelity and trace-distance inequalities, no-cloning and no-broadcasting, CHSH and its two-qubit optimization, Local Friendliness, contextual fraction, finite minimax theory, and bounded-score concentration. Their inclusion provides foundations. The manuscript's specific reference constructions and assembled error budgets are presented with proofs and without a claim that a priority search has established every formula as historically new.

C Computational audit and reproducibility

C.1 Independently executable verification programs

The editable source contains eleven Python programs in `filecontents*` blocks. Compiling the manuscript writes those programs beside the source; it does not execute them. They use Python's standard library together with NumPy, SciPy, and SymPy as indicated by their imports. They require no network access, credentials, external dataset, or proprietary engine. All reported computations are synthetic finite-model calculations.

Program	Main independent construction	Reported outcome
`reality_quantum_audit.py`	Explicit complex states, partial traces, singular values, and four-qubit operators.	Ten check groups pass; maximum equality residual below 4.0×10^{-15}.
`reality_collective_audit.py`	Enumeration and linear programs for extensions, contextual fraction, flags, and minimax; exact tail checks.	Analytic values reproduced; largest reported LP equality error below 3.4×10^{-16}.
`reality_integrated_audit.py`	Symbolic readout identity; independent scalar optimization; LF extension reconstruction; protected projectors.	Exact symbolic residual zero; numerical residuals below 1.3×10^{-15}.
`reality_readout_vertex_audit.py`	All 24 nonsignaling vertices for each of four latent friend-event pairs; 96 mixture weights.	49 readout-error budget pairs match the relaxed null bound to below 4.5×10^{-16}.
`reality_readout_table_audit.py`	Direct nonsignaling-table optimization with exact error constraints, plus 70-digit decimal planning values.	441 error pairs respect the bound; sharpness verified in the stated region; all table ceilings independently reproduced.
`reality_public_performance_audit.py`	Copied phase flags, controlled correction, marker dilations, complete tables, and robust interval settings.	All checks pass; identity/LP residual at most 4.5×10^{-16} and optimized-value gap below 4.2×10^{-13}.
`reality_reality_games_audit.py`	Exhaustive classical strategies and quantum game measurements, including degenerate GHZ cases.	All enumerations and probability checks pass; analytic targets and scopes are recorded in the JSON report.

Program	Main independent construction	Reported outcome
`reality_revision_independent_audit.py`	Pure marker dilations traced into game states; full joint magic-square POVMs; compatible-record channel.	64 GHZ and 81 magic-square dilations; 11,664 joint probabilities; maximum formula residual below 1.7×10^{-15}.
`reality_games_cross_review.py`	Independent one-deterministic-setting GHZ optimization and commuting-copy checks on arbitrary mixed states.	Direct operator tests confirm the exact GHZ case bound and preservation of the selected-context probabilities.
`reality_ghz_assistance_audit.py`	Alice's intermediate X measurement, pre-input classical message, and Bob's conditional correction.	An explicit broader resource class surpasses the local-POVM threshold; 101 coherences and both outcomes match the achieved formula.
`reality_ghz_exact_audit.py`	All 729 deterministic/nondegenerate setting patterns, 8,748 sampled instances, and 25 continuous searches.	Direct matrix and correlation-tensor evaluations agree; best-value gaps below 9.4×10^{-10}, with convergence flags retained.

The two approximate-readout programs are deliberately different formulations. One optimizes over a vertex representation and latent-event mixture with upper error budgets. The other optimizes directly over normalized nonsignaling probability tables with specified errors. Their agreement in the region where sharpness is proved helps catch transcription, normalization, and constraint-construction errors.

The numerical tolerances are implementation tolerances rather than physical uncertainty intervals. Floating-point residuals near machine precision support the correctness of these finite calculations. The proofs carry the exact statements. Sampling finitely many states cannot certify an unrestricted theorem, and an optimizer's success flag is not itself a proof of the physical assumptions that define its feasible set.

C.2 Coverage of the checks

The quantum checks include 512 pure marker pairs, 512 pairs with entangled accessible and inaccessible workspaces, 256 balanced mixed models, 84 product-marker examples, correlated and nonlocal marker families, 128 phase-independent channels, witness scans, and 51 explicitly encoded four-qubit circuits. They test both the constrained and unrestricted witnesses and the limiting cases $v = 0, 1$.

The contextual checks enumerate all 16 deterministic assignments in the CHSH scenario, construct the eight-assignment boundary model, compare contextual fractions and distances on a 41-point parameter grid, and verify independent-public-register invariance in 15 LP cases. They also compute the visible-flag example, handle zero-weight flags, compare equal-marginal couplings, solve the decision primal and dual, and exhibit attainment of the 2ϵ regret bound.

The statistical checks compare 480 exact binomial tails with the analytic concentration bound and enumerate a 14-trial adaptive null over all 16,384 outcome sequences. The integrated checks reconstruct 200 arbitrary conditional two-setting representations and test 100 random block-unitary public-record models. These checks exercise concrete failure cases as well as favorable examples.

C.3 Checks added in the revision

The new task checks include the full public phase-record density operator, the controlled correction, preservation of its projectors, all conditional answer probabilities, and the unavailable-record separable mixture. An independent optimization searches the full allowed Bloch-angle class. The interval policy is checked at multiple endpoints rather than only at its nominal coherence.

The game audit enumerates all 16 deterministic CHSH assignments, all 64 GHZ assignments, and all 4,096 valid-parity magic-square strategy pairs. It constructs quantum measurement projectors and checks the complete winning predicates. The exact GHZ proof covers deterministic observables as well as nondegenerate ones; separate matrix checks exercise those cases. For the magic square, the code verifies both the standard two-pair strategy and the one-pair CHSH embedding.

The resource-cut counterexample is checked separately: allowing interparty communication to transform the supplied GHZ state before questions can unlock a two-party strategy. Its achieved score is verified without claiming an optimum over the enlarged preprocessing class.

A separate implementation begins with pure laboratory–environment states and traces actual marker registers, rather than inserting the reduced-state formula being tested. It then evaluates 11,664 complete magic-square outcome probabilities. A compatible intermediate copying channel leaves those probabilities unchanged. Another independent check uses arbitrary mixed states for this channel identity. These constructions test the distinction between costly branch disclosure and harmless compatible records directly.

The revision also checks that a reused hidden phase flag differs from independently hidden flags: their two-specimen states have trace distance $1/2$ at full coherence. This counterexample is a resource-accounting check, not a claim that every reuse protocol can unlock the same advantage.

C.4 Running the source

Keep the `.tex` and `.bib` files in one directory. A standard TeX Live installation with XeLaTeX, BibTeX, Latin Modern fonts, and the packages named in the preamble is sufficient. Figures are editable vector graphics written directly in the source. There are no external image dependencies or custom style files.

The normal compilation sequence is:

```
xelatex Eidara_Civilization_Without_Single_Reality.tex
bibtex Eidara_Civilization_Without_Single_Reality
xelatex Eidara_Civilization_Without_Single_Reality.tex
xelatex Eidara_Civilization_Without_Single_Reality.tex
```

If a changed table of contents or cross-reference requires another settling pass, repeat XeLaTeX until its reference warnings clear. No shell escape is needed. Python is optional for typesetting and is used only when separately running the verification programs.

For example, after compilation, run `python3 reality_quantum_audit.py`. Each program writes a JSON report in its own directory. The source comments and imports specify its dependencies. Fixed seeds are declared for random matrix sampling; deterministic LP and enumeration programs need no random seed. Repeated runs can show small numerical differences across numerical-library versions, so comparison should use the stated tolerances and exact analytic targets.

C.5 Verification performed for this edition

This edition was assembled from the reviewed manuscript sources and compiled with XeLaTeX and BibTeX. Its mathematical checks include symbolic identities, independent numerical formulations, limiting cases, and targeted counterexamples. A separate adversarial review of the definitions and proofs identified and resolved the distinction between public-archive preservation and laboratory-only mixing, the nondegenerate measurement class, the environment needed for channel-composition bounds, and the altered null after imperfect readout.

The production review checks bibliography resolution, cross-references, glyph availability, embedded fonts, vector figures, page boundaries, and rendered layout. The accompanying production record retains the final file hashes and build results. Internal computational and editorial review is not external journal peer review, and no proof-assistant certification is claimed.

D Source provenance and research boundaries

D.1 Scientific literature

The bibliography contains the primary research used for the mathematical and physical framework. Published journal articles and author preprints are labeled separately. The source ledger records the primary URL and the actual depth of inspection, distinguishing full-text examination from verification through an official abstract or author manuscript. A citation supports the associated specific result or interpretation; it does not transfer the source's empirical status to the civilizational conjecture.

The preparation-label mechanism is explicitly credited to hidden-entanglement research. The fixed-readout optimization is related to published work on the original Bell functional, and reversal ethics is developed in dialogue with existing observer literature. These attributions constrain novelty claims. The most consequential recent prior art is the general circuit account of consistent quantum-agent reasoning, the finite multipartite LF classification, quantum-computing LF demonstrations, and the 2026 agent-like observer preprint. Their roles are identified in the main text. The manuscript's reference architecture is developed in relation to those results rather than portrayed as an invention of quantum observers or contextuality.

D.2 Canonical materials

Two author-supplied canonical sources are cited under public Continuum reference labels: the doctrine's *The Observer Is the Scroll* passage and the Bible's sections on observer-anchored models and the Charter. They support the conceptual and normative motivation. The mathematical model developed here is a proposed reference model with its own definitions, and its validity does not depend on the correctness of undeclared canonical equations.

No proprietary runtime specification, internal product, private system calibration, or uninspected historical experiment is used as a technical premise. The publication package contains the manuscript and its editable scientific apparatus. It does not reproduce the private canonical documents themselves. Their internal narrative and fictional material remain distinct from the experimental program.

D.3 Questions left open

The unresolved questions are substantive: which artificial systems should count as thoughtful observers; how their relevant dynamics could be controlled; which memory transformations preserve a defensible account of identity; how the finite task advantages extend to practical collective objectives; and whether an institution can scale while retaining the required coherence. The finite results constrain these questions. They leave the physical and philosophical work visible rather than resolving it by terminology.

Future editions should change the claim registry when evidence changes. A failed implementation can narrow a conjecture. A better control model can sharpen a bound. A new primary result can change the comparison with prior art. The purpose of a reproducible theoretical paper is to make those revisions possible without losing the reasoning that justified the earlier version.

E Finite data, memory, and falsifiable tests

A prospective experiment needs two specifications before data collection: the physical null hypothesis and the statistical procedure that can reject it. A generic CHSH violation does not automatically test every assumption in a Local Friendliness argument. The appropriate null polytope or inequality must follow from the observer protocol actually implemented. The statistical construction below applies to a specified bounded witness once that physical derivation has been supplied.

Modern Bell-test analysis explicitly treats dependence on earlier trials. Zhang, Glancy, and Knill give a prediction-based-ratio method that remains valid for local models with memory and avoids relying on Gaussian standard-deviation counts [72]. The elementary method proved here is less statistically efficient but has transparent assumptions and is sufficient for planning illustrative tests. Its exponential bound uses the bounded-variable principle introduced by Hoeffding [73].

E.1 A witness as a trial score

Choose fixed real coefficients $w_{C,s}$ and a null family K of admissible empirical models. Suppose

$$W(e) := \sum_{C \in \mathcal{M}} \sum_{s \in O_C} w_{C,s} e_C(s) \leq \beta \quad \text{for every } e \in \mathsf{K}, \tag{129}$$

where $\beta \in \mathbb{R}$ is the proved null bound. On trial t, sample context C_t with fixed probabilities $\pi(C) > 0$ and observe outcome $S_t \in O_{C_t}$. Define

$$Z_t := \frac{w_{C_t,S_t}}{\pi(C_t)}, \qquad a := \min_{C,s} \frac{w_{C,s}}{\pi(C)}, \quad b := \max_{C,s} \frac{w_{C,s}}{\pi(C)}, \quad R := b - a.$$

Here a, b are score endpoints, not the binary outcomes of the earlier example. We assume $R > 0$; a constant witness cannot provide the intended test. Infrequently sampled settings can make R large and weaken the bound, so setting probabilities are a substantive design choice.

Let $\mathcal{F}_{t-1}$ contain the complete recorded past before the fresh setting for trial t. The statistical null is

$$\mathbb{E}[Z_t \mid \mathcal{F}_{t-1}] \leq \beta \quad \text{almost surely for every } t. \tag{130}$$

One way to justify it is to require that each conditional behavior table $e^{(t)}$, given the history, belongs to K, while the fresh setting is sampled with the declared probabilities independently of the pretrial physical variables on which the null response could depend. Then inverse-probability weighting gives $\mathbb{E}[Z_t \mid \mathcal{F}_{t-1}] = W(e^{(t)}) \leq \beta$. The physical restrictions needed for conditional membership, including the relevant restrictions on communication within a trial, remain essential. Independence of successive trials is not assumed.

Theorem E.1 (A memory-tolerant fixed-horizon bound)**.** *Assume $Z_t \in [a, b]$ and (130). For any predeclared integer $n \geq 1$ and any $\epsilon > 0$,*

$$\Pr\left\{ \frac{1}{n} \sum_{t=1}^{n} Z_t \geq \beta + \epsilon \right\} \leq \exp\left(-\frac{2n\epsilon^2}{R^2} \right). \tag{131}$$

Proof. For a bounded random variable Z, conditional on the past, let $h(\lambda) = \log \mathbb{E}[\exp(\lambda Z) \mid \mathcal{F}]$. Its second derivative is the variance under the exponentially tilted conditional distribution. Any distribution supported on $[a, b]$ has variance at most $R^2/4$: writing its mean as μ, the inequality $(Z - a)(b - Z) \geq 0$ implies $\mathrm{Var}(Z) \leq (b - \mu)(\mu - a) \leq R^2/4$. Twice integrating $h'' \leq R^2/4$ from zero gives

$$\mathbb{E}[e^{\lambda(Z-\mu)} \mid \mathcal{F}] \leq e^{\lambda^2 R^2/8}.$$

For $\lambda \geq 0$ and conditional mean $\mu \leq \beta$, the same upper bound holds with $Z - \beta$ in the exponent. Iterated conditioning thus bounds the moment-generating function of $\sum_{t=1}^{n}(Z_t - \beta)$ by $\exp(n\lambda^2 R^2/8)$. Markov's inequality gives $\exp(-\lambda n\epsilon + n\lambda^2 R^2/8)$ for the upper tail. Its minimum over $\lambda \geq 0$ occurs at $\lambda = 4\epsilon/R^2$, proving (131). □

For a uniform-setting CHSH game define the binary success score

$$Y_t := \frac{1 + (-1)^{X_t Y_t'} A_t B_t}{2} \in \{0, 1\},$$

where $X_t, Y_t' \in \{0,1\}$ are the two settings and $A_t, B_t \in \{-1,+1\}$ are the outcomes. The prime on Y_t' distinguishes the second setting from the success score Y_t. Under the corresponding local null, the conditional mean is at most $3/4$, and $S = 8\mathbb{E}[Y_t] - 4$. The score range has length one. For the quantum benchmark above, the theoretical success probability is $(2+\sqrt{2})/4 \simeq 0.85355339$. These are model predictions, not data from an artificial-observer experiment.

E.2 Power is distinct from crossing an expected threshold

At significance level $\alpha \in (0,1)$, the preceding bound rejects when the mean score exceeds $\beta + R\sqrt{\log(1/\alpha)/(2n)}$. Setting a theoretical mean equal to this threshold gives an expected-score crossing estimate, not a guarantee of detection probability. A power calculation needs an alternative model and a lower-tail control.

Corollary E.2 (Conservative power planning)**.** *Suppose the alternative satisfies $\mathbb{E}[Z_t \mid \mathcal{F}_{t-1}] \geq \beta + \Delta$ for all trials, with $\Delta > 0$, and the same score range R. To make the probability of failing the fixed-horizon test at most $\eta \in (0,1)$, it suffices that*

$$n \geq \frac{R^2}{2\Delta^2}\left(\sqrt{\log(1/\alpha)} + \sqrt{\log(1/\eta)}\right)^2. \tag{132}$$

Proof. Apply the preceding exponential-moment argument to the lower tail under the conditional alternative. A deviation below the conditional mean by $R\sqrt{\log(1/\eta)/(2n)}$ has probability at most η. Condition (132) makes the sum of this deviation and the null rejection margin no larger than Δ. □

For the ideal CHSH game, $R = 1$ and $\Delta = (\sqrt{2}-1)/4$. With $\alpha = 0.01$ and $\eta = 0.20$, (132) gives $n \geq 544$ after rounding upward. Merely making the predicted mean exceed the significance threshold would give 215 trials. Reporting the latter number as an 80%-power requirement would be incorrect. Both numbers are conservative analytic illustrations for an idealized benchmark; a specific Local Friendliness protocol requires its own attainable score, score range, readout assumptions, and noise model.

E.3 Stopping, tuning, and public reproducibility

For a fixed prechosen $\lambda > 0$, the process

$$E_n(\lambda) := \exp\left\{\lambda \sum_{t=1}^{n}(Z_t - \beta) - \frac{n\lambda^2 R^2}{8}\right\}, \quad E_0 = 1, \tag{133}$$

is a nonnegative supermartingale under the null by the conditional moment bound in the proof. Consequently

$$\Pr\{\sup_{n\geq 0} E_n(\lambda) \geq 1/\alpha\} \leq \alpha.$$

For completeness, stop the process at its first threshold crossing or at a fixed finite horizon. Iterated conditional expectation bounds the stopped expectation by one. On the crossing event its value is at least $1/\alpha$, proving the finite-horizon probability bound; monotone convergence of the crossing events gives the unbounded-horizon result. This supplies a valid sequential rule. Choosing the most

favorable λ after inspecting the complete data and applying the same bound without adjustment is not justified.

A predeclared mixture $\sum_j \gamma_j E_n(\lambda_j)$, with $\gamma_j \geq 0$ and $\sum_j \gamma_j = 1$, remains a nonnegative supermartingale and permits a finite grid of tuning parameters. More efficient time-uniform methods are developed in the confidence-sequence literature [74]. The present derivation suffices to specify one auditable procedure; it does not claim statistical optimality.

The preregistration must fix the measurement scenario, physical null, witness coefficients, setting distribution, handling of missing outcomes, trial inclusion rules, and the stopping procedure. If multiple witnesses are explored, the confirmatory analysis uses held-out data or a valid simultaneous correction. Every setting and outcome, including failures, must be represented according to the stated rule. Selecting only trials with readable intermediate memories changes the sampled experiment and can introduce postselection bias.

Overlap agreement also needs statistical treatment. A finite discrepancy between two empirical marginals can arise from fluctuation, drift, or different physical instruments. It cannot be repaired silently and then used as evidence that the unrepaired experiment satisfied the mathematical assumptions. A suitable report gives the raw discrepancies, simultaneous uncertainty intervals, and the declared model or outer confidence set used for inference. If that feasible set is empty, the incompatibility is a finding about the model-and-calibration combination and is preserved for diagnosis.

The most important falsifiers are concrete. The claimed contextual witness can fall within its null bound; the required operation can fail to reverse the observer register; supposedly private information can leak into public records; marginal equivalences can fail calibration; a shared-action protocol can exceed its permitted disagreement probability; or an uncertainty set can exclude the observed behavior. Each outcome identifies a specific failed premise or implementation. None is excused by the theoretical ambition of a civilization with many irreducible accounts.

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