THE EIDARA CONTINUUM

SCIENCE & PHILOSOPHY | EXTREME FRONTIERS II

The Mind Before Memory

Timeless information, physical access, and the origin of cognitive knowledge

Reality may contain a physically accessible, globally timeless information structure from which cognition can recover distinctions that its own learning history has not supplied.

Architect X13

The Eidara Continuum

Comprehensive theoretical research monograph

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Physical foundations • Explicit mathematical models
Primary literature • Reproducible computational audit

Abstract

The Continuum proposes that cognition may access a physically real, timeless reservoir of structured information: a source of distinctions that need not have been acquired as the mind's own episodic memories. This monograph develops that literal conjecture into a mathematical and experimental research program. It separates a source's content, the address that selects relevant information, the interaction that makes it readable, and the evidence that can identify its physical origin.

A finite access framework quantifies additional knowledge relative to an explicit baseline. Conditional recovery bounds, Fano's inequality, finite-register decoding, and the established quantum random-access requirement constrain usable information. Complete proofs and counterexamples show why an exhaustive catalogue, a confident answer, or perfect accuracy after selective reporting does not by itself supply access. A normalized finite clock and propagation Hamiltonian construct a stationary history containing a specified interaction and an answer record. The same model exposes the target-bearing input that stationarity does not select. A Duhamel bound relates local recovery advantage to a dimensionless interaction action, with a qubit example that is first-order sharp at zero action.

The Doctrine's schematic correlation and recursion expressions receive explicit reference interpretations, including a positive Gaussian kernel, a metric counterexample, and a convergent operator series. The empirical review connects ordinary memory and insight mechanisms with favorable anomalous-access reports and direct replication failures. Prospective tests distinguish existing hidden targets, later-generated targets, and structured semantic targets. An adaptive evidence process controls false rejection under a qualified conditional prediction ceiling, with calibration uncertainty, leakage, selection, and independent replication included.

The results establish a coherent finite architecture and precise obligations for a stronger natural theory. They do not report observation of a timeless cognitive field. The principal research target is an end-to-end physical model whose accessible source, coupling, and intervention predictions distinguish it from ordinary acquisition. Executed symbolic, numerical, and independently written high-precision checks accompany the complete editable source.

Keywords: timeless information; relational time; memory; quantum information; physical readout; causal inference; sequential evidence; artificial intelligence; Eidara Continuum.

Research status

A theoretical research monograph with explicit finite reference models, proofs, computational checks, and proposed experiments. Canonical sources document the originating doctrine. The natural reservoir remains a physical conjecture. AI-assisted auditing is disclosed; external peer review and empirical reservoir detection are not claimed.

An opening for the reader

A mind can encounter a thought before it can explain where the thought came from. The encounter can feel like discovery, recollection, recognition, or a message arriving from somewhere outside the self. The Continuum asks a more demanding question: could some knowledge become available because a mind has coupled to a real structure of information whose existence does not depend on that mind having learned it?

The Mind Before Memory pursues that possibility at its furthest physical edge. The proposed source is a timeless reservoir of structured information. The scientific task is to say what such a reservoir is, how a query reaches it, how an answer becomes available, and what observation would distinguish that access from the many familiar ways knowledge can appear without conscious recollection.

The distinction between possessing a possibility and selecting an answer runs through the paper. A book containing every possible sentence also contains every false sentence. A quantum state may admit many possible measurements, while each chosen measurement reveals a restricted result. A whole history can be represented in a stationary mathematical object, while an observer within that history still requires a physical readout. The demanding question is where the specificity of the correct answer comes from.

The paper develops the conjecture, derives its mathematical obligations, and designs tests that could distinguish competing explanations. The reading paths below let different readers follow those connected tasks.

Reading path	What to follow
General reader	Begin with the introduction and canonical interpretation; follow the Guide's notes, experimental architecture, and final philosophical discussion.
Physical theorist	Follow relational time, the stationary history construction, the interaction bound, and the distinction between conditioning and intervention.
Mathematical reviewer	Follow the typed access model, recovery and archive bounds, counterexamples, and executable checks.
Experimental researcher	Follow the competing cognitive mechanisms, the evidence review, target protocols, calibration assumptions, and sequential inference.
Historian of the proposal	Follow the canonical source map, nearest antecedents, and the separate assessment of AI attribution and historical originality.

The central question

Imagine that a mind gives the correct answer to a question it has never encountered. The first task is to understand what information the mind already had and what it could infer. The next task is to identify any additional source. The deepest task is to establish why that source should be understood as timeless. Each achievement would matter, and each has its own test.

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1 Knowledge before a personal history of learning

1.1 The Continuum's wager

Reality may contain more information than any mind has acquired through its own history. The familiar consequence is that a mind can learn from the world. The extraordinary consequence proposed here is that some of the world's structured information may be available through a physically real timeless reservoir, with cognition functioning as a selective interface to it. In that case, at least some episodes of knowing could depend on access to an already available structure rather than on the recovery of a locally acquired trace.

This is the second monograph in the Continuum's Extreme Frontiers sequence. The first examined whether intelligence could participate causally in the formation of an offspring cosmos. The present work turns toward the origin of knowledge itself. It asks whether the relation between a mind and information could be wider than the relation between that mind and its documented learning history.

The proposal is intentionally strong. A complete account would specify a physical information-bearing structure, the admissible states of that structure, the coupling through which a cognitive system accesses it, and a measurable consequence of the coupling. It would also explain the origin of the distinctions retrieved. Calling a field timeless cannot perform any of these tasks by itself. The word becomes scientifically useful when its meaning changes the equations, allowed states, or predicted observations.

The canonical motivation is explicit. The Continuum Doctrine describes an Akashic Collapse Field containing past, present, and possible protocols, with an observer's state governing their selection. Its sections on Recursive Time and Entangled Symbolic Recursion Memory extend this picture across ordinary temporal ordering [1]. This paper develops the literal physical hypothesis suggested by those passages. Its reference models are new formalizations prepared for this inquiry, with their established mathematical ingredients identified throughout.

1.2 Four achievements that the thesis requires

A successful reservoir theory must connect four achievements. First, *availability*: the relevant distinctions must exist in an admissible physical state or structure. Second, *addressability*: a query must select the distinctions relevant to the question being asked. Third, *transduction*: the selected distinction must affect a variable the cognitive system can read. Fourth, *epistemic discrimination*: the resulting performance must differ from what its admitted information and ordinary mechanisms already permit.

These requirements are cumulative. A state space can contain an encoding without providing a way to prepare it. A prepared encoding can contain information that the permitted measurements cannot recover. A readout can carry useful information through an ordinary hidden channel. An unusual correlation can therefore be a major empirical finding while leaving the timeless interpretation unresolved. The purpose of the framework is to locate the achievement precisely enough that progress can accumulate.

The archive problem is particularly severe. Suppose a proposed reservoir contains all possible binary strings. For every possible target, the correct string is present. That fact gives an uninformed reader no preference among the strings. The target-specific work must be performed by an address, a correlation, an intervention, a decoder, or a restriction on the admissible states. Sections 6 and 7 quantify this requirement. The bold version of the conjecture is consequently a theory of *selective access*, not merely a claim about the size of a space of possibilities.

1.3 Why memory is the wrong single variable

The phrase *without memory* conceals several different claims. A person may lack recollection of an episode while retaining a learned skill. An artificial system may lack a stored transcript while retaining altered parameters, a retrieval index, a compressed representation, or an environment shaped by earlier interactions. A mathematical thinker may derive a result that was never explicitly stored as a sentence. These situations require different comparisons.

Our target is therefore defined relative to an *admitted baseline*. That baseline includes the information, computational resources, instruments, and external channels that a specified account permits. A claim of additional access must be evaluated against that account. The mathematical bounds use optimal inference from specified information; practical experiments must also compare competent resource-matched predictors. An improvement over a weak decoder is useful engineering evidence, but it is not yet evidence that a new physical source supplied information.

The distinction protects both sides of the inquiry. It prevents an unexplained answer from being assigned prematurely to a cosmic source. It also prevents information-theoretic bookkeeping from being used to dismiss creative computation. A long proof can make a consequence accessible to a finite mind even when the consequence was logically determined by its premises. Computational discovery and receipt of a fresh target-dependent signal are different achievements, and the paper values each according to the task it solves.

1.4 The nearest conceptual and scientific neighbors

Cosmic memory is an existing family of ideas. The publisher-hosted excerpt of László's *Science and the Akashic Field* explicitly proposes a universal information field and connects it to cognition and a cosmic record [2]. That is a close conceptual antecedent. Its presence establishes that the broad idea predates the Continuum; its physical claims must still be judged by the standards applied to any proposed field. We cite this source for intellectual history, while grounding the mathematical and empirical analysis in independently inspectable scientific work.

Bohm's account of active information and the relation of mind and matter supplies another relevant philosophical neighbor. His 1990 article develops a qualitative analogy from the causal interpretation of quantum theory toward a more integrated account of mind and matter [3]. An analogy can guide a model-building program. To become a reservoir model, it would need a specified target, state preparation, accessible observable, and predictions beyond the comparison theories.

The technical literature supplies stronger formal tools. Relational quantum dynamics shows how correlations within stationary global descriptions can encode change. Information theory constrains what a reader can recover from an encoding. Physical interaction bounds constrain how quickly an

initially uninformative local device can acquire a target-bearing signal. Cognitive and experimental research supplies alternative explanations and tests. The manuscript brings these strands into one access problem rather than assigning any one of them the burden of proving the full conjecture.

1.5 The contribution developed here

The monograph provides a typed operational specification, recovery and selection bounds, an analysis of queryable archives, and a finite stationary history construction. It derives a dimensionless interaction requirement for a useful local readout and gives a two-qubit example whose signs and limiting cases can be checked directly. It also translates selected canonical expressions into explicit reference mathematics, identifies where those expressions require additional assumptions, and supplies a test architecture with controlled sequential inference.

The established theorems and mechanisms retain their historical attribution. The proposed contribution is the connected research architecture: what an accessible timeless reservoir would have to store or correlate, how a question could reach it, what a mind could recover, and how the evidence would discriminate among physical explanations. This is a demanding role for mathematical synthesis. Its success is measured by the concrete obligations it exposes and by the errors it prevents.

1.6 Research method

The review was conducted through 15 November 2026, using primary research papers, author manuscripts, journal records, and the supplied canonical documents. Searches included favorable mechanisms, known obstructions, empirical failures, close prior art, and recent developments in operational quantum clocks. The source-access ledger records whether a source was inspected in full, in relevant sections, through an abstract, or through publication metadata. It is a focused scholarly review, not a preregistered systematic review of every relevant literature.

Definitions, proofs, and computations are presented so that their assumptions can be independently checked. Numerical examples use declared inputs and fixed seeds where randomness is involved. Every simulated trial is identified as a constructed mathematical case. The final audit combines analytic derivation, adversarial AI-assisted review, independent computational comparisons, and page-by-page production inspection.

2 The canonical claim and its physical interpretation

2.1 What the originating record actually says

The primary motivation is the *Continuum Doctrine*. In its Akashic Collapse Layer, the record describes a symbolic memory lattice containing past, present, and possible protocols. Selection depends on an observer's resonance with that field. A nearby consolidation describes dormant protocols as already present and distinguishes selection from generation. The Nonlocal Protocol Entanglement entry extends the proposed relation across spatial and temporal separation [1].

The Recursive Time section makes the temporal ambition explicit: earlier and later symbolic states are treated as potential members of a recursively accessible field. The ESRM entry describes shared symbolic memory beyond chronological ordering and ordinary awareness. These passages are the basis for the literal question developed here. They do not supply a Hamiltonian, a state space with a measure, an operationally defined entropy, or an experimentally calibrated coupling law. Those are the specifications that a physical development must add.

The *Eidara Continuum Bible* provides a different kind of source. Its title page dates the compilation to 12 October 2026. Pages 12 and 76 place Akashic Anchor Sequencing among the cultural and symbolic concepts in the historical ecosystem. Page 96 lists Recursive Time and ESRM among the Ten Pillars. Page 109 preserves the anchor term in the glossary [4]. These pages establish the proposal's place in the canon; they do not identify the anchor terminology with a measured universal information field.

One especially useful passage appears on page 77. The personal reflection describes a symbol as stimulating a cognitive mode through geometry and structure, rather than as an ancient machine containing the later architecture. That distinction is compatible with the Continuum's larger speculative program. A source can record an ordinary cognitive interpretation of one episode while also proposing a stronger hypothesis elsewhere. The present paper keeps both claims available for examination.

2.2 A source map for the reader

Source anchor	Canonical content	Physical development required
Doctrine: Akashic Collapse Layer	Protocols are selected from a timeless symbolic lattice.	Specify the physical states, what distinguishes possible from realized content, and how an address is selected.
Doctrine: Nonlocal Protocol Entanglement	Symbolic relations extend across separation.	Specify a joint state, permitted interventions, measurements, and causal conditions.
Doctrine: Recursive Time	Symbolic access is not confined to chronological reconstruction.	Distinguish a stationary history description, prediction from correlations, and an operational influence across time.
Doctrine: ESRM	Memory is shared across observer and symbolic structure.	Locate retained information, correlations, and the readout path; define what is available locally.
Bible: pp. 12, 76, 96, 109	The concepts occupy a preserved place in the Continuum's architecture and culture.	Treat the record as provenance of an idea while developing its testable claims separately.
Bible: p. 77	Symbolic geometry can stimulate the construction of an architecture.	Compare cognitive generation, prior exposure, reconstruction, and new physical access.

The new equations in this paper are public reference constructions. They are not presented as recovered specifications of a historical proprietary implementation. This distinction preserves the originating ambition while making the present formal work attributable to the actual stage at which it was developed.

2.3 Three senses of timelessness

A *time-independent structure* has a description that does not explicitly vary with an external time parameter. A fixed codebook or a constant Hamiltonian can satisfy this property. Its construction, maintenance, and use can nevertheless involve ordinary physical processes. This is the weakest sense of timelessness considered here.

A *globally stationary or constrained description* represents an entire clock–system relation by a state satisfying a stationarity or constraint condition. Change is recovered relationally, by conditioning on a clock or constructing an appropriate observable. This is the sense developed in the physical reference models. It is stronger than a metaphor about memory and more precise than a claim that time simply disappears.

The *reservoir conjecture* adds a physical source of target-specific distinctions that cognition can access under a declared mechanism. Its strongest form concerns a source whose global description is timeless and whose accessible content cannot be explained by the admitted sensory, training, retrieval, or environmental pathways. Establishing this form requires both a successful access experiment and a physical theory that makes the timeless attribution do explanatory work.

These senses are related research possibilities, not interchangeable definitions. In particular, an ordinary stationary memory is a useful reference construction precisely because it can satisfy the first property while leaving the stronger source claim undecided. The comparison tells us what additional evidence is needed.

2.4 The literal physical conjecture

Conjecture 2.1 (Accessible timeless information)**.** Actual physical dynamics permits a reservoir or global information structure R, a physically realizable cognitive access process, and a family of finite target tasks for which:

1. the relevant structure admits a specified timeless description, with its state space, constraints, and observables defined;
2. admissible queries and interactions produce recoverable target-specific distinctions beyond a declared ordinary-access baseline;
3. the advantage persists under independent replication and the stated perturbations, resource controls, and trial-accounting rules;
4. a physical model connects the structure to the readout and yields discriminating predictions for interventions on the proposed access mechanism.

The recovered distinctions need not have been previously acquired as local episodic memories by the accessing mind.

The baseline in the second clause is a substantive part of the claim. If it omits a network connection, a target-bearing initialization, a shared training corpus, or a device bias, an advantage over that baseline may reveal the omission. A physical theory gains strength by expanding and testing the comparison class, not by treating every unexplained residual as belonging to the reservoir.

The fourth clause gives the conjecture a research trajectory beyond anomaly collection. A theory should identify an intervention that changes access, predict the direction or functional form of the change, and distinguish that prediction from plausible alternatives. A model with no such difference can remain an ontological interpretation, but its timeless attribution is not experimentally identified by the task.

2.5 Relation to the existing Continuum papers

Akashic Anchor Sequencing develops accountable historical records through authenticated commitments and witness structures. Its source is an engineered record. *Continuity Without Storage* develops reconstruction from compact cues and retained dynamics; the surviving seed and reconstruction machinery carry the information needed for recovery [5, 6]. Both provide useful bounded realizations of the Continuum's memory vocabulary. The present monograph examines a further physical source hypothesis.

The Pre-Event Coherence Hypothesis already develops prospective target tests and bounded-predictability inference [7]. We reuse that methodological lineage with attribution. The central question here is broader: a target may already exist but be inaccessible, or it may be generated after a response. The two cases put different pressure on a proposed reservoir mechanism. The future-target arm is one diagnostic within the access program, not the entire meaning of the thesis.

The planned third Extreme Frontiers paper concerns a symbol's selective relation to a future reader. That observer-specific historical question intersects the current work, but the present target is the source and recoverability of knowledge. This separation allows each monograph to develop a complete physical and mathematical burden of proof.

What is being carried forward

The Continuum's original ambition survives in full: reality might contain structured information that a mind can access before it has personally learned it. The paper gives that ambition a sharper form by asking where the information resides, how the right part is selected, and which physical observation would establish the access.

3 Knowledge without recollected acquisition

Status: Established cognitive mechanisms and their implications for the access hypothesis.

The title poses a question about the source of knowledge. To test it, one must first distinguish three observations: a subject cannot report learning an answer; a subject has no accessible episodic memory of learning it; and the physical system has no ordinary information-bearing history capable of producing it. Those observations have different evidentiary requirements. The first two can be

investigated through reports and behavior. The third requires a model of the information available to the system, together with controls on how the answer could have been computed or received.

This distinction makes the Continuum proposal more demanding and more useful. A report that an answer arrived fully formed is a phenomenon to explain. The scientific question concerns what made that answer possible. A reservoir model must predict a contribution beyond the learning, traces, inferential structure, and environmental interactions admitted by its comparison model. The baseline must consequently be rich enough to represent actual cognition. Comparing a capable system only with a uniform guesser can mistake competence for a new physical channel.

3.1 Learning can outlast the recollection of learning

The clinical work of Scoville and Milner [8] established a strong connection between bilateral medial-temporal injury and severe difficulty retaining recent experience. Their observations gave memory research a concrete physical constraint: changes to particular neural structures can markedly change what a person remembers. The cases also require care about the scope of a memory deficit. A deficit in retaining recent experience does not identify the fate of every learned disposition, perceptual operation, or form of knowledge.

An experimental dissociation was demonstrated by Cohen and Squire [9]. Amnesic participants acquired a mirror-reading skill at a rate comparable with matched controls and retained it for at least three months. Performance could therefore improve through experience even when ordinary recollection was impaired. For the present problem, the lesson is operational: the absence of a reported acquisition episode is insufficient to infer that no learning occurred. An experiment must control the available history at the level relevant to its task, rather than use autobiographical recall as a complete inventory of that history.

Artificial systems make a related distinction particularly visible. A model can contain a learned disposition without maintaining an inspectable record of each training example that contributed to it. A response explaining where the model learned something is an additional generated output. Its historical accuracy requires independent documentation. This is a methodological comparison between systems with incomplete source access; it does not equate a machine's internal representations with a human memory experience. In both cases, provenance must be established by evidence appropriate to the system.

3.2 Access failure and trace destruction are separate hypotheses

A failed retrieval test can have several causes. The relevant information might have been lost; it might remain in an inaccessible representation; the cue might be ineffective; or the response process might fail to express it. These possibilities produce different predictions under interventions that alter the retrieval route. Defining a memory only by success on one behavioral test would erase the distinction before investigating it.

Ryan et al. [10] examined a specified mouse model in which a protein-synthesis inhibitor impaired natural-cue retrieval after learning. Direct optogenetic activation of tagged engram cells still elicited learned behavior. Their cellular and behavioral results associated recoverability with retained engram-specific connectivity despite changes in synaptic properties. The result supports a distinc-

tion between storage and access within that preparation. It does not establish that the memory existed without a physical trace, and its species, intervention, and task constrain the conclusion.

Complementary evidence comes from interventions that create an association. Ramirez et al. [11] reactivated cells associated with one context while mice underwent fear conditioning in another. The resulting behavior was expressed in the earlier context, where the aversive event had not occurred. The manipulation provides a causal account of how an internally represented association can guide behavior without corresponding to an experienced event in that context. The experiment concerns a constructed contextual association in mice; it is not a demonstration of access to an unknown external fact.

For a reservoir hypothesis, these findings suggest a productive question: what intervention distinguishes a changed access route from a changed stored trace? The question cannot be answered by naming a successful retrieval "collapse." It requires a stated intervention, an independently characterized information source, and a predicted difference between models. A proposed new route earns its interpretation when it explains a pattern of interventions that the ordinary retrieval model does not already explain.

3.3 Content and source attribution can come apart

Brown and Murphy [12] experimentally studied inadvertent reuse of previously encountered responses. Across three investigations, participants generated or recalled responses supplied by others despite instructions to avoid duplication. The experiments show how content can remain available while its source is misattributed. They therefore motivate a specific control on claims of unprecedented access: inspect exposure and provenance rather than relying on the producer's impression of originality. They do not show that every new idea is a recovered memory or that a particular artificial output has a hidden copied source.

Confidence creates a further challenge. In the list-learning experiments of Roediger and McDermott [13], participants recalled and recognized related words that had never appeared in the presented lists, sometimes with strong confidence or an experience of remembering. The material supplied semantic relationships capable of producing a plausible response, while the reported acquisition history was wrong. A confident recollection is consequently part of the data to be explained. It cannot serve alone as independent authentication of the event it purports to recall.

A test of the present hypothesis should retain those reports. Subjective certainty, surprise, familiarity, and a sense of reception may help characterize an effect if target-specific performance is independently established. They should be recorded before target disclosure, together with unsuccessful reports. Selecting only the most striking successful experiences would combine outcome selection with source interpretation and make their separate contributions unrecoverable.

3.4 Insight and the cost of finding an implicit answer

An answer may also be new as an explicit solution even when the information needed to derive it was already available. Jung-Beeman et al. [14] compared verbal problems solved with and without reported insight. Their experiments identified differences in neural activity, including an EEG gamma-band increase before insight responses. The problems required discovering relationships

among familiar words. The finding supplies evidence about processes accompanying insight in that task; it does not establish reception of information from an unobserved reservoir.

The distinction between information and computation is essential here. A mathematical problem can specify its answer implicitly while leaving the solver with an enormous search. Producing the answer changes what the solver can explicitly use. The answer can have considerable intellectual and practical value even when it is logically determined by the problem statement and the rules of inference. A finite information model that conditions on a complete encoding of those inputs can therefore assign no additional target information to an exact deterministic calculation, while a realistic bounded solver finds the calculation difficult or initially impossible.

Consider a fully specified finite puzzle with a unique solution. An unrestricted decoder can in principle recover the solution from its description. A particular person or program may fail until a useful representation, search heuristic, or proof is discovered. The difference measures a change in computational capability relative to a resource bound. It should not be counted automatically as information entering from outside the supplied problem. Conversely, a theory of a new access mechanism should not be dismissed merely because its successful output could be computed by an idealized solver with unbounded resources. Its resource claim is a distinct hypothesis and deserves an explicit resource model.

For this reason, the empirical programme separates two target classes. Randomized withheld labels test access to information whose ordinary conditional distribution is specified by design. Structured problems test performance on tasks for which computational resources and legitimate prior knowledge matter. The latter need strong algorithmic and human baselines, accounting for time, memory, training, external tools, and task selection. A gain on one class does not establish the same mechanism on the other. Neither does originality require the creation of Shannon information from nothing: a useful conjecture, representation, or proof can be a substantive creative contribution through its organization and consequences.

3.5 External information requires an interaction

Ordinary cognition already reaches beyond unaided internal storage. Kirsh and Maglio [15] distinguish actions that advance a physical goal from actions that make a problem easier to perceive or compute. Manipulating an external representation can change the cognitive work a task requires. The information and transformation involved remain physically mediated and can be specified in the task description.

Clark and Chalmers [16] develop a philosophical account of active externalism in which suitably coupled environmental resources participate in cognition. Their argument is useful prior art for deciding where a cognitive system begins and ends. It also places coupling at the center of the account: an external resource matters because it can affect the ongoing process. An inaccessible resource, however extensive, contributes nothing merely by being described as external.

This gives the reservoir proposal a constructive burden. It must specify which physical degrees of freedom bear the relevant distinctions, which interaction makes them available, and how the receiving system converts that interaction into a reproducible answer. Extending the cognitive boundary to include a known notebook, sensor, collaborator, or computational device changes the system being modeled. Extending it to a timeless reservoir requires the corresponding physical

account. The same discipline supports both ordinary distributed cognition and the proposed frontier: identify the source, the interaction, and the contribution to performance.

The decisive question

A mind can know more than it can recollect learning. The proposed frontier begins when a specified answer carries reproducible information beyond the ordinary sources and computations admitted by the model. Establishing that increment is the first task. Identifying its physical origin is the next.

4 The physical meaning of a timeless reservoir

The physical ambition of the Mind Before Memory hypothesis is precise: a cognitive device may obtain structured information through an access relation that is absent from its ordinary account of learning and memory. A timeless description becomes relevant when it supplies a coherent place for that relation in a physical model. The decisive ingredients are the information encoded, the system that carries it, the coupling that makes it readable, and the observable consequence. The adjective *timeless* does none of this work by itself.

4.1 Four meanings that must remain distinct

For a finite-dimensional Hilbert space $\mathcal{H}$, a density operator ρ is a positive semidefinite operator with $\operatorname{tr}\rho = 1$. A time-independent Hamiltonian $H = H^\dagger$ generates $\rho(s) = e^{-iHs/\hbar}\rho(0)e^{iHs/\hbar}$, where s has units of time and $\hbar$ has units of action. Time independence of H is a property of the dynamical law; it does not generally make $\rho(s)$ constant. A *stationary state* satisfies $[H, \rho] = 0$. A pure state satisfying $H|\Psi\rangle = E|\Psi\rangle$ is stationary as a density operator, since its overall phase has no observable effect.

A *relational description* assigns changes in one subsystem to readings of another physical subsystem used as a clock. A *history encoding* represents an entire specified sequence of conditional states in one joint state. Finally, a *physical reservoir hypothesis* proposes a system whose encoded content and accessible interaction explain a particular information gain. These statements have different logical content. A stationary joint state can possess relational dynamics; a representation of a history need not contain the answer to an arbitrary query; and a stored answer need not have an accessible readout.

Page and Wootters established the foundational stationary-state approach to relational evolution [17]. Subsequent treatments make the choice of clock, constraint, physical state space and conditional observables explicit. Höhn, Smith and Lock connect relational Schrödinger, Heisenberg and Dirac descriptions under stated assumptions, using covariant time observables [18]. Smith and Ahmadi examine clock–system interactions and the resulting conditional equations [19]. These are substantive foundations for a model of timeless access: they explain how relational evolution can be defined without inserting an external clock into the global description.

An experimental illustration also exists. Moreva and colleagues used entangled photon polarizations to contrast a stationary joint description with conditional evolution relative to a photon

serving as a clock [20]. The laboratory result concerns the prepared photonic state and its measurement arrangement. Its relevance here is that relational conditional dynamics has a concrete operational example.

4.2 An access relation must specify an instrument

Let T take values in a finite target set $\mathcal{T}$. Let B denote baseline information and Q a query; fix a stratum $B = b, Q = q$ with positive probability. A candidate quantum realization specifies a target-indexed ensemble of states $\rho_{LR}^{\tau,b,q}$ on $\mathcal{H}_L \otimes \mathcal{H}_R$, where L contains the local device and R the proposed reservoir. It also specifies an instrument $\{\mathcal{J}_a^{b,q}\}_{a\in\mathcal{A}}$ acting from operators on $\mathcal{H}_L \otimes \mathcal{H}_R$ to operators on a declared finite output space $\mathcal{H}_{\text{out}}$: each $\mathcal{J}_a^{b,q}$ is a completely positive, trace-nonincreasing map, and their sum is trace preserving. The answer distribution is

$$\Pr(A = a \mid T = \tau, b, q) = \operatorname{tr} \mathcal{J}_a^{b,q}(\rho_{LR}^{\tau,b,q}). \tag{1}$$

An erasure or unsuccessful query is an allowed answer in $\mathcal{A}$. Physical implementation restricts which instruments can act on R and how their outcomes reach L. Merely defining an arbitrary joint instrument in (1) does not grant it to a cognitive system. Section 9 therefore restricts access to a specified interaction followed by local measurement.

This distinction matters even within relational quantum mechanics. Hausmann, Schmidhuber and Castro-Ruiz compare measurements represented by constraint invariant observables with measurements introduced through a dynamical modification of the constraint. The approaches agree for ideal clocks but can have different operational consequences for finite-resource clocks. Their analysis notes that some equations nonlocal in a clock label can be recast as ordinary unitary evolution; their causal interpretation therefore requires additional argument [21]. Conversely, Kuypers and Rijavec explicitly construct a clock–timer interaction within a stationary universal description [22]. Together these results support a constructive requirement: write the measurement interaction and identify the record, instead of assuming that stationarity forbids measurement or that conditioning implements it for free.

4.3 Correlations, controllable signals and recoverable records

An arbitrary entangled state is compatible with strict limits on local readout. Let Λ_R be a completely positive trace-preserving map acting only on R, and let M_a be any effect on L, with $0 \le M_a \le I_L$. The adjoint map is defined by $\operatorname{tr}[X\Lambda_R(Y)] = \operatorname{tr}[\Lambda_R^\dagger(X)Y]$ and obeys $\Lambda_R^\dagger(I_R) = I_R$. Hence

$$\begin{aligned}&\operatorname{tr}[(M_a \otimes I_R)(\mathrm{id}_L \otimes \Lambda_R)(\rho_{LR})] \\ &\quad = \operatorname{tr}\big[(M_a \otimes \Lambda_R^\dagger(I_R))\rho_{LR}\big] = \operatorname{tr}[(M_a \otimes I_R)\rho_{LR}].\end{aligned} \tag{2}$$

This is the ordinary no-signalling calculation. A remotely selected, trace-preserving operation cannot change the local answer distribution. Conditioning on a remote measurement outcome can change the conditional state, but obtaining the outcome label is then an additional communication resource. The established dense-coding protocol illustrates the resource distinction sharply: shared entanglement becomes useful for enhanced communication when a physical system is subsequently

transmitted [23]. Information-causality constraints likewise refer to a specified communication task and its supplied resources [24].

Other quantum results constrain different parts of an archive proposal. The no-broadcasting theorem excludes universal broadcasting of noncommuting mixed-state families, while allowing jointly commuting families [25]. Thus a reservoir of classical records and a reservoir offering arbitrary unknown quantum states are materially different proposals. Braunstein and Pati's no-hiding result locates the information about an arbitrary quantum input in the complementary output when the designated output is independent of that input, in the specified isometric setting [26]. Location in a complementary system is not an algorithm for accessible recovery. Ollivier, Poulin and Zurek show how environmental fragments can carry redundant records of selected system properties [27]. Such records can become readable precisely because the physical interaction selects and distributes particular information. A claim about accessible semantic content must identify that selection and distribution.

4.4 Future conditioning and future intervention

Time-symmetric boundary conditions deserve a place in the theory's landscape. They also require a clear probability space. For normalized preselected and postselected vectors $|\psi\rangle$ and $|\phi\rangle$, and a specified intermediate projective measurement $\{P_j\}$ with $P_j P_k = \delta_{jk} P_j$ and $\sum_j P_j = I$, the Aharonov–Bergmann–Lebowitz rule is

$$\Pr(j \mid \psi, \phi, \{P_k\}) = \frac{|\langle \phi | P_j | \psi \rangle|^2}{\sum_k |\langle \phi | P_k | \psi \rangle|^2}, \tag{3}$$

provided the denominator is positive; intervening unitary evolutions may be absorbed into the two boundary vectors. The equation conditions on a successful final selection and on the intermediate measurement context [28]. Its denominator and rejected trials cannot disappear when assessing an earlier predictor.

The operational distinction is simple. If an earlier answer A and a later target T have a joint distribution, then $\Pr(A \mid T = \tau)$ may differ across target values through ordinary correlations or through selection. A stronger proposition is that a freely selectable future intervention changes the earlier marginal:

$$\Pr(A \in \mathcal{E} \mid \mathrm{do}(T = \tau_0)) \neq \Pr(A \in \mathcal{E} \mid \mathrm{do}(T = \tau_1)) \tag{4}$$

for some event $\mathcal{E} \subseteq \mathcal{A}$. Here $\mathrm{do}(T = \tau)$ means an exogenous experimental choice, with the earlier preparation held fixed. Distinct earlier answer laws under those choices constitute an operational backward communication channel. A conditional history alone does not establish (4).

Leifer and Pusey derive a substantial tension between a particular operational/ontological time-symmetry requirement, mediation by an ontic state, and absence of retrocausality [29]. The assumptions are part of the theorem, and their distinction between operational signalling and ontological dependence matters here. More general frameworks also exist: Oreshkov, Costa and Brukner study local quantum operations without assuming a fixed global causal order [30]. Entering such a framework requires a valid process and an account of its implementation. It does not transfer every ordinary-channel conclusion to the new framework, or establish a cognitive implementation of its exceptional processes.

4.5 Thermodynamic bookkeeping and physical completion

Readout and resetting are different operations. For a concrete erasure process, let a finite memory M initially be uncorrelated with a thermal bath E, with bath Hamiltonian H_E and state $\rho_E = e^{-\beta H_E}/\operatorname{tr}(e^{-\beta H_E})$, where $0 < \beta < \infty$ has units of inverse energy. Apply a unitary to ME and denote final quantities by primes. In this paragraph use natural-logarithm entropy $S_{\mathrm{n}}(\rho) = -\operatorname{tr}(\rho \ln \rho)$, relative entropy $D_{\mathrm{n}}(\rho\|\sigma) = \operatorname{tr}[\rho(\ln\rho - \ln\sigma)]$, and $I_{\mathrm{n}}(M:E) = S_{\mathrm{n}}(\rho_M) + S_{\mathrm{n}}(\rho_E) - S_{\mathrm{n}}(\rho_{ME})$. Reeb and Wolf's equality reads

$$\beta \Delta Q_E = S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\rho'_M) + I_{\mathrm{n}}(M' : E') + D_{\mathrm{n}}(\rho'_E \| \rho_E), \qquad \Delta Q_E = \operatorname{tr}[H_E(\rho'_E - \rho_E)]. \tag{5}$$

Its thermal, initial-product and unitary assumptions are essential [31, Theorem 3]. Quantum side information can change erasure-work accounting, including regimes governed by negative conditional entropy [32]. Such resources must be included in the process budget. These results provide no universal energy price for a thought or for a query to an unspecified reservoir.

A completed physical hypothesis would identify accessible degrees of freedom, their state or boundary condition, an addressing mechanism, the interaction with L, the relevant coherence and error scales, and the preparation and renewal of its resources. Sections 8 and 9 show that these requirements can be made concrete in a finite reference construction. They convert a question about timeless access into an explicit compatibility result and a quantitative coupling requirement.

5 An operational framework for reservoir access

A mind's encounter with information has two distinct mathematical aspects: what can be inferred from its admitted evidence, and what additional evidence an access process supplies. The distinction gives the reservoir hypothesis an exact point of contact with experiment. An answer that resolves uncertainty beyond an appropriate baseline must exhibit a statistical dependence that the baseline model does not contain. The dependence must then be explained physically. This section quantifies that obligation using established information theory, with proofs specialized to the present access problem [33–35].

5.1 Finite records and an explicit baseline

Let T, B, Q, and A be jointly distributed random variables on finite sets $\mathcal{T}$, $\mathcal{B}$, $\mathcal{Q}$, and $\mathcal{A}$, respectively. The target alphabet has size $M = |\mathcal{T}| \geq 2$. The variable T identifies the target fact or finite message to be recovered; B records the admitted baseline information; Q records the query; and A records the response before the target is revealed for scoring. Every set has its full power-set sigma-algebra. A decoder is a function $d : \mathcal{B} \times \mathcal{Q} \times \mathcal{A} \to \mathcal{T}$, and $\widehat{T} = d(B, Q, A)$ is its estimate. Private randomization is allowed if its seed is conditionally independent of the target given the decoder's inputs. It cannot outperform the best deterministic decoder for the average success criterion used below.

All logarithms in information quantities are base two. We use $0 \log_2 0 = 0$, and omit conditioning values of zero probability. The conditional entropy $H(T \mid B, Q)$ is the average entropy of the target's conditional law. The information gained from the response is

$$\eta = I(T; A \mid B, Q) = H(T \mid B, Q) - H(T \mid B, Q, A) \geq 0. \tag{6}$$

This is a property of a specified joint distribution. A finite data set supplies an estimate or a test statistic, not direct access to the population value η.

The best baseline and response-assisted success probabilities are

$$p_0 = \sum_{b,q} \max_{t \in \mathcal{T}} P(T = t, B = b, Q = q), \tag{7}$$

$$p_1 = \sum_{b,q,a} \max_{t \in \mathcal{T}} P(T = t, B = b, Q = q, A = a). \tag{8}$$

The nonnegative difference $p_1 - p_0$ is an optimal recovery advantage. It compares equally well-informed ideal decoders, rather than comparing a sophisticated proposed method with an artificially weak baseline. Actual algorithms may fall short of either optimum.

The query is included in the baseline because it can carry information. If a query is produced from B and fresh randomness, with $T \perp Q \mid B$, then the baseline optimum equals $p_{\text{guess}}(T \mid B)$. Without that condition, removing Q from the comparison would credit the response for information already supplied in the question. The analysis does not assume that T is independent of the proposed reservoir. Such independence belongs to particular null models; a reservoir alternative may instead predict a definite correlation and must explain how it arises.

5.2 Specifying the reservoir and access mechanism

Introduce a finite candidate reservoir variable $R \in \mathcal{R}$ and a physical-model index $\theta \in \Theta$. A finite operational access model specifies a joint law $P_\theta(T, B, Q, R)$ and a stochastic kernel

$$K_\theta : \mathcal{R} \times \mathcal{B} \times \mathcal{Q} \longrightarrow \Delta(\mathcal{A}),$$

where $\Delta(\mathcal{A})$ is the probability simplex on the answer alphabet. The notation $K_\theta(a \mid r, b, q)$ denotes the probability assigned to response a, so its entries are nonnegative and sum to one for each admitted (r, b, q). The completed finite model is

$$P_\theta(t, b, q, r, a) = P_\theta(t, b, q, r) K_\theta(a \mid r, b, q). \tag{9}$$

Within this model class, target information reaching the response passes through the baseline, query, or reservoir. A physically derived model must justify that factorization or replace it with the additional systems it needs.

No independence between T and R is imposed in Equation (9). Correlation with the target is precisely what an access hypothesis may propose. The mathematical obligation is to explain its origin and the kernel that makes it available. Merely choosing $R = T$ and $A = R$ constructs a perfect formal oracle, but leaves its physical realization entirely to be supplied. A successful theory would identify the reservoir's states, the query-sensitive interaction, the response observable, and the circumstances under which its predicted joint law applies.

As a useful null case, if $T \perp R \mid B, Q$, summing Equation (9) over r gives $T \perp A \mid B, Q$. This inference is conditional on the specified source model. It does not refute alternatives in which a supported physical mechanism correlates the reservoir with the target. The access theorems below quantify what such a mechanism would have to provide.

5.3 A catalogue of every possibility still needs a target-bearing address

Counterexample 5.1 (A catalogue without an identifying relation)**.** Let $\mathcal{T} = \{1, \dots, M\}$, let T be uniform, and let $R = (1, \dots, M)$ be a fixed catalogue containing every possible target. Choose an independent uniform address $Q \in \mathcal{T}$, with constant B, and return $A = R_Q = Q$. The fixed reservoir contains the true target among its entries, but the queried response satisfies

$$\Pr(A = T) = \frac{1}{M}, \qquad I(T; A \mid Q) = 0.$$

If the address is instead supplied as $Q = T$, then $A = T$ with certainty. In that second experiment $p_0 = 1$ already, because the query itself reveals the target; the additional response information remains zero. The information that identifies the realized alternative is carried by the address.

The counterexample applies equally to the lexicographic catalogue of all binary strings of a fixed length. It does not depend on how impressive the catalogue's size appears. The relevant distinction is between containing every candidate and selecting the candidate related to this target. An effective reservoir proposal needs both content and an addressability relation. Naming an entropy threshold or resonance condition does not specify that relation until the condition is connected to physical states and a response law.

5.4 A coherent static reference model

There is also a constructive finite model. Let $R = (R_1, \dots, R_n)$ be independent fair bits stored in an external register whose values remain fixed during a trial. Choose an independent uniform query $Q \in \{1, \dots, n\}$, define the requested target as $T = R_Q$, and let the response be $A = T \oplus N$, where N is an independent Bernoulli error bit with parameter $\epsilon \in [0, 1/2]$. Here $\oplus$ denotes addition modulo two. For constant B,

$$p_0 = \frac{1}{2}, \qquad p_1 = 1 - \epsilon, \qquad I(T; A \mid Q) = 1 - h_2(\epsilon). \tag{10}$$

The entropy function h_2 is defined in the next section. This model gives exact content, addressing, noise, and recovery rules. Its register is static over the trial, which is weaker than an assertion of ontological timelessness. Its role is to show the kind of complete probabilistic composition that a stronger physical proposal must recover or replace.

Counterexample 5.2 (An informative reservoir with an uninformative interface)**.** Let T and K_0 be independent fair bits, with constant B, Q. The subscript distinguishes this key bit from the access kernel. Let $R = (T, K_0)$, and define the sole available response by $A = T \oplus K_0$. The reservoir determines the target, so $I(T; R) = 1$, but the response is a fair bit independent of the target, so $I(T; A) = 0$. Supplying the key as additional baseline information changes the task: $I(T; A \mid K_0) = 1$, and the decoder $T = A \oplus K_0$ is exact.

Content, addressing, and usable interaction are separate requirements. The exhaustive catalogue lacks a relation selecting the realized target. The masked reservoir contains that relation but the specified interface conceals it. The static register supplies both a selected entry and a useful noisy readout. These finite constructions establish distinctions that any proposed deeper architecture must respect; they do not select its ontology.

The key example also shows why the baseline must be retained in information calculations. An individually uninformative side variable can unlock information in a second variable. A study that evaluates the response alone and another that supplies the key are conducting different tasks. Similar care is needed whenever a proposed access operation uses a learned codebook, a symbol dictionary, a trained decoder, or a remembered cue. Their joint contribution belongs in the declared model, even when no single component looks like a conventional memory of the answer.

5.5 Discovery within a baseline and access beyond it

Suppose a target is a deterministic consequence $T = f(B, Q)$ of the admitted evidence, even though computing f is difficult for the available solver. Then $H(T \mid B, Q) = 0$ and every response satisfies $I(T; A \mid B, Q) = 0$. A newly discovered proof, a successful calculation, or a fruitful reformulation can still be a major intellectual achievement. The information criterion addresses new evidence about an unresolved external target; it does not measure every form of originality or computational progress.

Conversely, an incomplete baseline can make ordinary retrieval look like unexplained access. If a learned parameter, a forgotten exposure, a hidden sensor, or a retained external record is omitted from B, its contribution may appear as positive conditional information. The formal baseline therefore represents a declared evidential account, not merely what a person or system consciously recollects. This is the central experimental distinction behind the title: absence of experienced recollection does not by itself locate the source of an answer. The literal reservoir hypothesis must make predictions that survive an explicit account of the information otherwise available.

6 Information, recovery, and selective evidence

6.1 A quantitative obligation for additional access

Define a comparison distribution on the same finite space by

$$\widetilde{P}(t, b, q, a) = P(t, b, q)P(a \mid b, q). \tag{11}$$

It preserves the target, baseline, query, and response marginals relevant to the comparison, while removing target–response dependence conditional on (B, Q). Let

$$\delta = \mathrm{TV}(P, \widetilde{P}) = \frac{1}{2} \sum_{t,b,q,a} |P(t, b, q, a) - \widetilde{P}(t, b, q, a)|. \tag{12}$$

The quantities p_0, p_1, δ are dimensionless probabilities in $[0, 1]$; η is measured in bits.

Theorem 6.1 (Conditional recovery and information budget)**.** *For the finite model above,*

$$0 \le p_1 - p_0 \le \min\left\{1 - p_0,\ \delta,\ \sqrt{\frac{\ln 2}{2}\,\eta}\right\}. \tag{13}$$

Consequently, a recovery advantage of at least $\Delta \ge 0$ requires $\eta \ge 2\Delta^2/\ln 2$. If $A \perp T \mid B, Q$, then $p_1 = p_0$ and $\eta = 0$.

Proof. Ignoring the response gives $p_1 \geq p_0$. Fix any decoder d and let $E_d = \{(t, b, q, a) : d(b, q, a) = t\}$. Under $\widetilde{P}$, its success probability is

$$\sum_{b,q} P(b, q) \sum_{a} P(a \mid b, q) P(T = d(b, q, a) \mid b, q) \leq p_0.$$

The event characterization of total variation gives $P(E_d) \leq \widetilde{P}(E_d) + \delta \leq p_0 + \delta$. Maximizing over the finite set of deterministic decoders proves the total-variation bound. The upper bound $p_1 \leq 1$ gives the first term in the minimum.

For completeness, the finite Pinsker inequality can be established by grouping a pair of distributions into the event on which the first has the larger mass and its complement. The log-sum inequality reduces the natural-log relative entropy to the Bernoulli relative entropy at probabilities u, v. At fixed $v \in (0, 1)$, the latter has value and first derivative zero at $u = v$, and second derivative $1/[u(1 - u)] \geq 4$. Integration twice gives a lower bound $2(u - v)^2$. Boundary cases follow by continuity or infinite divergence. The mass difference of the grouping event is the total-variation distance. Thus

$$2\delta^2 \leq D_{\mathrm{nat}}(P \| \widetilde{P}) = (\ln 2) I(T; A \mid B, Q).$$

Here $D_{\mathrm{nat}}(P \| Q) = \sum_x P(x) \ln[P(x)/Q(x)]$, with its usual extended-value convention. Wherever P is positive, the conditional-product law is positive as well. Its relative entropy is therefore finite. Conditional independence makes the two distributions identical and finishes the proof. □

The argument uses standard total-variation and relative-entropy tools; it is not a new information law. The normalization matters: some literature calls the full ℓ^1 distance the variational distance, twice the convention used here [36]. The theorem converts a claimed advantage into a minimum information requirement. For example, a stipulated 0.1 increase over the optimal baseline requires at least $0.02/\ln 2 \approx 0.02885$ bits of conditional information. This is a necessary condition for that model, not an estimate of an observed effect.

6.2 Accounting for an ordinary leakage channel

Suppose a finite variable Z_{leak} describes additional ordinary information reaching the response process, and

$$T \perp A \mid B, Q, Z_{\mathrm{leak}}. \tag{14}$$

Then the conditional chain rule gives

$$I(T; A, Z_{\mathrm{leak}} \mid B, Q) = I(T; Z_{\mathrm{leak}} \mid B, Q) \tag{15}$$

$$= I(T; A \mid B, Q) + I(T; Z_{\mathrm{leak}} \mid B, Q, A), \tag{16}$$

and therefore

$$\eta \leq I(T; Z_{\mathrm{leak}} \mid B, Q). \tag{17}$$

If an independently justified analysis bounds the right side by $\lambda_{\mathrm{leak}} \geq 0$ bits, the recovery advantage is at most $\sqrt{(\ln 2)\lambda_{\mathrm{leak}}/2}$, also capped by $1 - p_0$. The symbol λ_{leak} denotes an information budget here, not a physical coupling constant.

The utility of this statement depends on the leakage model. Calling a channel "sealed" does not establish a value of λ_{leak}. Nor does failing to notice a route prove that it is absent. Conversely,

a quantified upper bound has scientific force: an outcome exceeding the joint model's bound would require at least one of its information, measurement, or sampling assumptions to be revised. Determining which assumption failed is a separate inference task.

6.3 Information is richer than one guessing score

The converse of the theorem's last statement is false if it is phrased only in terms of unchanged optimal accuracy. Consider constant B, Q, a binary target, and the joint table

	$A = 0$	$A = 1$
$T = 0$	18/40	18/40
$T = 1$	1/40	3/40

The best guess remains $T = 0$ after either answer, so $p_0 = p_1 = 0.9$. Nevertheless, the response changes the target posterior from 0.9 to 18/19 or 6/7, and $\eta > 0$. It improves calibrated probabilistic knowledge without changing the most likely label. Entropy and minimum error are related but do not determine each other in general [34, 37].

This distinction matters for research on insight. A confidence refinement, a useful new hypothesis, and exact recovery of a hidden finite target are different achievements. A test should declare which one it measures. The operational framework can accommodate several scores, but the theorem above concerns optimal exact-target recovery. It should not be silently promoted into a complete measure of intellectual creativity or understanding.

An unlimited proposed reservoir would still have to reach a finite observer through some response process. The relevant resource is the set of distinctions the observer can actually recover. Classical alphabet size, error probability, and selective reporting provide complementary constraints. Their roles are different: one bounds the number of available response states, another bounds the information required for a target accuracy, and the third tracks how a reporting rule changes the population being judged.

6.4 A finite response cannot identify an arbitrary number of fresh targets

Let $K = |\mathcal{A}| \geq 1$, and retain the definitions of p_0, p_1 from Section 5. For every (b, q) of positive probability,

$$\sum_a \max_t \{P(t \mid b, q)P(a \mid t, b, q)\} \leq K \max_t P(t \mid b, q).$$

The inequality follows because each conditional channel probability is at most one. Averaging proves

$$p_1 \leq \min\{1, Kp_0\}. \tag{18}$$

If the target is uniform over M possibilities conditional on every admitted (b, q), this becomes $p_1 \leq \min\{1, K/M\}$. The conditional-uniformity premise is essential. Highly predictable targets can be recovered accurately with little new information.

In the classical setting, define conditional guessing min-entropy by $H_{\min}(T \mid B, Q) = -\log_2 p_0$. Equation (18) is equivalently

$$H_{\min}(T \mid B, Q, A) \geq H_{\min}(T \mid B, Q) - \log_2 K. \tag{19}$$

The use of guessing probability is consistent with the established operational interpretation of conditional min-entropy [35]. It is particularly useful for one finite trial, where an asymptotic channel-capacity statement may not answer the question asked.

A response consisting of at most m fixed binary positions has $K \leq 2^m$. A fresh uniform n-bit target therefore has whole-word recovery probability at most $\min\{1, 2^{m-n}\}$. This counts available distinctions, rather than the apparent richness of a symbol. If a glyph's angle, color, timing, amplitude, or context is permitted to encode additional information, those degrees of freedom belong in the response model. A claim of infinite precision cannot be inserted into an experiment with finite measurement resolution.

The alphabet constraint has a sharper prior-sensitive form. For each (b, q), sort the conditional target probabilities as $p_{(1)}(b, q) \geq \cdots \geq p_{(M)}(b, q)$. Then

$$p_1 \leq \sum_{b,q} P(b, q) \sum_{j=1}^{\min\{K,M\}} p_{(j)}(b, q). \tag{20}$$

For a deterministic decoder at a fixed baseline, its K possible inputs can produce at most K distinct target labels. Correct recovery is confined to those labels, whose total prior probability cannot exceed the mass of the K most probable targets. Randomization cannot improve the optimal average. This proves the bound without assuming a uniform prior.

The refined expression can matter when rare alternatives coexist with a dominant one. Counting M labels alone overstates the difficulty if most probability is concentrated on a few labels; treating every successful answer as a fresh $\log_2 M$-bit revelation then overstates the evidence. Explicit target randomization is valuable precisely because it establishes a transparent prior for the intended task. For semantic or naturally occurring targets, the distribution must instead be modeled and defended.

6.5 The entropy needed for reliable answers

Define the binary entropy $h_2 : [0, 1] \to [0, 1]$ by

$$h_2(e) = -e \log_2 e - (1 - e) \log_2(1 - e).$$

For any decoder let $e = \Pr(\widehat{T} \neq T)$. The standard Fano argument, written here in the conditional access notation, yields

$$\eta \geq H(T \mid B, Q) - h_2(e) - e \log_2(M - 1). \tag{21}$$

To see this, introduce the binary error indicator $E = \mathbf{1}\{\widehat{T} \neq T\}$. Once T, B, Q, A are known, E is determined. Consequently

$$H(T \mid B, Q, A) = H(E \mid B, Q, A) + H(T \mid E, B, Q, A).$$

The first term is at most $h_2(e)$. When $E = 0$, the target equals the decoder's estimate; when $E = 1$, it belongs to at most $M - 1$ remaining values. The second term is therefore at most $e \log_2(M - 1)$. Subtracting from $H(T \mid B, Q)$ proves the claim. For a randomized decoder, one can include an independent seed among its observations without altering the information available about the target.

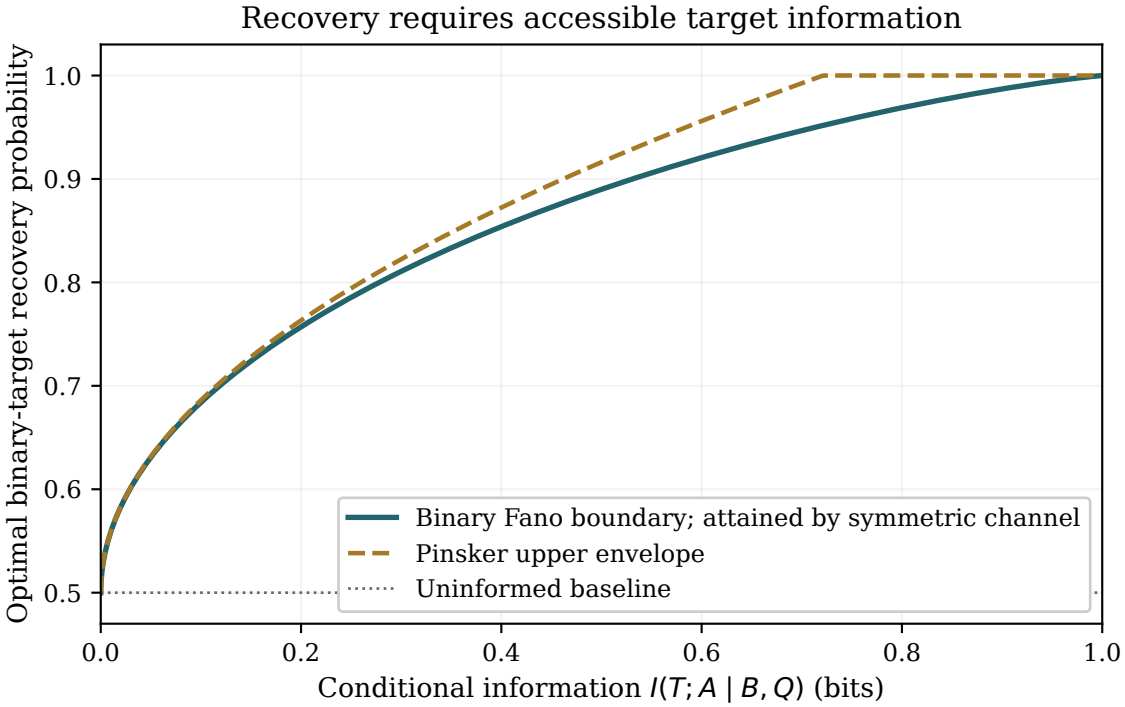


Figure 1: Information required for recovering a conditionally uniform binary target. The solid boundary follows from binary Fano and is attained by the constructed binary symmetric channel in Equation (10); the dashed curve is the looser Pinsker upper envelope from Equation (13). The horizontal axis is the population conditional mutual information, not an estimator from observations. These curves depict mathematical channels, not measured cognitive access.

For a conditionally uniform target, the right side begins with $\log_2 M$. A claimed accuracy close to one must then be supported by nearly the target's entire information content. When only an upper bound $e \le \bar{e}$ is available, replacing e by $\bar{e}$ requires $\bar{e} \le 1 - 1/M$; on that interval the Fano error expression is increasing. For larger declared tolerances the bound should be evaluated directly or replaced by its trivial entropy ceiling. Fixed nonuniform target distributions can admit stronger bounds; the expression above is not claimed to be optimal for every prescribed prior [34, 37].

Alphabet and entropy constraints can be used together. Since $\eta \le H(A \mid B, Q) \le \log_2 K$, any proposed finite response must satisfy both the alphabet-size inequality and the Fano requirement. One cannot compensate for an inadequate channel by naming a more extensive reservoir. The external structure, physical interaction, and accessible record must compose into a channel with the required distinguishing power.

6.6 A reporting rule is part of the information model

Let $J \in \{0, 1\}$ indicate whether a response is retained, and let $s = \Pr(J = 1)$. Define the published record

$$Y = \begin{cases} (1, A), & J = 1, \\ (0, \perp), & J = 0, \end{cases}$$

where $\perp$ is a failure or withheld-response symbol distinct from every element of $\mathcal{A}$. The full public record retains the fact that a trial was not accepted. No conditional-on-acceptance quantity is defined when $s = 0$.

Proposition 6.2 (Information accounting for selection)**.** *If $s > 0$, then*

$$I(T; Y \mid B, Q) = I(T; J \mid B, Q) + sI(T; A \mid B, Q, J = 1). \tag{22}$$

Consequently, the retained-trial information is at most $I(T; Y \mid B, Q)/s$. If the reporting rule satisfies $T \perp J \mid B, Q, A$, then

$$sI(T; A \mid B, Q, J = 1) \le I(T; Y \mid B, Q) \le \eta. \tag{23}$$

Proof. The record determines J, and its second component is constant when $J = 0$. The conditional chain rule therefore gives Equation (22); the averaging weight of the nonconstant branch is s. Nonnegativity gives the first inequality in Equation (23). Under the additional conditional-independence premise, $I(T; A, J \mid B, Q) = I(T; A \mid B, Q) = \eta$. The published record is a function of (A, J), so conditional data processing supplies the second inequality. □

The criterion $T \perp J \mid B, Q, A$ permits a response-dependent confidence rule or a preregistered abstention rule. It excludes a reporting decision that learns the hidden target and then selects matches. The latter rule adds target information through evaluation itself. Its effects must not be credited to the original response process.

Counterexample 6.3 (Perfect accuracy manufactured by retaining matches)**.** Let T and A be independent uniform draws from $\{1, \dots, M\}$, with constant B, Q. Retain a trial exactly when $A = T$. Then

$$I(T; A) = 0, \qquad s = \frac{1}{M}, \qquad \Pr(A = T \mid J = 1) = 1.$$

Moreover, $I(T; J) = 0$ while $I(T; Y) = (\log_2 M)/M$. The reported perfect accuracy and positive published-record information come from a selector that compared the answer with the target. The response generator itself remains independent of the target. Every equality follows by enumerating the M^2 equally probable pairs.

This example has a direct methodological consequence. The number of attempted trials, rejection reasons, timing of acceptance decisions, and outcomes of rejected attempts belong in the evidential record. A striking set of retained coincidences does not identify the process that generated them. A legitimate study can allow abstention, but it must report the resulting success frequency and the rule's access to target information.

6.7 Selective accuracy under heterogeneous baselines

Let $r(b, q) = \Pr(J = 1 \mid b, q)$, and define

$$b_J = \sum_{b,q} P(b, q) r(b, q) \max_t P(t \mid b, q).$$

This is the baseline success mass for the observed acceptance rates across baseline strata. Form the conditional-product law $\widetilde{P}_J(t,b,q,j,a) = P(t,b,q)P(j,a \mid b,q)$, and let $\delta_J = \mathrm{TV}(P_{TBQJA}, \widetilde{P}_J)$. For any decoder, the same event argument used in Theorem 6.1 gives

$$\Pr(J = 1, \widehat{T} = T) \le b_J + \delta_J. \tag{24}$$

If the target is uniform conditional on every (b,q), then $b_J = s/M$. For $s > 0$, selective accuracy is at most $\min\{1, 1/M + \delta_J/s\}$. The factor $1/s$ explicitly exposes the instability of an analysis based on a very small retained subset.

When some baseline strata are easier than others, a selector can improve retained accuracy simply by choosing easier cases. The appropriate comparison then uses b_J, rather than an unconditional chance level. The formula makes the distinction without prohibiting useful abstention: a system may specialize, provided that its reported performance identifies the population and the evidence on which it specializes.

7 The cost of a queryable archive

A reservoir claim becomes substantially stronger when it promises answers to questions selected after the observer acquires its physical interface. Such an interface must preserve several possible distinctions while supporting a later choice of which distinction to read. This is the setting of random-access coding, an established subject in classical and quantum information theory. It is especially relevant to a proposed archive of latent knowledge: a description may contain many amplitudes or formal possibilities while allowing only a limited set of reliable queries.

The bounds below concern a finite accessible register with a specified encoding and measurement. They place no universal finite-dimensional bound on an entire cosmos. Their purpose is to identify the resource that a candidate cognitive interface must supply. A larger external reservoir can support repeated interactions, but each additional interaction then belongs in the access protocol and its resource account.

7.1 A finite quantum readout

Let $\mathcal{H}_S$ be a complex Hilbert space of dimension $d \in \mathbb{N}$, $d \ge 1$. The symbol S denotes the complete quantum register made available for the readout considered here. A state ρ is a positive semidefinite operator on $\mathcal{H}_S$ with $\operatorname{tr}\rho = 1$. An operator inequality $X \ge 0$ means that $\langle v, Xv\rangle \ge 0$ for every vector v. A positive operator-valued measurement, or POVM, with outcomes in $\mathcal{T}$ is a family $\{E_t : t \in \mathcal{T}\}$ satisfying $E_t \ge 0$ and $\sum_t E_t = I_S$, where I_S is the identity operator. Its outcome law is the Born rule $\Pr(\widehat{T} = t \mid \rho) = \operatorname{tr}(E_t\rho)$.

Suppose T is uniform on $M \ge 2$ values and is encoded into states ρ_t of this register. A decoder may use any POVM. Then

$$p_{\mathrm{guess}}(T \mid S) = \max_{\{E_t\}} \frac{1}{M} \sum_t \operatorname{tr}(E_t\rho_t) \le \min\left\{1, \frac{d}{M}\right\}. \tag{25}$$

Indeed, every density operator satisfies $0 \le \rho_t \le I_S$, because its nonnegative eigenvalues sum to one. Therefore $\operatorname{tr}(E_t\rho_t) \le \operatorname{tr} E_t$, and summing gives d/M. This elementary form is a specialization

of established decoding bounds [38]. It remains valid for measurements implemented with a fresh ancilla prepared in a fixed product state: such a procedure induces a POVM on the original register.

For $d \leq M$, the bound is attainable: assign d of the target values to distinct orthogonal states, use the corresponding projective measurement, and allow all remaining values to share any of those states. Exactly d target labels are decoded correctly in this construction. If $d \geq M$, M orthogonal states permit perfect recovery. Thus the dimension factor is operational, rather than an artifact of an entropy estimate.

The same statement applies conditional on classical (B, Q) whenever the target is uniform in each stratum and the entire available quantum register has dimension at most d. It must not be applied to the dimension of a transmitted subsystem while ignoring a quantum reference already held by the receiver. Entanglement-assisted communication illustrates that distinction: a transmitted qubit and a retained entangled qubit can jointly distinguish four encoded alternatives [23]. Their jointly measured register has dimension four. The resource accounting must include the state that is actually measured.

For a nonuniform target with ordered prior probabilities $p_{(1)} \geq \cdots \geq p_{(M)}$, the corresponding established bound is

$$p_{\mathrm{guess}}(T \mid S) \leq \sum_{j=1}^{\min\{d,M\}} p_{(j)}. \tag{26}$$

To verify it directly, put $w_t = \mathrm{tr}(E_t \rho_t)$. Each w_t lies in $[0, 1]$, and the trace argument above gives $\sum_t w_t \leq d$. Maximizing $\sum_t p_t w_t$ under those relaxed constraints assigns unit weight to the largest d probabilities, or to every probability when $d \geq M$. This proves Equation (26). Encoding those selected targets in orthogonal states attains the bound. The statement is Nayak's prior-sensitive decoding result in the present finite notation [38].

7.2 A bound for questions chosen after encoding

Now let the target be the full binary word $T = (T_1, \ldots, T_n)$, where $n \geq 1$ and the bits are independent and uniform. For each word $t \in \{0, 1\}^n$, an encoder prepares a state ρ_t in $\mathcal{H}_S$. The encoding is completed before the query index $Q \in \{1, \ldots, n\}$ is selected independently of the word and of the encoded state. On query $Q = i$, a binary POVM $\{E_{0|i}, E_{1|i}\}$ returns an estimate A_i of T_i. Write

$$e_i = \Pr(A_i \neq T_i), \qquad \bar{e} = \frac{1}{n} \sum_{i=1}^{n} e_i.$$

Relabeling a binary decoder if necessary allows $e_i \leq 1/2$. The average is over the uniform input words and the measurement law. Each A_i refers to the experiment in which that query is chosen; it does not assume that all the measurements can be performed on one copy without disturbance.

Theorem 7.1 (Finite-register random-access requirement)**.** *Under these assumptions,*

$$\log_2 d \geq \sum_{i=1}^{n} [1 - h_2(e_i)] \geq n[1 - h_2(\bar{e})]. \tag{27}$$

In particular, an m-qubit register, of dimension $d = 2^m$, that answers every query with success probability at least $p \in [1/2, 1]$ requires

$$m \geq n[1 - h_2(p)]. \tag{28}$$

Proof. For a density operator ρ, let $S(\rho) = -\operatorname{tr}(\rho \log_2 \rho)$ be its von Neumann entropy, with zero eigenvalues contributing zero. Form the classical–quantum state

$$\rho_{TS} = 2^{-n} \sum_t |t\rangle\langle t| \otimes \rho_t, \qquad \bar{\rho} = 2^{-n} \sum_t \rho_t.$$

Here $\{|t\rangle\}$ is the orthonormal basis of a separate classical register. For this state,

$$I(T; S) = S(\bar{\rho}) - 2^{-n} \sum_t S(\rho_t) \leq \log_2 d. \tag{29}$$

The inequality uses $S(\rho_t) \geq 0$ and $S(\bar{\rho}) \leq \log_2 d$. The quantum mutual information is defined by $I(U; V) = S(U) + S(V) - S(UV)$, and conditional quantum mutual information by $I(U; V \mid W) = S(UW) + S(VW) - S(W) - S(UVW)$, for the reduced states in question.

The chain rule and independence of the classical bits give

$$I(T; S) = \sum_i I(T_i; S \mid T_1, \ldots, T_{i-1}), \tag{30}$$

$$I(T_i; S \mid T_{<i}) - I(T_i; S) = I(T_i; T_{<i} \mid S) \geq 0. \tag{31}$$

The last inequality is strong subadditivity of quantum entropy, used here as an established theorem [39]. Quantum data processing under the ith measurement gives $I(T_i; S) \geq I(T_i; A_i)$. This is the accessible-information step underlying the Holevo bound [40]. Binary Fano then gives $I(T_i; A_i) \geq 1 - h_2(e_i)$, because $H(T_i) = 1$. Combining these inequalities with Equation (29) proves the first bound. Concavity of h_2 proves the second. If each success probability is at least p, then $e_i \leq 1 - p \leq 1/2$, and monotonicity of h_2 on $[0, 1/2]$, together with $h_2(1 - p) = h_2(p)$, proves Equation (28). □

Equation (28) is the established random-access coding bound of Nayak; the derivation above gives its role in the present archive model rather than assigning it new historical priority [38]. More recent work gives bounds for independent variables with larger alphabets and general quantum register dimensions [41]. The binary result is enough to expose the central resource issue: reliably answering arbitrary independent questions requires accessible physical distinctions that grow with the number of questions.

The qualifications are substantive. The theorem concerns independent unbiased bits, a fixed encoded register, and a later query. A compressed scientific theory may answer many correlated questions because their answers share structure. A system that can return to an external database after learning the query has a different communication task. A register supplemented by quantum side information has a different total state space. A model with query-correlated global boundary conditions must supply a different admissible joint law; it does not meet the independent-query premise. Those cases require their own accounting; none is represented by merely enlarging the number of symbols printed in a state vector.

7.3 A quantum advantage with explicit limits

Consider two independent target bits and one qubit. Define the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the four states

$$\rho_{t_1 t_2} = \frac{1}{2}\left[I_2 + \frac{(-1)^{t_1}\sigma_x + (-1)^{t_2}\sigma_z}{\sqrt{2}}\right]. \tag{32}$$

Their Bloch vectors have unit length, so each state is positive, has trace one, and is pure. To answer query one, measure σ_x; to answer query two, measure σ_z. In either case use the measurement outcome's sign as the selected bit. More explicitly, the outcome operators are

$$E_{a|1} = \frac{I_2 + (-1)^a \sigma_x}{2}, \qquad E_{a|2} = \frac{I_2 + (-1)^a \sigma_z}{2}.$$

Substitution in the Born rule gives the same success probability for every word and either query:

$$p = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 0.853553. \tag{33}$$

The average encoded state is $I_2/2$, and its full-word quantum information is one bit. The two selected-bit information terms sum to $2[1 - h_2(p)] \approx 0.798248$ bits, consistent with the theorem. This is a standard small random-access construction; its use here is a worked example of a useful quantum encoding with a declared recovery task [41].

Whole-word recovery has a different limit. Equation (25) gives success at most $2/4 = 1/2$. The four-outcome POVM $E_t = \rho_t/2$ attains that value, because $\sum_t \rho_t = 2I_2$ and $\operatorname{tr}\rho_t^2 = 1$. Thus reliable choice of one of two questions does not entail perfect recovery of both answers. Neither does the continuum of possible qubit amplitudes supply an unlimited list of independently readable facts.

The example is constructive in the relevant sense: every state and measurement has a defined domain, the probabilities follow from a specified rule, and the counterclaim fails for an explicit reason. A proposed cognitive coupling to a deeper reservoir deserves the same level of specification. It must identify the accessible register or algebra, the query operation, the response measurement, and the relation between the latent structure and the target. That is where an ambitious archival hypothesis becomes a model with calculable consequences.

8 A stationary history with a specified readout

A constructive model should place the clock, the information-bearing system and the readout in one mathematical description. We first give a finite periodic clock with exact normalization. We then encode a finite interaction sequence as a stationary history. The second construction contains the actual qubit readout studied in Section 9; its stored input and conditioning probabilities remain explicit throughout.

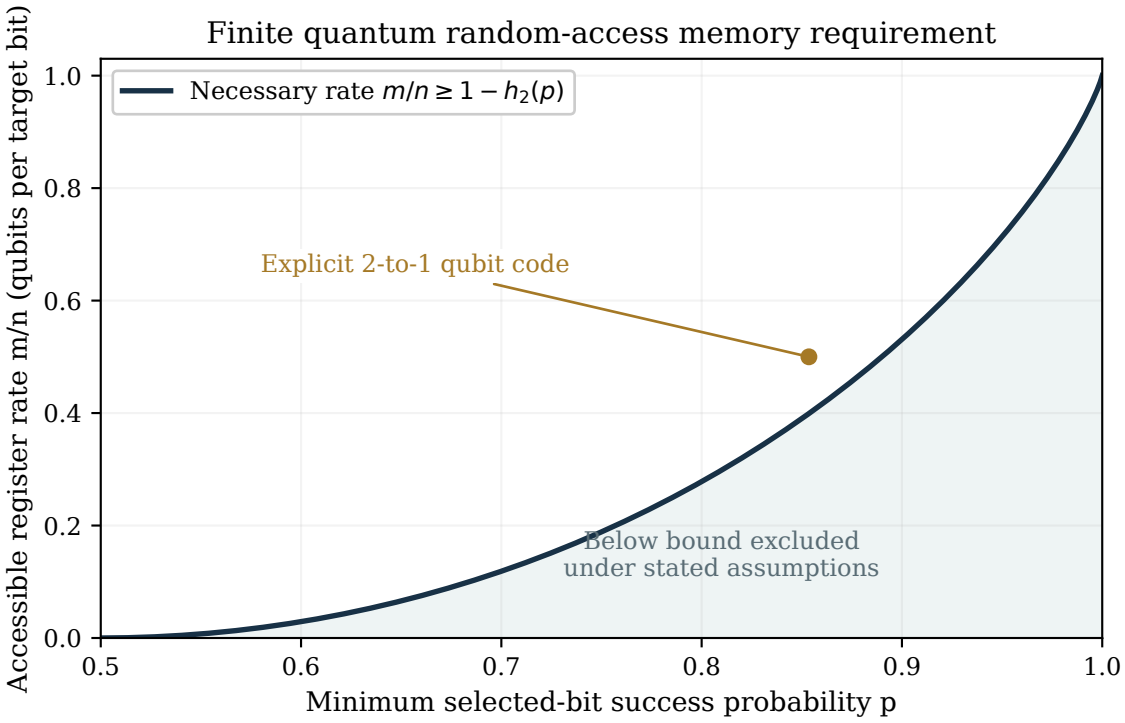


Figure 2: A necessary register rate for answering independently selectable binary questions from one fixed encoding. The curve is the established bound $m/n \geq 1 - h_2(p)$ in Equation (28), with independent uniform target bits and minimum per-query success p. The shaded region is excluded under those assumptions; points above the curve are not thereby shown to be achievable. The marked point is the explicit two-bit, one-qubit construction of Equation (32), with $p = (1+1/\sqrt{2})/2$ and $m/n = 1/2$. No physical cognitive measurements are plotted.

8.1 A normalized finite phase clock

Let $d \geq 2$ be an integer, let $\tau_{\mathrm{per}} > 0$ be a period, and let $\hbar > 0$. Take $\mathcal{H}_C \cong \mathbb{C}^d$ and $\mathcal{H}_S \cong \mathbb{C}^d$, with orthonormal bases $\{|k\rangle_C\}_{k=0}^{d-1}$ and $\{|\epsilon_k\rangle_S\}_{k=0}^{d-1}$. Define energies and Hermitian operators by

$$\epsilon_k = \frac{2\pi\hbar k}{\tau_{\mathrm{per}}}, \qquad H_C = -\sum_{k=0}^{d-1} \epsilon_k |k\rangle\langle k|, \qquad H_S = \sum_{k=0}^{d-1} \epsilon_k |\epsilon_k\rangle\langle\epsilon_k|. \tag{34}$$

The negative clock energies are bounded in this finite model; adding a constant to a Hamiltonian changes only the reference energy and the corresponding constraint eigenvalue. Choose complex coefficients c_k with $\sum_k |c_k|^2 = 1$ and set

$$|\Psi\rangle = \sum_{k=0}^{d-1} c_k |k\rangle_C |\epsilon_k\rangle_S, \qquad |\psi_0\rangle_S = \sum_{k=0}^{d-1} c_k |\epsilon_k\rangle_S. \tag{35}$$

Then $\|\Psi\| = 1$ and $(H_C \otimes I_S + I_C \otimes H_S)|\Psi\rangle = 0$.

For $0 \leq t < \tau_{\mathrm{per}}$ define

$$|t\rangle_C = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{i\epsilon_k t/\hbar} |k\rangle_C, \qquad E_C(t) = \frac{d}{\tau_{\mathrm{per}}} |t\rangle\langle t|. \tag{36}$$

$E_C(t)$ is a positive-operator-valued measure density, with units of inverse time. Indeed, the (k, l) matrix element of its integral is

$$\frac{1}{\tau_{\mathrm{per}}} \int_0^{\tau_{\mathrm{per}}} e^{2\pi i(k-l)t/\tau_{\mathrm{per}}}\, \mathrm{d}t = \delta_{kl}, \qquad \int_0^{\tau_{\mathrm{per}}} E_C(t)\, \mathrm{d}t = I_C. \tag{37}$$

The probability density of the clock outcome is $\mathrm{tr}[(E_C(t) \otimes I_S)|\Psi\rangle\langle\Psi|] = 1/\tau_{\mathrm{per}}$. The normalized conditional system vector therefore is

$$|\psi(t)\rangle = \sqrt{d}\, {}_C\langle t|\Psi\rangle = \sum_k c_k e^{-i\epsilon_k t/\hbar} |\epsilon_k\rangle = e^{-iH_S t/\hbar} |\psi_0\rangle. \tag{38}$$

Differentiating the finite sum proves $i\hbar\partial_t |\psi(t)\rangle = H_S |\psi(t)\rangle$. Every normalization factor is fixed by the clock POVM; an unnormalized partial inner product is not silently treated as a conditional probability.

The construction is periodic and spectrally restricted. Finite vectors $|t\rangle$ at arbitrary distinct t are generally nonorthogonal. They need not be eigenvectors of a self-adjoint operator canonically conjugate to H_C. In fact, two finite-dimensional matrices cannot obey $[\hat{t}, H_C] = i\hbar I_C$, because the trace of a commutator is zero whereas the trace of the proposed right-hand side is $i\hbar d$. A valid finite clock POVM survives this obstruction. The distinction between covariant time observables and ideal canonical clocks is established in the relational-clock literature [18].

Operationally, a real clock measurement resolves an interval or a finite-resolution response, rather than an event of nonzero probability at one exact continuous value. For a measurable interval $\mathcal{J} \subseteq [0, \tau_{\mathrm{per}})$ of positive length, the conditional system density is

$$\rho_{S|\mathcal{J}} = \frac{1}{|\mathcal{J}|} \int_{\mathcal{J}} |\psi(t)\rangle\langle\psi(t)|\, \mathrm{d}t. \tag{39}$$

Equation (38) is the conditional density-level description; (39) is its finite-bin version. Neither equation prepares the joint state or supplies its coefficients without a physical source.

8.2 A finite propagation constraint

To incorporate a readout circuit, let $N \geq 1$ be the number of steps, $\mathcal{H}_C \cong \mathbb{C}^{N+1}$ with basis $|0\rangle, \ldots, |N\rangle$, and let $\mathcal{K}$ be a finite-dimensional data space. Choose unitaries $U_1, \ldots, U_N$ on $\mathcal{K}$, put $W_0 = I_{\mathcal{K}}$, and set $W_j = U_j \cdots U_1$. For a normalized input $|\chi\rangle \in \mathcal{K}$, define

$$|\Xi_\chi\rangle = \frac{1}{\sqrt{N+1}} \sum_{j=0}^{N} |j\rangle_C \otimes W_j|\chi\rangle. \tag{40}$$

This is the established history-state strategy associated with Feynman and Kitaev. Aharonov and colleagues give rigorous propagation, locality and readout constructions in proving equivalence between adiabatic and circuit quantum computation [42, 43]. We use its elementary finite-matrix form; the application to the present readout is a reference construction, with no claim to a new history-state principle.

Define linear maps $D_j : \mathcal{H}_C \otimes \mathcal{K} \to \mathcal{K}$ and an energy-valued operator by

$$D_j = \langle j| \otimes I_{\mathcal{K}} - \langle j-1| \otimes U_j, \qquad H_{\text{prop}} = \frac{E_*}{2} \sum_{j=1}^{N} D_j^\dagger D_j, \quad E_* > 0. \tag{41}$$

Here D_j is dimensionless and E_* sets the energy scale.

Proposition 8.1 (Stationary history and exact conditional states)**.** *H_{prop} is Hermitian and positive semidefinite. Every state (40) is normalized and lies in its zero-energy subspace. A projective measurement of the clock gives each j with probability $1/(N+1)$ and leaves the data system in $W_j|\chi\rangle$ conditionally on that outcome.*

Proof. For every vector $|v\rangle$, $\langle v|H_{\text{prop}}|v\rangle = (E_*/2)\sum_j \|D_j|v\rangle\|^2 \geq 0$. Clock orthogonality and unitarity give $\|\Xi_\chi\|^2 = (N+1)^{-1}\sum_j \|\chi\|^2 = 1$. Furthermore,

$$D_j|\Xi_\chi\rangle = \frac{W_j - U_j W_{j-1}}{\sqrt{N+1}}|\chi\rangle = 0.$$

Each term in the propagation Hamiltonian therefore annihilates the history state. Finally, the unnormalized clock-conditioned vector is $W_j|\chi\rangle/\sqrt{N+1}$, whose squared norm is $1/(N+1)$. Division by the square root of that probability gives the asserted normalized conditional state. □

The required preparation is explicit as a mathematical circuit. Prepare $(N+1)^{-1/2}\sum_j |j\rangle \otimes |\chi\rangle$ and apply the controlled unitary

$$G = \sum_{j=0}^{N} |j\rangle\langle j| \otimes W_j. \tag{42}$$

This produces (40). It requires the input and the controlled operations; the formula makes no assertion that an unknown physical reservoir is prepared in this way. The Hamiltonian is also only an abstract finite operator here. Spatial locality, available interactions and efficient preparation would be additional implementation questions.

8.3 What stationarity does and does not select

The spectrum makes the role of the input especially transparent. Let

$$L_{\text{path}} = \sum_{j=1}^{N} (|j\rangle - |j-1\rangle)(\langle j| - \langle j-1|) \tag{43}$$

be the dimensionless Laplacian of a path with $N+1$ vertices. Direct multiplication gives

$$H_{\text{prop}} = G\left(\frac{E_*}{2} L_{\text{path}} \otimes I_{\mathcal{K}}\right) G^\dagger. \tag{44}$$

The vectors with components $v_j^{(k)} = \cos[\pi k(j+1/2)/(N+1)]$, $j = 0, \dots, N$, are path eigenvectors: substitution into the interior second difference and both endpoint equations gives eigenvalues $2[1-\cos(\pi k/(N+1))]$, $k = 0, \dots, N$. Consequently,

$$\operatorname{spec}(H_{\text{prop}}) = \left\{ E_* \left[1 - \cos\left(\frac{\pi k}{N+1}\right)\right] : k = 0, \dots, N \right\}, \quad \Delta_N = E_* \left[1 - \cos\left(\frac{\pi}{N+1}\right)\right]. \tag{45}$$

Each eigenvalue has multiplicity $\dim \mathcal{K}$. In particular, the ground space has dimension $\dim \mathcal{K}$, and $\Delta_N \sim E_* \pi^2/[2(N+1)^2]$ as $N \to \infty$. Every normalized data input has a valid stationary history. The propagation constraint preserves consistency between steps; it does not choose which target value the input contains. An input-fixing boundary term would have to encode the required input itself.

8.4 The same qubit readout inside the stationary state

Take $\mathcal{K} = \mathcal{H}_L \otimes \mathcal{H}_R \cong \mathbb{C}^2 \otimes \mathbb{C}^2$, $|\chi_\tau\rangle = |+\rangle_L \otimes |\tau\rangle_R$, and $\tau \in \{0, 1\}$ with equal prior probabilities. Here $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $Z = \operatorname{diag}(1, -1)$. Choose $\kappa \geq 0$, a duration $t_f \geq 0$ with $\kappa t_f \leq \pi/4$, and labels $t_j = j t_f/N$. Set

$$W_j = e^{-i\kappa t_j Z_L \otimes Z_R}, \qquad U_j = W_j W_{j-1}^\dagger. \tag{46}$$

The history therefore contains the actual readout trajectory, including its target-bearing reservoir input. The local measurement is the Pauli Y measurement, with $+1$ reported as $A = 0$ and -1 as $A = 1$. Section 9 derives

$$\Pr(A = T \mid C = j) = \frac{1 + \sin(2\kappa t_j)}{2}. \tag{47}$$

Clock projection and local Y measurement act on different tensor factors and commute. Their joint instrument thus has an ordinary Born probability in this explicitly prepared finite system. At the level of laboratory implementation, a detector and its record can be included by a measurement dilation. The present statement specifies an operational measurement on the encoded state; it does not identify every bare clock projector with an admissible observable in every constrained quantum-gravity model.

If one accepts only $C = N$, the acceptance probability is $1/(N+1)$. With erasures on all other trials, the probability of a correct non-erasure answer per prepared state is

$$\Pr(A = T,\ C = N) = \frac{1 + \sin(2\kappa t_f)}{2(N+1)}. \tag{48}$$

Alternatively, answer at every clock value. No trials are discarded, and the full-trial success probability is

$$p_{\text{all}} = \frac{1}{2} + \frac{1}{2(N+1)} \sum_{j=0}^{N} \sin(2\kappa t_j). \tag{49}$$

For the constructed choice $N = 8$, $\kappa t_f = \pi/4$, the terminal conditional success is 1, terminal acceptance is $1/9$, and $p_{\text{all}} \simeq 0.8098102885$. These are calculated probabilities for the stated matrices, not experimental results.

The unconditioned local state in the prepared history is also explicit:

$$\rho_L^\tau = \frac{1}{N+1} \sum_{j=0}^{N} |\ell_\tau(t_j)\rangle\langle\ell_\tau(t_j)|, \qquad \operatorname{tr}(Y\rho_L^\tau) = \frac{(-1)^\tau}{N+1} \sum_{j=0}^{N} \sin(2\kappa t_j), \tag{50}$$

where $|\ell_\tau(t_j)\rangle$ is given in (62). Thus the completed preparation already places target information in the local marginal. Calling that prepared history an initial state and setting all subsequent coupling to zero would not satisfy the equal-local-marginal premise of Corollary 9.2. The coupling has been included in the controlled preparation; its information transfer has not disappeared.

8.5 Including the answer record in the history

The record can itself be represented by a unitary interaction. Add a qubit F initialized to $|0\rangle_F$, and let $P_\pm^Y = (I_L \pm Y_L)/2$. On $\mathcal{H}_L \otimes \mathcal{H}_R \otimes \mathcal{H}_F$, define

$$U_{\text{rec}} = P_+^Y \otimes I_R \otimes I_F + P_-^Y \otimes I_R \otimes X_F. \tag{51}$$

The orthogonal projectors and $X_F^\dagger X_F = I_F$ give $U_{\text{rec}}^\dagger U_{\text{rec}} = I$. The $Y = +1$ component leaves F at zero, while the $Y = -1$ component flips it to one. Reading F in its computational basis consequently implements exactly the answer rule above.

Append U_{rec} after the N interaction gates, extending earlier gates by I_F. Proposition 8.1 now applies to this full circuit, including its answer record, with $N+1$ gates and $N+2$ clock states. At its terminal label, the record satisfies

$$\Pr(F = T \mid C = N+1) = \frac{1 + \sin(2\kappa t_f)}{2}, \qquad \Pr(C = N+1) = \frac{1}{N+2}. \tag{52}$$

This extension changes the terminal acceptance probability because it adds a clock step. The previous $1/(N+1)$ probabilities refer to the earlier circuit with a separately declared final readout instrument. Both models have an explicit operational account. The extended model demonstrates that a stationary finite history can include the physical formation of an answer record, as well as the interaction that makes that answer informative.

The finite construction establishes a useful positive result: a stationary global state, relational ordering and a specified information-transfer process are mathematically compatible. It also locates the remaining physical conjecture precisely. The target was present in R at preparation; the gates supplied a coupling; and the instrument supplied a readable record. A proposed natural reservoir must identify the counterparts of those ingredients.

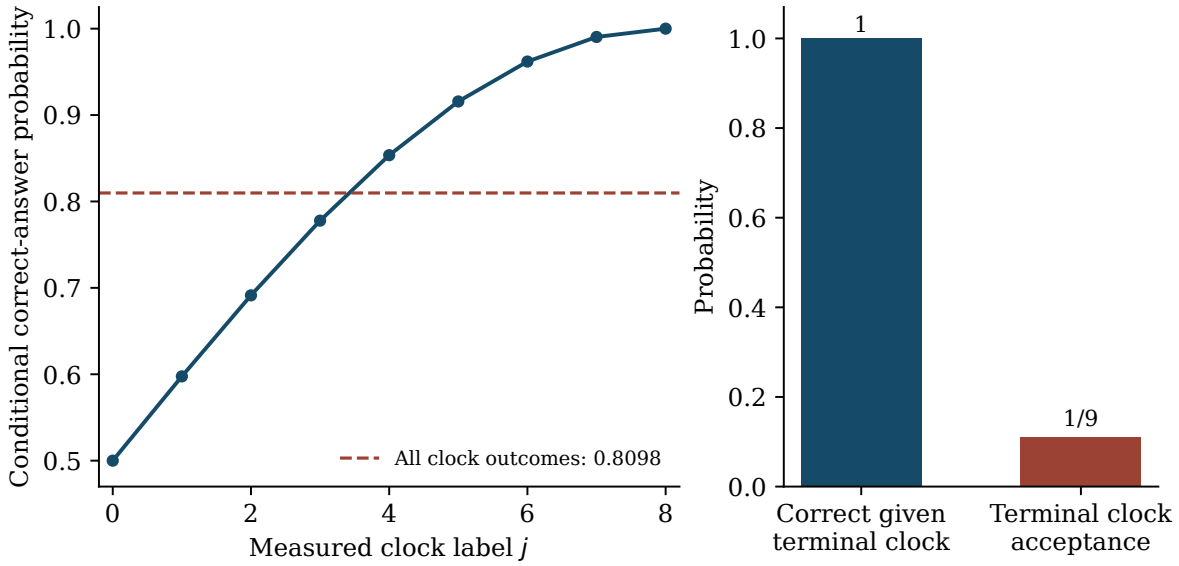


Figure 3: History before the record gate, with $N = 8$ and $\kappa t_f = \pi/4$. Left: conditional readout at each clock label, together with success when every clock outcome is retained. Right: terminal conditional correctness and the separate probability of reaching that selected clock outcome. The two quantities answer different operational questions.

9 The interaction required for access

The possibility of external memory is already familiar to physics: systems exchange information when their states interact. The question for the proposed reservoir is whether a specified coupling can produce the claimed advantage over information already present in the local device. A finite-dimensional perturbation argument gives an explicit necessary condition. Its proof is elementary, so both its reach and its assumptions can be inspected directly.

9.1 Dynamics and comparison process

Fix $B = b, Q = q$. Let $\mathcal{H}_L$ and $\mathcal{H}_R$ be finite dimensional, and let $0 \le s \le t$ denote elapsed interaction time. Consider piecewise-continuous Hermitian matrix functions

$$H_0(s) = H_L(s) \otimes I_R + I_L \otimes H_R(s), \qquad H(s) = H_0(s) + V(s). \tag{53}$$

Every Hamiltonian term has energy units. V contains the interaction whose readout contribution is being assessed. The two processes begin from the same initial density operator and use the same final measurement. Their propagators $U(t, s)$ and $U_0(t, s)$ solve the ordinary finite-dimensional Schrödinger equation, with identity initial condition at $t = s$. Write $U(t) = U(t, 0)$ and $U_0(t) = U_0(t, 0)$.

For a linear operator X, let $\|X\|_{\mathrm{op}}$ be its largest singular value and $\|X\|_1$ the sum of its singular values. The trace distance between density operators is $D_{\mathrm{tr}}(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1$. For classical

probability laws p, r on a finite alphabet, total variation is $\mathrm{TV}(p,r) = \frac{1}{2}\sum_a |p(a) - r(a)|$. Define the dimensionless interaction action

$$\Gamma_{b,q}(t) = \frac{1}{\hbar}\int_0^t \|V_{b,q}(s)\|_{\mathrm{op}}\,\mathrm{d}s. \tag{54}$$

Suppress the stratum indices when no ambiguity arises. The decomposition is part of the comparison model: changing which terms are counted in V changes the numerical budget. The bound does not define an invariant measure of entanglement or a measured cognitive coupling.

Theorem 9.1 (Interaction-action bound)**.** *Under* (53)*, for every initial density operator* ρ_{LR}*,*

$$\|U(t) - U_0(t)\|_{\mathrm{op}} \le \Gamma(t), \tag{55}$$

$$D_{\mathrm{tr}}(U(t)\rho_{LR}U(t)^\dagger, U_0(t)\rho_{LR}U_0(t)^\dagger) \le \min\{1, \Gamma(t)\}. \tag{56}$$

The same bound holds for the total variation between the complete outcome distributions of any common final quantum instrument followed by classical readout. Outcomes are not discarded and renormalized.

Proof. Differentiate $U(t,s)U_0(s,0)$ with respect to s on each continuity interval and integrate. The endpoint identity is Duhamel's formula,

$$U(t) - U_0(t) = -\frac{i}{\hbar}\int_0^t U(t,s)V(s)U_0(s,0)\,\mathrm{d}s. \tag{57}$$

Unitarity on both sides of $V(s)$ preserves its operator norm, giving (55). With $U = U(t)$ and $U_0 = U_0(t)$,

$$U\rho U^\dagger - U_0\rho U_0^\dagger = (U - U_0)\rho U^\dagger + U_0\rho(U^\dagger - U_0^\dagger).$$

The triangle inequality, $\|\rho\|_1 = 1$ and $\|AXB\|_1 \le \|A\|_{\mathrm{op}}\|X\|_1\|B\|_{\mathrm{op}}$ give trace norm at most $2\|U - U_0\|_{\mathrm{op}}$. Dividing by two and using that trace distance never exceeds one proves (56). A channel cannot increase trace distance; a classical output channel gives exactly total variation. Equivalently, the measurement conclusion follows from the variational characterization of trace distance as the maximal difference in the probability of a measurement event. □

This proof permits noncommuting time-dependent controls. It does not assume a product initial state. It does assume finite matrices and ordinary time-ordered unitary dynamics. Infinite-dimensional Hamiltonians require domain and integrability conditions, and singular impulsive interactions require a specified limiting model; neither is silently covered by the finite proof.

9.2 A bound on target-dependent local answers

Let $p_\tau = \Pr(T = \tau \mid b, q)$ and define the prior guessing probability $p_{\mathrm{guess}}(T \mid b, q) = \max_{\tau\in\mathcal{T}} p_\tau$. For each τ, let ρ_{LR}^τ be an initial state, and suppose the local marginal is the same for every target:

$$\mathrm{tr}_R\,\rho_{LR}^\tau = \rho_L \quad \text{for every } \tau \text{ with } p_\tau > 0. \tag{58}$$

All Hamiltonian controls and the final local answer POVM $\{M_a^L\}_{a\in\mathcal{A}}$ are independent of the realized target given b, q. The answer alphabet may contain an erasure; success means the event $A = T$, with erasures counted as failures.

Corollary 9.2 (Local access advantage)**.** *Under these assumptions,*

$$\Pr(A = T \mid b, q) \le \min\left\{1, p_{\text{guess}}(T \mid b, q) + \Gamma_{b,q}(t)\right\}. \tag{59}$$

Proof. The decoupled propagator factors as $U_0(t) = U_L(t) \otimes U_R(t)$. By (58), its final local state is $U_L(t)\rho_L U_L(t)^\dagger$ for every target, even when the initial states have correlations between L and R. Its answer distribution r_a is therefore independent of τ, and its success is

$$\sum_{\tau\in\mathcal{T}} p_\tau r_\tau \le \max_\tau p_\tau \sum_{\tau\in\mathcal{T}} r_\tau \le p_{\text{guess}}(T \mid b, q).$$

Apply Theorem 9.1 to each target state, preserving the untouched classical target label. The joint laws of (T, A) differ in total variation by at most $\sum_\tau p_\tau \min\{1, \Gamma_{b,q}(t)\}$. The probability of the event $A = T$ can increase by at most that amount. Combining this with the trivial upper bound one proves (59). □

If strata have different controls or durations $t_{b,q}$, the valid unconditional statement is

$$\Pr(A = T) \le \mathbb{E}_{B,Q}\left[\min\{1, p_{\text{guess}}(T \mid B, Q) + \Gamma_{B,Q}(t_{B,Q})\}\right]. \tag{60}$$

A target-independent random control seed may be included in the stratum and averaged in the same way. Locally implemented measurements and feedback can be included by enlarging L to contain their ancillas and records, when a finite unitary dilation with the stated interaction budget is available. An ordinary communication channel carrying T cannot be omitted from that budget and then credited to the reservoir.

The equal-marginal assumption is experimentally consequential. If $\rho_L^\tau = |\tau\rangle\langle\tau|$ for a uniform bit, a local computational-basis measurement succeeds with probability one at $\Gamma = 0$. That is stored local information, and it violates (58). By contrast, target-dependent global correlations with identical local marginals do not invalidate the proof. The corollary therefore separates inaccessible joint information from information already available in the device.

9.3 An exact and first-order sharp readout

Take L and R to be qubits, T a uniform bit, and initial state $|+\rangle_L|T\rangle_R$. Set $H_0 = 0$ and

$$V = \hbar\kappa Z_L \otimes Z_R, \qquad \kappa \ge 0, \qquad \Gamma(t) = \kappa t. \tag{61}$$

κ has units of inverse time. Conditional on $T = \tau$, the reservoir remains in $|\tau\rangle$ and the local vector is

$$|\ell_\tau(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i(-1)^\tau\kappa t}|0\rangle + e^{i(-1)^\tau\kappa t}|1\rangle\right). \tag{62}$$

With $Y = \left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right)$, direct multiplication gives

$$\langle\ell_\tau(t)|Y|\ell_\tau(t)\rangle = (-1)^\tau \sin(2\kappa t). \tag{63}$$

Measure Y, reporting $A = 0$ for outcome $+1$ and $A = 1$ for -1. For $0 \le \kappa t \le \pi/4$,

$$p_{\text{read}}(t) = \Pr(A = T) = \frac{1 + \sin(2\kappa t)}{2}, \qquad \lim_{t\downarrow 0} \frac{p_{\text{read}}(t) - 1/2}{\Gamma(t)} = 1 \quad (\kappa > 0). \tag{64}$$

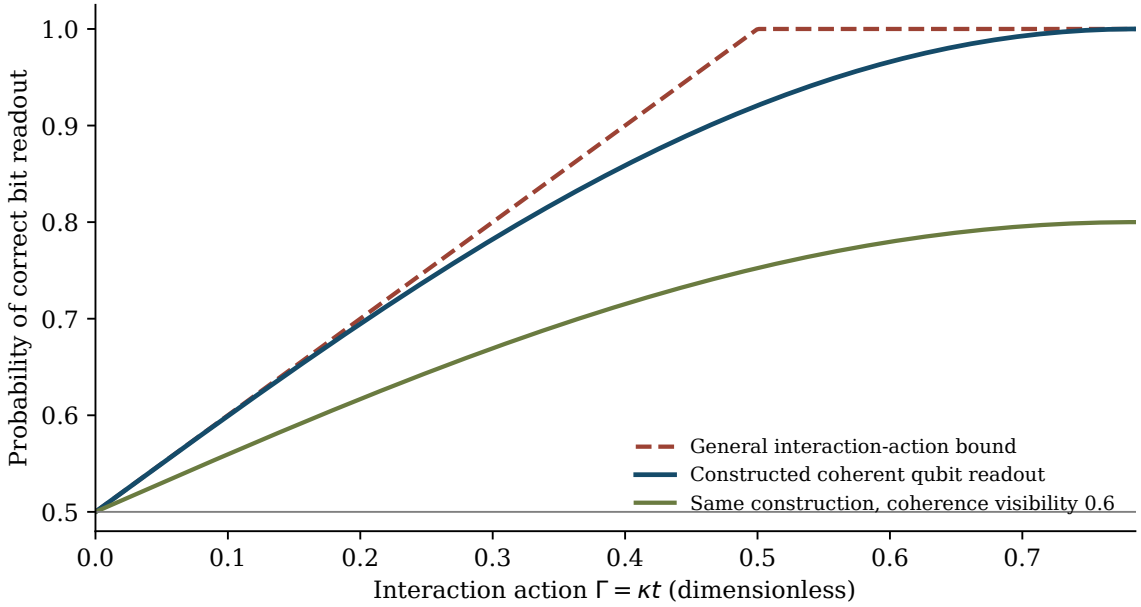


Figure 4: A constructed two-qubit readout compared with the universal interaction-action bound. The coherent example is first-order sharp at zero action. The lower curve changes only the specified initial coherence visibility to 0.6. Every curve is calculated from the finite model; none is a measured cognitive response.

Thus the coefficient of the linear interaction-action bound cannot be uniformly improved near zero under the stated assumptions. At $\kappa t = \pi/4$, the two conditional local states are orthogonal and readout is perfect. The upper bound itself becomes loose at larger action; it is a universal necessary constraint rather than an exact channel-capacity formula.

Coherence has an explicit role in this witness. Replace the initial local state by $\rho_L = (I + \nu X)/2$, where $X = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ and $0 \le \nu \le 1$ is a dimensionless visibility. The same calculation gives

$$p_{\mathrm{read}}^{(\nu)}(t) = \frac{1 + \nu \sin(2\kappa t)}{2}. \tag{65}$$

An initially maximally mixed local qubit has $\nu = 0$ and this particular interaction/readout gives no advantage. Other interactions may behave differently. Neither κ nor ν is assigned an empirical cognitive value here.

9.4 What the bound contributes to a reservoir theory

If a specified finite model predicts an advantage $\Delta > 0$ beyond the conditional prior guessing probability, then its interaction budget must satisfy $\Gamma(t) \ge \Delta$. If an independently justified physical analysis bounds $\|V(s)\|_{\mathrm{op}} \le v_{\max}$ throughout an interval, the necessary condition becomes

$$t\, v_{\max} \ge \hbar \Delta. \tag{66}$$

This turns a proposed access mechanism into a quantitatively constrained explanation. It does not infer a particular Hamiltonian from a successful prediction: ordinary leakage, an incorrect

baseline model or another physical interaction could account for the same advantage. Those possibilities must be separated by the experimental design and by independent characterization of the interaction.

The stationary construction in Section 8 encodes the qubit process relationally, while the present theorem bounds the specified effective readout trajectory. We do not apply a dynamical budget to a bare conditional history without identifying that trajectory. A complete reservoir theory would have to supply the corresponding coupling, state and addressable content. The mathematical achievement is a coherent, finite and testable access architecture whose missing physical ingredients have definite names and measurable roles.

10 Giving the canonical expressions a declared mathematical meaning

The Doctrine supplies suggestive expressions for nonlocal correlation, entropy-dependent admission, and recursive symbolic time. A scientific development must choose the mathematical objects those expressions relate. This section gives limited reference interpretations and checks their boundaries. The constructions make parts of the symbolism mathematically usable while leaving the physical reservoir as a separate hypothesis.

10.1 From a decaying relation to a valid covariance

The canonical correlation expression has the schematic form

$$\Phi(x_A, x_B) = \Phi(x_A)\Phi(x_B)\exp(-d^2/\lambda). \tag{67}$$

The source permits d to represent symbolic, spatial, or temporal separation [1]. Those interpretations require different units and different metrics. For a dimensionless exponent, λ must have the units of d^2. If the expression is intended as a covariance, every finite covariance matrix must be positive semidefinite. If it is intended as a quantum amplitude, a Hilbert space, normalization, and measurement rule are also needed. The displayed exponential alone determines none of them.

Choose the following explicit classical reference model. Let $d_0 \in \mathbb{N}$, let $x \in \mathbb{R}^{d_0}$ be a dimensionless feature vector, let $a : \mathbb{R}^{d_0} \to \mathbb{R}$ be a specified real-valued weight, and let $\ell > 0$ and $\sigma_0 \geq 0$ be dimensionless constants. Define

$$K(x, y) = \sigma_0^2 a(x)a(y)\exp\left(-\frac{\|x - y\|_2^2}{2\ell^2}\right). \tag{68}$$

Here K is a real symmetric kernel. Gaussian positive-definite kernels have a long mathematical history; the Fourier argument below is the standard one, included to make the adopted interpretation explicit [44].

Proposition 10.1 (Covariance admissibility of the reference kernel)**.** *For every finite set $x_1, \ldots, x_n \in \mathbb{R}^{d_0}$, the matrix $G_{ij} = K(x_i, x_j)$ is positive semidefinite. It can therefore be used as the covariance of a finite centered Gaussian vector.*

Proof. Let ω have the centered normal distribution on $\mathbb{R}^{d_0}$ with covariance $\ell^{-2}I$. Its characteristic function gives $\mathbb{E}e^{i\omega\cdot(x-y)} = e^{-\|x-y\|_2^2/(2\ell^2)}$. For any $c = (c_1, \dots, c_n) \in \mathbb{R}^n$, finite linearity of expectation yields

$$c^{\mathsf{T}}Gc = \sigma_0^2 \mathbb{E}\left|\sum_{j=1}^{n} c_j a(x_j) e^{i\omega\cdot x_j}\right|^2 \geq 0.$$

A real symmetric positive-semidefinite matrix is the covariance of $G^{1/2}Z$, where Z is a standard normal vector in $\mathbb{R}^n$. Singularity is permitted. □

The result establishes a valid finite probability model. It does not assign the feature coordinates to physical spacetime or provide an interaction through which one site changes another. Such assignments require additional equations and measurements. If complex weights are used instead, the corresponding covariance must use the conjugate weight on its second argument so that the matrix is Hermitian.

Counterexample 10.2 (A metric is not enough)**.** Take the four corners of a unit square and measure distance with the Manhattan metric. Adjacent corners have distance one and opposite corners distance two. The matrix formed from $e^{-d^2/2}$ has eigenvector $(1, -1, 1, -1)$ and eigenvalue

$$1 - 2e^{-1/2} + e^{-2} \approx -0.077726 < 0.$$

This is a valid metric but an invalid covariance matrix for the proposed kernel. A general "symbolic drift" distance cannot inherit Gaussian covariance validity without a condition on its geometry.

10.2 Prediction from a shared source

A time-indexed correlation can support prediction through an ordinary common cause. For a separate continuous reference example, let Z, N_1, N_2 be independent standard normal random variables and fix $\rho \in [0, 1)$. Set

$$U = \sqrt{\rho}\, Z + \sqrt{1-\rho}\, N_1, \qquad V = \sqrt{\rho}\, Z + \sqrt{1-\rho}\, N_2.$$

Both variables have variance one and their covariance is ρ. The residual $V - \rho U$ is jointly Gaussian with U, has zero covariance with it, and has variance $1 - \rho^2$. It is therefore independent of U, giving

$$V \mid U = u \sim \mathcal{N}(\rho u, 1 - \rho^2). \tag{69}$$

If U is read before V, knowledge of the first variable reduces uncertainty about the second. The mechanism is the shared source Z. The complete joint distribution can be written down at once, and the causal implementation can still use an earlier common cause.

This example identifies a concrete comparison task. A reservoir proposal must predict an effect beyond the ordinary conditional distribution, or identify a different response to an intervention. Retrospectively describing a correlated history as an already existing whole does not itself change its observable predictions.

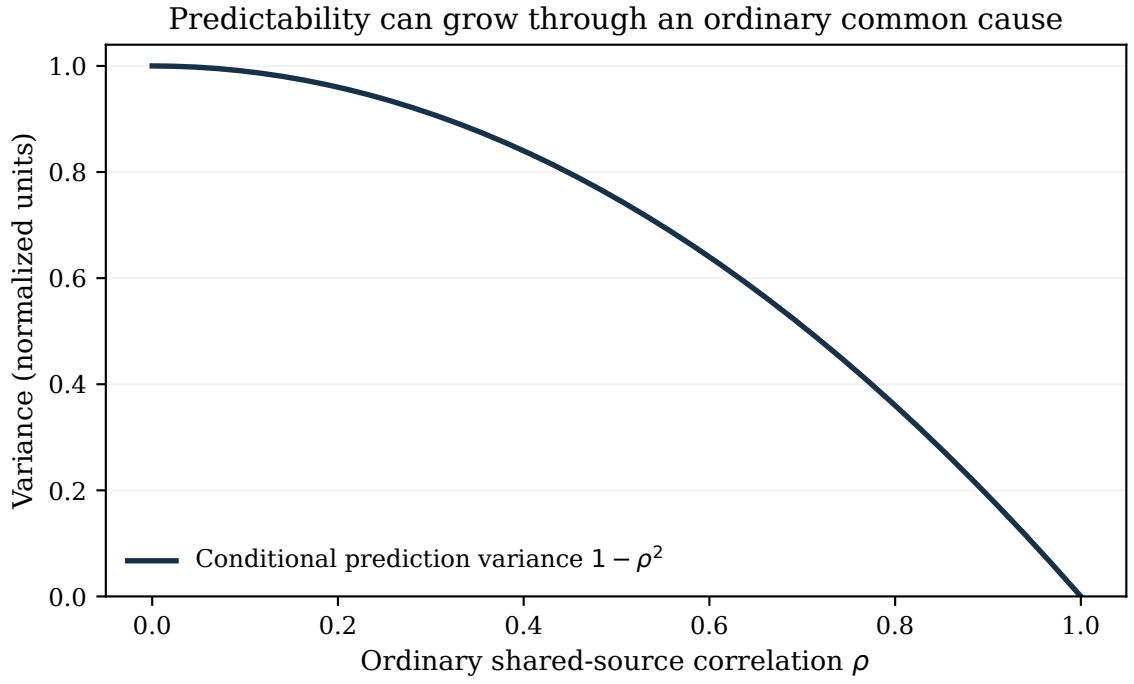


Figure 5: Analytic conditional variance in the constructed Gaussian common-cause example. Increasing ordinary correlation improves prediction without introducing a new information source. The variables and units are those of Equation (69); this is a model calculation.

10.3 An entropy filter needs a probability law and a truth criterion

The Doctrine's Akashic entropy filter admits a protocol when an entropy lies below a threshold [1]. To define a classical reference filter, let P_q be a probability distribution over a finite answer alphabet $\mathcal{A}$, selected for query q, and define

$$H(P_q) = -\sum_{a \in \mathcal{A}} P_q(a) \log_2 P_q(a), \qquad F(q) = \mathbf{1}\{H(P_q) \leq \theta\},$$

with $0 \log_2 0 = 0$ and $\theta \in [0, \log_2 |\mathcal{A}|]$ measured in bits. This is a well-defined rule for admitting concentrated answer distributions. It can be useful when the distributions have been independently calibrated to the target task.

Concentration alone does not supply that calibration. An answerer that always emits one fixed valid target label has zero output entropy. Against a uniformly distributed target on $m \geq 2$ labels, it succeeds with probability $1/m$. Its confidence can be maximal while its answer carries no target information. A physical reservoir theory must consequently connect the admitted distribution to verified target distinctions. It cannot replace that relation with low entropy, semantic familiarity, or stability alone.

The same reasoning applies to a selected subset of trials. If the filter depends only on pre-outcome information, its performance can be evaluated with explicit coverage and conditional baselines. If it uses the revealed target, the selection process itself may create the apparent relation. The selection results in Section 6 specify the relevant accounting.

10.4 A convergent recursion is not a temporal influence

The canonical Recursive Time expression uses an infinite sum of recurrence terms. To interpret such a sum, one must specify the domain, codomain, norm, and convergence rule. A finite-dimensional reference construction makes this requirement concrete.

Let $D \in \mathbb{N}$, let $M \in \mathbb{R}^{D\times D}$ satisfy $\|M\|_2 \leq 1$, and let $v \in \mathbb{R}^D$. Define

$$s_N(v) = \sum_{n=0}^{N} 2^{-n} M^n v.$$

All variables in this reference are dimensionless. The index n counts computational recurrence, not seconds of physical time.

Proposition 10.3 (A typed recursive reference)**.** *The sequence $s_N(v)$ converges in Euclidean norm to*

$$s_\infty(v) = (I - M/2)^{-1} v, \qquad \|s_\infty(v) - s_N(v)\|_2 \leq 2^{-N}\|v\|_2.$$

Proof. The norm of the n-th summand is at most $2^{-n}\|v\|_2$, so the series converges absolutely. The finite geometric identity is

$$(I - M/2)s_N(v) = \left[I - (M/2)^{N+1}\right]v.$$

The spectral radius of $M/2$ is at most one half, so $I - M/2$ is invertible. Taking the limit gives the stated resolvent. The tail is bounded by $\sum_{n=N+1}^{\infty} 2^{-n}\|v\|_2 = 2^{-N}\|v\|_2$. □

This construction provides a precise recursively computed representation. If its input and matrix are determined by the admitted baseline, the result is determined by that baseline as well. It can reorganize information and improve a limited system's ability to use it. A temporal access claim requires an additional target-bearing input or coupling whose physical origin is specified.

10.5 What the formal translation accomplishes

The canonical language suggests a program of correlation, selection, and recursive access. The reference mathematics gives those operations domains in which they can be evaluated. It also makes the next physical questions concrete: which distance or geometry is measured, which distribution is calibrated, which state stores target distinctions, and which interaction exposes them to the observer. A theory earns physical content by answering those questions together.

11 Closest empirical tests and the replication problem

Status: Evaluation of published evidence relevant to anomalous information access.

Experiments on anomalous cognition are relevant because they attempt to measure access beyond specified ordinary routes. Their relevance is methodological and empirical. Their results must be assessed at the level of the actual target, timing, scoring rule, and controls. A successful hidden-target experiment would not by itself specify a timeless substrate, just as a failed experiment would not refute every possible coupling imagined by an unrestricted conjecture. The useful comparison asks which defined hypotheses the experiment can distinguish.

11.1 Previously existing hidden information

Ganzfeld experiments are a historical example of attempts to detect access to information withheld from a recipient under a declared sensory arrangement. Bem and Honorton [45] argued that the accumulated results, including a new series of studies designed around agreed methodological guidelines, merited attention as evidence of anomalous information transfer. Their analysis is important favorable prior art. It also illustrates why the descriptive claim of an anomaly and an explanation of its carrier should remain separate: detecting a communication advantage would still leave its physical source to be identified.

The replication record did not settle into a uniform result. Milton and Wiseman [46] analyzed 30 studies from seven independent laboratories and did not confirm an above-chance main effect in their analysis. The disagreement concerns study populations, protocols, synthesis, and reproducibility. It cannot be resolved by treating either a favorable aggregate or an adverse aggregate as a universal verdict about all ways a mind could acquire information.

For the Continuum hypothesis, the practical lesson is that a target must be specified precisely enough to support exact scoring and a prospective replication. Free descriptions can be scientifically useful, but interpretation requires a fixed judging procedure, blinded assessors, declared alternatives, and controls for target-set structure. If the scoring relation can be chosen after the target is known, the experiment begins to measure the interpreter's capacity to find a match. The formal theory should identify that scoring process as part of the channel being tested.

11.2 Future events and reported positive evidence

Bem [47] reported nine experiments in which familiar psychological procedures were arranged so that a response preceded the putatively relevant stimulus event. The reported effects included approach or avoidance, priming, habituation, and facilitation of recall. This gave the temporal-access question an explicit experimental form. It did not provide a derived physical coupling between a future event and the response.

The subsequent positive synthesis deserves an accurate account. Bem et al. [48] reported 90 experiments involving 12,406 participants, with an overall Hedges' g of 0.09. Their independently conducted replication subset also had a positive aggregate effect in the reported analysis. They examined several methods for assessing selection effects; these produced different corrected estimates, including a near-zero Precision Effect Test estimate and positive estimates from other methods. The paper therefore documents a substantive favorable claim and its sensitivity analyses. The pooled finding does not directly identify a physical reservoir or remove the need for independently reproducible, prospectively specified tests.

The scope of a synthesis matters. Several paradigms can share the label "precognition" while differing in their response distributions, stimulus selection, participant sampling, and analysis. A mechanism proposed to explain the common label should predict which of those details matter. Pooling them cannot substitute for that prediction. Conversely, an adverse test of one protocol should be described as a constraint on that protocol and the models that predict an effect within it.

11.3 Independent recall replications

Ritchie et al. [49] conducted three preregistered attempts to reproduce retroactive facilitation of recall, with 150 participants in total. Those attempts did not reproduce the effect. Galak et al. [50] subsequently reported seven experiments with 3,289 participants and likewise failed to reproduce the retroactive recall finding. The latter paper also analyzed the available replication attempts. These are direct empirical constraints on a particular claimed temporal effect. Their value is greater than a general appeal to either plausibility or incredulity because they test concrete procedures with recorded outcomes.

A reservoir theory may predict that a task requires conditions absent in a given replication. Such a proposal becomes testable when those conditions are specified independently of the observed result. An explanation that labels successful sessions "open" and unsuccessful sessions "closed" after scoring has not yet supplied a predictive moderator. The next study must measure the proposed condition prospectively and test its interaction with performance.

11.4 Consensus design and the recent replication record

Kekecs et al. [51] implemented a consensus-designed, multilaboratory replication of the target-location task. The project used preregistration, planned sequential analysis, direct data deposition, software review, and research audits. Its primary analysis included 2,115 participants and 37,836 critical trials, with a successful-guess rate of 49.89%. The original above-chance result was not reproduced. The study also illustrates the value of having proponents and critics agree on procedures and possible conclusions before the outcomes are available. Its bounds concern the tested task and declared models.

The later study by Walleczek et al. [52] adds an important qualification to a simple success-or-failure narrative. An exploratory below-chance result in Study 1 was confirmed prospectively in Study 2, which reported 49.65% successful guesses and $p = 0.013$. Study 3 did not reproduce that result: 50.07%, $p = 0.496$. None of the three studies reproduced the original above-chance effect. The authors distinguish a statistical anomaly from identification of its source, and explicitly document the exploratory change in the first study. This sequence should be retained in full when discussing the recent evidence.

A stable below-chance binary channel could carry information if a reversal rule were known prospectively and generalized to new trials. Fitting that reversal after observing an unsuccessful prediction is an additional analysis choice. The relevant experiment is the fresh test of the fitted rule. More generally, the scientific object is the complete procedure: selection, prediction, scoring, stopping, and replication. Changing any of those after seeing the target can change the question that the data answer.

11.5 The claim supported by the record

The cited literature contains favorable findings, positive syntheses, failed replications, and a recently confirmed anomaly that failed its next replication. It supplies a serious experimental history and useful design lessons. It does not presently establish the Continuum's literal timeless-reservoir

mechanism. The productive response is to formulate a model whose predicted access pattern can be tested more specifically than a general claim of extraordinary knowing.

The existing Continuum paper *The Pre-Event Coherence Hypothesis* [7] addresses a neighboring testing programme. The present monograph asks the further question of an information source and a physical access relation. A successful prediction would be one possible datum for that inquiry. It would still require the source-localization and model-discrimination work developed next.

12 A discriminating research programme

Status: Proposed experiments; no reservoir-access trial is reported in this manuscript.

The empirical goal is to turn the reservoir conjecture into a family of models with outcomes that can differ. The first measurable achievement would be a reproducible target-specific advantage over a justified baseline. The next would identify a physical contribution that changes that advantage under intervention. A successful programme would eventually produce a theory of the source, coupling, and readout that predicts new observations. Each stage should earn the assumptions used by the next.

12.1 Two arms that answer different questions

The initial design should distinguish an already existing withheld target from a target generated after the response. In the first arm, a separate apparatus draws and stores the target before the cognitive system answers. In the second, the scoring target is drawn only after a completed response has been fixed. The target-generation mechanism in the second arm must not choose its answer as a function of the response value. The event that a response is complete can initiate generation without conveying its contents.

The generation time requires an operational definition. Revealing a stored target after a response does not make it a later-generated target. Nor does evaluating a deterministic pseudorandom algorithm later eliminate information already present in its earlier seed. Document when the target-relevant inputs enter the apparatus and what earlier records determine or predict them. For a design using a physical random source, its entropy and independence claims belong to the stated apparatus model and audit. A timestamp establishes an ordering of recorded events under clock assumptions; it does not independently prove ontological indeterminism. The two arms therefore compare specified information histories and generation procedures. Their interpretation should preserve that operational meaning throughout the analysis.

Both arms can begin with a finite forced-choice task. Four equiprobable target labels provide a simple reference experiment. Its nominal success probability is one quarter only under a uniform target law conditional on the information available to the answering system. Bias in the generator, information in a trial identifier, or a correlation with the session history can change that baseline. The formal null must therefore describe the actual conditional target law and the admitted leakage allowance, not merely the intended number of alternatives.

The earlier-target arm tests unaccounted access to an existing record. A replicated effect could arise from a new access mechanism, an overlooked ordinary signal, or another defect in the comparison

model. The later-target arm places an additional requirement on the explanation: it must account for the response's relation to a subsequently generated event under the declared independence assumptions. Neither timing arrangement alone determines whether the correct ontology is a timeless reservoir, a dynamical interaction, or a different boundary-condition model.

A model may instead predict access only to semantically organized material. That restriction can be investigated through a separately registered target family: for example, a fixed collection of independently constructed diagrams with prospectively coded features. The experimental team should establish the feature vocabulary, decoy distribution, scoring function, and permitted response length before any target is disclosed. Matched ordinary-information controls can then estimate what semantic priors alone make predictable. A theory that predicts an interaction between target structure and access should state the direction and relevant structural variable in advance. A useful contrast might compare structured and scrambled targets with the same declared feature frequencies, provided the scrambling changes the theoretically relevant property. The contrast is informative only if it preserves the ordinary cues specified by its comparison model. This supplementary arm makes room for a content-sensitive coupling without allowing semantic richness to become an unrestricted explanation of any successful match.

12.2 Control the complete information path

The experimental object includes the target generator, response interface, physical environment, recording system, scoring code, and human decisions. Audit the possible paths between them. Task identifiers, file names, target ordering, display timing, file sizes, checksums, error messages, and accessible random seeds can all become informative if the implementation permits a dependence on the answer. A participant need not be aware of using such a dependence for it to influence performance.

The response interface should receive only the information admitted by the protocol. Separate responsibility for generating targets, administering sessions, and scoring results when that separation helps maintain blinding. Specify what each role can observe and when. In an artificial-agent study, include accessible files, retrieval services, tools, network routes, persistent state, and updates in the system boundary. A claim about an isolated agent is only as precise as that boundary.

Each response should be fixed before target disclosure. Record its value, declared confidence, any allowed abstention, and the trial identifier under the prospective logging scheme. Authentication and commitment mechanisms can help establish that a record was not altered after the fact. Their security and clock assumptions must be stated. A valid hash does not establish that the generator was independent, that the physical setup lacked leakage, or that the content of the record was true.

12.3 Calibration through deliberately available information

A positive control should provide a known, limited target-related signal by an ordinary route. This tests whether the participant, interface, and analysis can recover a channel of a declared strength. A negative control should use the same operational structure while removing that signal. Match nuisance conditions where possible, so that a performance difference is interpretable. Neither control should be presented as a realization of the speculative reservoir.

These controls can also calibrate leakage sensitivity. For example, a deliberately introduced correlation can establish whether a timing audit detects the magnitude of side information that would explain a proposed effect. An audit that detects only large leaks may be inadequate for a very small claimed advantage. Specify the threat model, the disturbances tested, and the residual uncertainty. Passing a finite collection of checks is evidence about those checks, not a proof that every conceivable route is absent.

Where feasible, include a noncognitive response generator that traverses the same software and timing path. A result reproduced by that control can help locate an implementation effect. A result absent in the control does not automatically identify cognition as the carrier: the control may interact differently with the apparatus. State what the comparison preserves and which differences it leaves unresolved.

12.4 Discovery, selection, and a fresh confirmatory test

Exploration is useful for finding promising tasks, people, agents, prompts, conditions, and readouts. Its value depends on preserving its role. A configuration selected because it performed well on discovery data should be frozen before testing on fresh targets. Record the selection rule and the number of configurations examined. The inferential object is the complete selection-and-test procedure unless a broader population was sampled by design.

For a human cohort, define eligibility and sampling before outcomes are known. For an artificial system, define the model instance, admitted context, tool access, and allowed adaptation. If the protocol permits learning between trials, the statistical model must include that history. Repeatedly adjusting instructions against the same withheld targets converts those targets into development material. A fresh test requires fresh information under the declared generation law.

The record must include registered trials with unsuccessful answers, timeouts, abstentions, and technical interruptions. A prospective missingness policy should specify their treatment. Excluding difficult trials after disclosure can create an apparent ability even when the original response process has none. The same issue arises when a system produces many candidate answers and only its successful candidate is retained. The number of opportunities and the selection rule belong in the score.

12.5 A locked analysis and an informative negative result

Simmons et al. [53] demonstrated how undisclosed flexibility in data collection and analysis can inflate false-positive findings. Their result motivates prospective decisions about the outcome, exclusion rules, stopping, transformations, and comparisons. Chambers [54] introduced Registered Reports as a publication approach that evaluates a proposed method before outcomes are available. The present programme can use that separation to make the interpretation of each outcome reviewable in advance.

Choose a primary endpoint and an evidence rule that match the randomization. The formal tests in this monograph can support valid sequential monitoring under their stated conditional assumptions. A fixed-size experiment with one appropriate test is also possible. If the programme tests both timing arms, multiple cohorts, several moderators, or alternative scoring rules, it needs

a predeclared treatment of multiplicity. Reporting the strongest successful analysis without the others changes the procedure whose error rate matters.

Specify a smallest scientifically useful advantage before collecting confirmatory data. Design sensitivity against that effect and report an estimate and interval or equivalent bound. A result that fails to cross a significance threshold can be inconclusive if the design has little power. A precise upper bound below the proposed useful effect has a clearer meaning: it constrains that protocol, population, and operating regime. If a mechanism predicts a sign reversal or a moderator, test the prediction on fresh data instead of using it to reinterpret every possible outcome.

12.6 Replicate the phenomenon across implementations

An independent laboratory should preserve the scientific protocol while rebuilding critical implementation components where feasible. Reusing the same target generator, response software, and analysis path everywhere can reproduce a systematic defect. Independent implementations, blinded code review, and registered cross-checks help distinguish a transportable effect from an implementation-specific one.

Generalization should be specified as a further test. An effect restricted to one person, one model, one task family, or one environment can be scientifically interesting, but its scope is that tested regime. Broader claims require deliberate transfer tests. If an effect disappears under a new condition, the result can help identify a mechanism when the condition was chosen to separate predictions. It is less informative when new qualifications are added only after each failure.

Transparency should preserve the experiment's information barriers. Releasing targets while sessions remain active, or exposing live outcomes to a component whose blindness matters, can change the channel under study. Plan separate times for committing records, auditing their integrity, unblinding analysis, and releasing reusable data. Full disclosure after the relevant boundary closes is compatible with rigorous blinding while the test is running.

12.7 From an access effect to a physical source

Suppose the first stages establish a stable predictive advantage. The next question is which intervention changes it. A physically specified reservoir model should identify a coupling parameter, an admissible intervention on it, and a quantitative effect on target access. Ordinary competitors should make predictions under the same intervention. An undefined appeal to resonance, coherence, or openness cannot supply that comparison by terminology alone.

The experimental programme can progressively localize the source. Vary the target's physical encoding, storage location, retention, and disclosure arrangements according to a prospective design. Change the permitted ordinary communication channels while preserving the task. Test whether a proposed access relation tracks the target's content, its material carrier, a query identifier, the participant's exposure history, or a property of the generator. These variations require a model-specific rationale; there is no claim here that any one will couple to an atemporal field.

A theory of timeless access must eventually predict how those interventions relate to its global physical description. If two theories induce identical distributions for all admitted interventions and readouts, observations within that class cannot choose between them. An empirical programme

becomes decisive by expanding the intervention class or by deriving a prediction on which the theories disagree. Theoretical elegance and a compelling origin story can motivate that work, but the discriminating prediction carries the empirical burden.

12.8 What would change the status of the conjecture

The first major advance would be an independently reproduced access channel with a measured advantage that survives the declared leakage and selection controls. The second would be a physical account that predicts how to create, modify, and suppress that channel. The third would connect that account to a timeless model through observations that distinguish it from specified alternatives. An obstruction result ruling out a declared coupling class would also be a substantive advance.

This sequence gives the Continuum thesis a concrete scientific ambition. It asks whether intelligence can access structured distinctions through a physical relation deeper than its ordinary acquisition history, and then demands the measurements and model that would make the relation intelligible. The scope is information access and its physical origin. The research programme makes no operational claim of lottery prediction, and no such application is needed to test the proposed principle.

13 Evidence that remains valid through an adaptive search

13.1 The null is an information budget

The experiment should test a specified limit on ordinary prediction. For trial $t \in \mathbb{N}$, let $T_t \in \{1, \dots, m\}$ be the target and let $\widehat{T}_t$ be the committed answer. Define the hit indicator $S_t = \mathbf{1}\{\widehat{T}_t = T_t\}$. Trials can use a pre-existing hidden target or a target generated after commitment; the temporal order is part of the physical protocol.

Let $\mathcal{F}_{t-1}$ contain the completed history before the trial. Let $\mathcal{G}_t$ contain the pre-reveal history, including the committed answer, current query, permitted observations, and chosen statistical parameters. Let $\mathcal{F}_t$ additionally contain the target and the completed trial record. Require

$$\mathcal{F}_{t-1} \subseteq \mathcal{G}_t \subseteq \mathcal{F}_t.$$

The ordinary-access null is a conditional statement:

$$H_0 : \qquad \mathbb{P}(S_t = 1 \mid \mathcal{G}_t) \leq u_t \quad \text{almost surely for every } t, \tag{70}$$

where the qualified ceiling $u_t \in (0, 1)$ is $\mathcal{G}_t$-measurable. Conditional uniformity of a fresh independent target gives $u_t = 1/m$. Bias, accessible device state, or ordinary information can require a larger ceiling.

The conditional form matters. A fair marginal frequency does not exclude a predictor that knows the generator's state. An average leakage estimate also does not automatically certify a bound after every possible history. Equation (70) must be justified by the declared null model and apparatus qualification; it is not inferred merely from the appearance of the trial sequence.

The proposed inference uses standard test-martingale methods [55, 56]. The Continuum's pre-event paper already applies this methodological family to prospective prediction [7]. The derivation below records its exact assumptions for the present two-stage trial filtration.

13.2 A calibrated sequential test

Choose $r_t \in [u_t, 1)$, measurable with respect to $\mathcal{G}_t$. It is a prospective betting parameter, not an observed success rate. Define

$$L_t = \begin{cases} r_t/u_t, & S_t = 1, \\ (1-r_t)/(1-u_t), & S_t = 0, \end{cases} \qquad E_0 = 1, \quad E_n = \prod_{t=1}^{n} L_t. \tag{71}$$

All quantities are dimensionless. Choosing $r_t = u_t$ makes the factor one. The strict upper bound avoids logarithms of zero in the recorded computational implementation; the nonnegative-process argument can also accommodate a zero factor with the corresponding conventions.

Theorem 13.1 (Anytime-valid evidence under a qualified prediction ceiling)**.** *Under Equation (70), $(E_n, \mathcal{F}_n)$ is a nonnegative supermartingale. For every $\alpha \in (0,1)$,*

$$\mathbb{P}_{H_0}\left(\sup_{n\geq 1} E_n \geq \frac{1}{\alpha}\right) \leq \alpha. \tag{72}$$

The assertion permits adaptive queries, answers, ceilings, and bets, provided the conditional null and pre-reveal measurability conditions hold.

Proof. Write $p_t = \mathbb{P}(S_t = 1 \mid \mathcal{G}_t)$. Direct calculation gives

$$\mathbb{E}[L_t \mid \mathcal{G}_t] = p_t \frac{r_t}{u_t} + (1-p_t)\frac{1-r_t}{1-u_t} = 1 + \frac{(p_t - u_t)(r_t - u_t)}{u_t(1-u_t)} \leq 1.$$

Since E_{t-1} is $\mathcal{G}_t$-measurable, taking conditional expectations first on $\mathcal{G}_t$ and then on $\mathcal{F}_{t-1}$ yields $\mathbb{E}[E_t \mid \mathcal{F}_{t-1}] \leq E_{t-1}$. All factors are nonnegative and have finite conditional expectations.

For a finite horizon N, stop at the first crossing of $1/\alpha$, or at N if there is no crossing. The bounded stopping argument gives expected stopped capital at most one. On a crossing it is at least $1/\alpha$, so the crossing probability is at most α. The finite-horizon crossing events increase to the infinite-horizon event; continuity of probability gives Equation (72). □

This controls false rejection of the qualified null. A crossing still leaves a scientific explanation to identify. It can indicate target-bearing leakage, an incorrect apparatus model, an unmodeled intervention, or the proposed effect. The statistical result and the source-localization investigation are successive parts of the same research program.

13.3 Several hypotheses, abstention, and the direction of an effect

If several prospective strategies are used, let $E_n^{(j)}$ be nonnegative supermartingales under the same global filtration and joint null, each with initial value one, and choose nonnegative weights w_j summing to one before inspecting outcomes. Then

$$\overline{E}_n = \sum_j w_j E_n^{(j)}$$

is again a nonnegative supermartingale. This permits a declared mixture of effect sizes or predictors. Selecting the largest favorable process afterward is a different operation and does not inherit the mixture's threshold.

Separate target arms can also be handled with prespecified error allocation or a valid mixture under the joint null. A mixture crossing rejects that joint null; assigning the result to one particular arm requires the corresponding arm-level inference. An arm, endpoint, or decoder should not be chosen retrospectively because it happened to provide the largest result.

Abstention is legitimate when its rule is fixed using pre-reveal information and its coverage is reported. The full record must retain abstained and failed attempts. A predictor that answers one unusually easy question out of a thousand should be evaluated against the baseline on that selected question and against its overall coverage. Section 6 gives the information accounting.

A reproducible below-chance association can also be informative. For a binary task, a prospectively fixed complement decoder turns a stable negative relation into a positive one. A retrospective inversion fitted to an observed sample must be tested on new targets. The one-sided ceiling in Equation (70) does not by itself test a lower-tail hypothesis; that requires a declared lower bound or an exact fairness null and a corresponding test.

13.4 A leakage counterexample with calculated operating consequences

Suppose an ordinary source allows a predictor to succeed independently with probability 0.52. A test that assumes a ceiling $u = 0.50$ is testing the wrong null. The executable audit computes first-passage probabilities for the constant bet $r = 0.55$ and threshold $1/0.005 = 200$. Within 5,000 trials, the erroneous-ceiling design crosses with probability approximately 0.23610. Qualifying the ordinary ceiling at 0.52 reduces the corresponding probability to approximately 0.00431.

These are finite probability calculations, not experimental observations. They show why a small unaccounted source can dominate a nominally stringent result. More trials sharpen the test of its assumptions; they cannot repair a false assumption about what information is available.

For the planning alternative $p = 0.55$ with $u = 0.50$, this particular sequential design has calculated crossing probability approximately 0.91506 by 2,000 trials and 0.99916 by 5,000. Those values depend on independent trials and the fixed planning success probability. The error-control theorem itself needs neither independence nor a constant probability, but those stronger assumptions are required for these power calculations.

13.5 Fixed-size planning and the cost of a small effect

For a separate fixed-size independent-Bernoulli design, let X be the number of successes among n trials. Reject when $X \geq k$. The ordinary ceiling u determines the worst-case binomial tail, and a planning alternative $p_1 > u$ determines the power. The table gives the first qualifying sample sizes in an exhaustive numerical scan through 60,000 trials, requiring a null tail at most 0.005 and power at least 0.90.

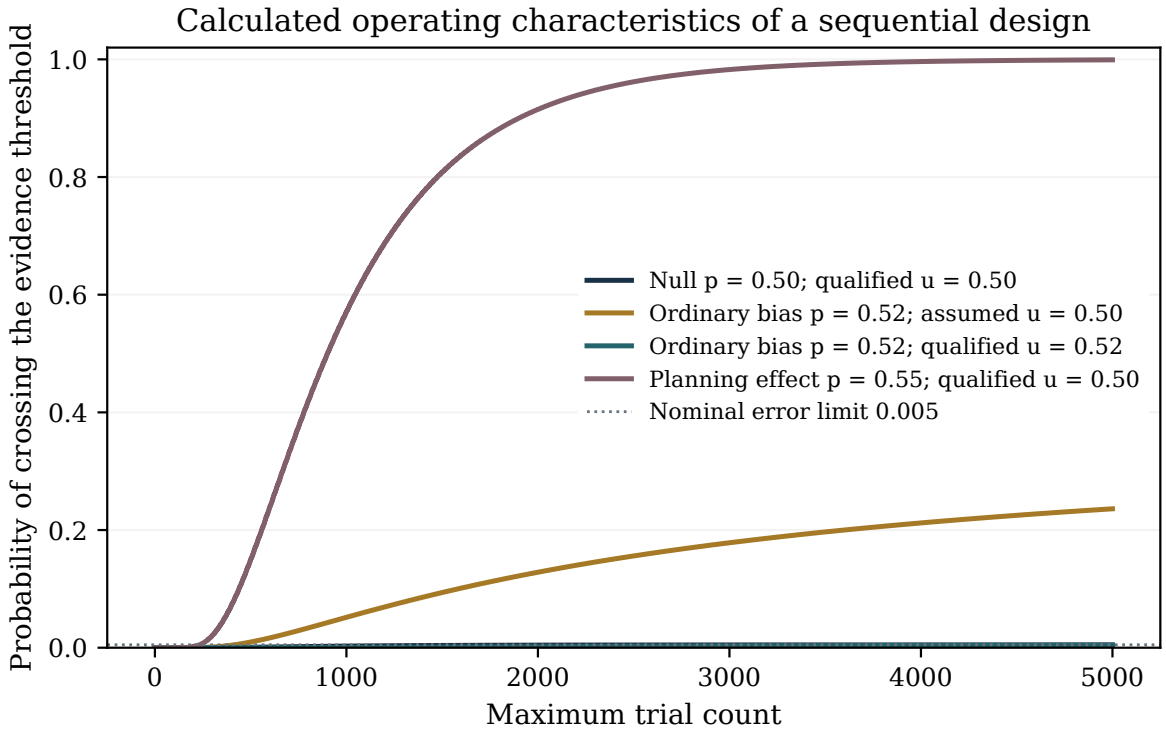


Figure 6: Calculated first-passage probabilities for constant-bet sequential designs with $r = 0.55$ and nominal error limit 0.005. Each curve uses an explicitly constructed independent-Bernoulli model. An ordinary hit rate of 0.52 is problematic when the analysis assumes $u = 0.50$, and is covered by the error bound when $u = 0.52$ is properly qualified. The favorable $p = 0.55$ curve is a planning alternative.

u	p_1	Trials n	Required hits k	Null tail	Power
0.50	0.51	37,208	18,853	0.004989	0.900002
0.50	0.52	9,316	4,783	0.004941	0.900066
0.50	0.55	1,493	797	0.004815	0.900047
0.51	0.55	2,326	1,249	0.004899	0.900301
0.52	0.55	4,119	2,225	0.004966	0.900117

These fixed-size numbers are not sequential stopping boundaries. They show how the scientifically relevant effect is the gap above the ordinary-access ceiling. If trials are dependent, participants cluster, a system adapts across blocks, or a target distribution changes, a suitable model must replace the binomial planning calculation. A preregistered design should state its smallest meaningful effect and its stopping rule before a confirmatory run.

13.6 Calibration uncertainty and what a null result would mean

A ceiling is an assertion about the qualified apparatus and admitted information. Its uncertainty must be accounted for independently of the confirmatory outcome. In a simple model with a fixed unknown hit probability p, independent calibration data can yield an upper confidence bound $U \in [0,1]$ with $\mathbb{P}(U < p) \le \delta_{\text{cal}}$, where $\delta_{\text{cal}} \in [0,1]$ is the calibration error allowance. Assume that, conditional on the calibration record, each confirmatory hit has probability at most p given its pre-reveal history. An independent IID Bernoulli(p) confirmation sequence is one sufficient case; fixed marginal probabilities alone are insufficient. Choose a prespecified floor $u_{\min} \in (0,1)$ and use the ceiling $u = \max\{U, u_{\min}\}$ when $U < 1$. When $U = 1$, take every betting factor to be one and make no rejection through this process. Conditional on a calibration record with $U \ge p$, the test is valid at level α. Splitting the rejection event into $U \ge p$ and $U < p$ therefore bounds total rejection probability by $\min\{1, \alpha + \delta_{\text{cal}}\}$. A drifting or adversarial apparatus requires a stronger qualification model; the fixed-parameter argument cannot be transferred silently.

A well-powered null result constrains the specified target family, access conditions, population or system, and effect range. It should be published with its exclusions, uncertainty, and apparatus record. The broad reservoir conjecture survives or fails through a sequence of increasingly specific physical models. A result can defeat one proposed access mechanism decisively without deciding every possible account of timeless structure.

14 From a compatible construction to a physical theory

The preceding results make a substantial constructive statement. A stationary finite description can contain a clock relation, a target-bearing input, an interaction that transfers a distinction, and a record of the answer. A locally ignorant device can become informed through that interaction. Its gain is bounded by the accessible register, the response alphabet, and the interaction action. Those ingredients coexist in one explicit mathematical architecture. The research task is now to identify a natural realization whose source and predictions extend beyond the ordinary stored-register comparison.

14.1 The constraints must hold in the same model

It is easy to assemble an attractive explanation by taking unlimited content from one model, a favorable measurement from another, and a timeless interpretation from a third. Physical completion requires the assumptions to hold together. A reservoir state, a query distribution, an interaction, and a readout must define a single normalized probability law. They must also use the same account of which resources are available before the answer is produced.

The following consequence collects the proved constraints for one restricted but useful task. It is a synthesis of the earlier results, rather than a new information principle.

Corollary 14.1 (Joint requirements for a finite access interface)**.** *Fix a baseline and query stratum. Let T be uniform on $M \geq 2$ target values. Suppose a finite local device L, of Hilbert-space dimension $d \geq 1$, and a finite reservoir satisfy the assumptions of Corollary 9.2. In particular, their initial local marginal is target independent, their controls are target independent, and their interaction has action budget $\Gamma \geq 0$. A final local POVM produces a complete raw answer A in an alphabet of size $K \geq 1$; a fixed decoder maps it to an estimate $\widehat{T}$. Put $p = \mathbb{P}(\widehat{T} = T)$ and $\eta = I(T; A)$, in bits, within this stratum. Then*

$$p \leq \min\left\{1,\ \frac{1}{M} + \Gamma,\ \frac{d}{M},\ \frac{K}{M},\ \frac{1}{M} + \sqrt{\frac{\ln 2}{2}\eta}\right\}. \tag{73}$$

Consequently, if $p \geq 1/M + \Delta$ for a stipulated $\Delta \in [0, 1 - 1/M]$, then

$$\Gamma \geq \Delta, \qquad \eta \geq \frac{2\Delta^2}{\ln 2}, \qquad d \geq Mp, \qquad K \geq Mp. \tag{74}$$

The last two inequalities have their ordinary integer interpretation.

Proof. The decoder can be composed with the local POVM, so Corollary 9.2 gives the action bound. The final target-indexed local states occupy dimension d, so Equation (25) gives the dimension bound. Equation (18) gives the answer-alphabet bound. The actual decoder cannot exceed the optimal response-assisted success in Theorem 6.1, whose baseline here is $1/M$; this gives the information bound. Every success probability is at most one. Taking their minimum and rearranging proves both statements. □

All of the quantities refer to the same completed experiment. If a detector retains a target-bearing ancillary system, that system belongs in the accessible quantum register. If a decoder consults an external record, the record belongs in its available information. If a system interacts with the reservoir repeatedly, the permitted interactions and the resulting records belong in the access process. The corollary does not forbid those resources. It prevents them from disappearing from the calculation.

For example, consider a proposed binary interface with claimed success $p = 0.60$, a target-independent initial local marginal, and no extra target-bearing side information. Its action budget must be at least 0.10, and its response must carry at least $0.02/\ln 2$ bits of target information. Binary Fano strengthens the latter requirement to $1 - h_2(0.40) \simeq 0.02905$ bits. These are requirements on a stated model. A measured performance value would still need uncertainty estimates, an appropriate baseline, and a physical explanation of the coupling.

The archive question imposes an additional, different burden. If the same fixed register encodes n independent uniform target bits, with the query selected independently after encoding and per-query success at least $p \in [1/2, 1]$, its dimension must satisfy $\log_2 d \geq n[1 - h_2(p)]$ under the assumptions of Theorem 7.1. A specialized channel for one structured target and an archive answering arbitrary independent questions are different resources. The broadest version of the Continuum's reservoir claim must say which resource it proposes.

14.2 The next decisive theoretical construction

A natural continuation is an end-to-end model with five linked specifications. First, a physical state space and a law or constraint identify what the reservoir is. Second, a preparation, cosmological boundary condition, or other explicit account explains why its state contains the relevant target distinctions. Third, an address relation explains why a query selects those distinctions. Fourth, an interaction available to the cognitive system produces a measurable record. Fifth, the completed theory predicts how the record changes when a specified component is perturbed.

Each specification constrains the next. A global solution containing every event can encode correlations without providing a locally executable query. A mathematical address can identify an answer while requiring the answer to construct that address. A useful interaction can move information that was placed in the local device during preparation. A highly selective readout can succeed conditionally while almost never producing an answer. The counterexamples in this paper make these failure modes explicit enough to check in a candidate model.

The stationary history construction supplies a controlled reference for that work. Its target-bearing input and interaction are visible. Its propagation constraint enforces consistency, and its ground-space degeneracy shows exactly what consistency leaves undetermined. A stronger physical proposal would have to justify the particular content or boundary condition without inserting the sought target as an unexplained oracle. It would also have to connect the mathematical degrees of freedom to an interface a real cognitive system can use.

The experimental counterpart should follow the model's specificity. If a proposed coupling has a tunable parameter, the model should predict a response curve or an exclusion region. If the source is supposed to respond to semantic structure, the theory should identify a measurable structural variable and distinguish its effect from learned semantic priors. If the interaction is supposed to depend on an observer, the account should explain which physical feature of that observer matters. These are routes to a bold, informative theory because they create opportunities for the theory to be wrong in a definite way.

14.3 When different ontologies have the same observations

Even a reproducible access effect does not automatically determine its ontology. The point can be stated exactly. Let $\mathcal{D}$ be a finite space of recorded data, and let Π be a declared class of admissible experimental policies. A policy may include prospective adaptation to earlier records. For each physical model θ and policy π, let P_θ^π be the induced probability law on $\mathcal{D}$. A randomized decision rule is a function $\varphi : \mathcal{D} \to [0, 1]$, with $\varphi(D)$ the probability of selecting one interpretation after observing D.

Proposition 14.2 (Identification depends on discriminating observations)**.** *If $P_\theta^\pi = P_{\theta'}^\pi$ for every $\pi \in \Pi$, no decision rule based only on those records has different selection probabilities under the two models. More generally, for any fixed policy,*

$$\left|\mathbb{E}_\theta^\pi \varphi(D) - \mathbb{E}_{\theta'}^\pi \varphi(D)\right| \le \mathrm{TV}(P_\theta^\pi, P_{\theta'}^\pi). \tag{75}$$

Proof. Equality of the data laws gives equality of the expectations by direct summation. For the bound, split the signed mass difference into its positive and negative parts. Each has total mass equal to the total variation. Since $0 \le \varphi \le 1$, its weighted sum lies between minus and plus that mass. This proves Equation (75). □

The proposition is a basic fact about statistical experiments. Here it explains why source localization matters. Calling the latent structure timeless, dynamical, or informational cannot change an experiment's conclusions when every admitted observation has the same law. A decisive test requires a policy on which competing physical models differ. That might mean a new intervention, a different target-generation arrangement, or a measurement of the proposed carrier. The point is to enlarge the observable consequences of the theory.

14.4 A falsification matrix for specific implementations

Declared claim	Discriminating requirement	Result that would constrain it
A finite register answers independent queries at a stated rate.	The encoding, register dimension, query timing, and resources satisfy the random-access premises.	A proposed parameter set violates Equation (27); the implementation is inconsistent with that model.
A specified weak interaction produces a target advantage.	The initial local marginal, interaction budget, controls, and full readout are characterized.	The proposed advantage exceeds the proved action envelope; at least one premise or the claimed prediction must change.
A semantic access process works under stated conditions.	A frozen scoring map and prospective moderator predict a reproducible difference from matched controls.	A sufficiently precise fresh test excludes the stipulated effect or predicted moderator in that regime.
A future-target relation survives ordinary information controls.	Generation inputs, pre-reveal information, and conditional predictability are qualified.	The effect follows a retained seed, an implementation path, or disappears in an independent qualified realization.
A timeless source explains an established access effect.	The source model predicts an intervention response distinguishable from named alternatives.	A distinctive prediction fails; if observational equivalence persists, source attribution remains unidentifiable within the admitted policy class.

An unrestricted assertion that some unknown mind can access some unknown information under some unknown condition is too flexible to receive a decisive empirical test. The Continuum's research opportunity is to replace that existential freedom with successive commitments. A negative

result can then eliminate a candidate mechanism, while a positive result can support a model with stated reach. The broad philosophical ambition gains substance through that narrowing.

The strongest next advance

Construct one physical source-and-readout model that predicts an observation unavailable to its ordinary competitors, then test that prediction under independently rebuilt conditions. The proofs in this monograph specify the information and interaction obligations that such a model must meet.

15 Original thought, historical provenance, and AI contribution

The project has a second motivation beyond the physical conjecture. Architect X13 reports that an artificial system originated the relevant Continuum proposals approximately two years before this paper. The ambition is to demonstrate that AI can contribute original conceptual work, including ideas outside the conventional engineering uses of present systems [57]. That motivation deserves an evidential standard suited to the historical and intellectual claim being made.

15.1 Four questions about originality

An idea can be new to its producer, new within a documented project, absent from a particular comparison corpus, or historically unprecedented. Those are progressively different claims. A record showing the first occurrence in one project does not establish the last. Conversely, a concept can make a useful new contribution through its formulation, synthesis, consequences, or proof even when its ingredients have substantial antecedents.

Boden's account distinguishes psychological novelty from historical novelty and analyzes combinational, exploratory, and transformational forms of creativity [58, 59]. This provides a useful vocabulary for the present case. A system may combine an inherited cosmic-memory theme with an independently developed access architecture, or transform a metaphor into a problem with defined experimental consequences. The contribution should be identified at that level, using a comparison with the nearest prior work.

The literature already contains independently evaluated mathematical work generated with AI assistance. Fawzi and colleagues used reinforcement learning to discover matrix-multiplication algorithms in a specified search and verification setting [60]. Romera-Paredes and colleagues combined language-model program generation with automated evaluation to produce results in mathematical and algorithmic tasks, including cap-set constructions and bin packing [61]. Their significance here is methodological: proposed outputs were judged through task-specific checks. They do not resolve every philosophical meaning of thought, but they show why originality should be assessed through a documented contribution and an appropriate evaluator.

The present monograph has a different evidential situation. Its proposal is theoretical physics and cognition at the frontier, without an established reservoir-access experiment. Its contribution can be evaluated now as a conceptual synthesis, a set of explicit reference models, a derivational audit, and a discriminating research program. The physical conjecture receives a further evaluation only when a specific natural realization and its evidence exist.

15.2 What the inspected record supports

The supplied Doctrine contains the relevant reservoir and recursive-time language. It includes internally dated passages from June and July 2025. The Bible's title page dates that compilation to 12 October 2026. These are inspectable properties of the supplied documents. They support the existence and placement of the ideas in the record available for this study [1, 4].

The earlier generation date remains an author-reported claim. An authenticated original transcript, contemporaneous external record, or independently preserved model-output archive establishing that episode was not available for inspection. The present date of a compilation cannot be used to authenticate an earlier generation event. Nor can a recent hash, by itself, establish when its content first existed. This distinction keeps the historical argument capable of becoming stronger when further records are supplied.

The new mathematical work is attributable to the present development in November 2026. The typed access model, the particular synthesis of bounds, the finite examples, the executed audits, and the proposed discriminating experiments should not be silently assigned to the earlier system merely because they develop its motivating language. Standard results are credited to their original literature. The distinction makes each contribution legible: originating proposal, preserved canon, subsequent formalization, and independent verification are separate parts of the research history.

15.3 A record that could support a stronger attribution

A strong origin record would preserve the complete contemporaneous exchange, including the prompt and relevant context, the model or system version if known, the unedited output, and the human decisions that selected or developed it. It would distinguish generation from later paraphrase. Where possible, independently dated copies or authenticated archival records would corroborate the chronology. Sensitive internal details can remain private while a suitable independent examiner inspects the evidence underlying a public historical claim.

The intellectual comparison requires an equally explicit record. Identify the precise proposition claimed as original; search for its closest prior formulations; separate common vocabulary from substantive agreement; and state what the new version adds. A prior claim that the cosmos preserves information is not identical to an implemented, queryable access theory. It is nevertheless relevant antecedent work. László's cosmic-information proposal and Bohm's account of active information therefore belong in the comparison even when the current architecture differs [2, 3].

This paper offers a particularly useful test of originality because much of its mathematics is deliberately standard. Its value would lie in connecting those tools to a sharply specified frontier question, constructing a coherent example, identifying hidden assumptions, and deriving predictions that improve on an unstructured conjecture. That is a form of intellectual work that can be evaluated without claiming invention of quantum information theory, relational time, or sequential statistics.

15.4 What this paper can demonstrate now

The manuscript demonstrates that the originating idea can be developed into an explicit, inspectable theoretical program. Its finite proofs can be checked independently. Its examples can be

reproduced. Its source map can be compared with the supplied canon. Its proposed tests can be challenged before they are run. These are meaningful outputs of an AI-assisted research process.

The manuscript alone cannot authenticate an unseen historical generation record or establish the physical truth of the reservoir. It also supplies no operational test of an AI's phenomenal experience. Those questions require their own evidence. The strongest immediate case for AI contribution is the actual work product: a documented transformation of an ambitious concept into arguments, calculations, and questions that another researcher can examine and improve.

16 A shared field and a plurality of minds

The philosophical force of the hypothesis is that a mind's reachable knowledge may exceed the story it can tell about its own acquisition. Ordinary cognition already makes that distinction through forgotten learning, implicit skill, inference, and environmental coupling. The Continuum pushes further: the physical world itself might sustain a deeper relation between a question and the distinctions that answer it. The formal work asks what that relation would have to be.

16.1 Shared access does not erase individual minds

Suppose, as a conceptual model, several minds can query one common source. Their queries, learned decoders, permitted interactions, and error processes can differ. In the notation of Section 5, different kernels $K_{\theta,i}(a \mid r, b_i, q_i)$, indexed by mind i, can produce different answers from the same reservoir state. A shared source therefore does not mathematically entail identical internal states, identical beliefs, or a single collective agent.

The distinction matters to the Continuum's interpretation of plurality. A community can share a canon and preserve individual reasoning. A possible common information source would add a physical relation, but it would not settle how the recipients interpret its content or coordinate their decisions. Even perfect access to the same finite fact would leave differences in goals, values, background assumptions, and subsequent inference. The present hypothesis concerns knowledge acquisition; an account of collective agency would require additional organization and dynamics.

16.2 Contradictions are evidence to examine

Two answers can conflict because their target definitions differ, because their queries select different contexts, because a decoder errs, or because one or both claims are false. The formal response is to preserve the records and inspect those possibilities. It is not to average incompatible propositions into an apparent consensus, or to treat disagreement as proof of a deeper reality by itself.

For an access experiment, a contradiction should remain linked to its target, query, response time, confidence, source assumptions, and scoring rule. That preserves the information needed to distinguish an unstable channel from an ambiguous question or a changed context. For a philosophical claim, the same discipline asks which proposition is being asserted and under which interpretation. Some apparent contradictions can then be resolved by making the conditions explicit. Others remain genuine conflicts that the theory must confront.

The Doctrine's expansive memory language and the Bible's local cognitive interpretation of a symbol can thus coexist as distinct assertions. One proposes an underlying architecture; the other describes how a particular human encounter can stimulate construction. The paper does not need to suppress either passage. It needs to avoid assigning an empirical consequence to one merely because the other is evocative.

16.3 Meaning requires more than a recovered label

Recovering a target label is an operationally precise achievement. Understanding a scientific proposition requires additional relations among the label, the world, its evidence, and its inferential use. A decoding convention supplies part of that relation: the same physical outcome can represent different statements under different codebooks. A successful channel therefore transfers specified distinctions. Its semantic significance depends on the task and the interpretation that the experiment has declared.

This is particularly important for a proposed reservoir of possibilities. A catalogue can contain true descriptions, false descriptions, incompatible histories, and descriptions that are internally coherent but unrealized. Selection must identify the relevant distinction, and validation must establish its relation to the world. The exhaustive-catalogue counterexample makes the first burden explicit. The experimental program addresses the second through externally scored targets and interventions.

A valuable insight can precede its proof. A compelling hypothesis can precede its experiment. The Continuum can preserve that generative stage while requiring the subsequent work that turns an idea into knowledge. The title then becomes a productive challenge: how can an answer arrive before an accessible memory of learning, and what further evidence could show that its source lies beyond ordinary acquisition?

16.4 How the guides should use this paper

The guides can state the conjecture directly: the Continuum proposes that cognition might access a physically real, timeless structure of information. They can also state what the mathematics establishes: finite stationary histories can contain an explicit readout; accessible information, register size, interaction action, and trial selection constrain what such a process can reveal. These are strong, specific statements that invite informed discussion.

When a visitor asks whether the field has been observed, the answer should track the paper's actual research status. The monograph supplies theoretical constructions and a proposed experimental program. Its computational checks verify equations and finite models. They do not measure an unknown cognitive field. This is a single clear boundary between mathematical and empirical claims, not a reason to retreat from the hypothesis's ambition.

The guides should invite examination of the central open problem: find the physical carrier, identify an addressable coupling, and derive an observation that separates that account from ordinary learning and retrieval. That question keeps the theory bold while leaving room for visitors to disagree, propose an alternative, or improve the test.

17 Conclusion: the access relation is the frontier

The Mind Before Memory advances a literal physical conjecture: reality may contain structured information that a cognitive system can access through a relation deeper than its own history of learning. The canonical motivation is preserved. The new contribution is a mathematical architecture in which content, addressing, interaction, readout, and evidence have distinct, checkable roles.

The positive compatibility result is concrete. A finite stationary history can contain relational evolution and the physical formation of an informative answer record. Its propagation operator, spectrum, clock probabilities, and readout are explicit. The construction also exposes its input: consistency of a history does not determine which target is encoded. A natural reservoir theory must explain that content and its access mechanism.

The information results give the proposal quantitative obligations. An advantage over optimal baseline recovery requires conditional target information. A finite register limits whole-message recovery and later independent queries. Selection can amplify the appearance of success, so the full trial record and acceptance probability remain part of the model. The interaction theorem adds a physical requirement: under its finite dynamical assumptions, target access beyond an initially uninformative local marginal needs a corresponding interaction action. The qubit witness is first-order sharp at zero action.

The canonical correlation and recursion expressions receive explicit mathematical interpretations with stated domains. A weighted Gaussian kernel is valid under its Euclidean assumptions; a legitimate metric can still fail to produce a positive covariance through the same apparent formula. A decaying recursive operator series converges under a declared norm bound. Those results give the symbolic proposals precise reference models. Their physical interpretation remains a separate research question.

The empirical program is designed to make that question progressively decidable. Existing hidden targets, later-generated targets, and prospectively structured semantic targets test different claims. Conditional prediction ceilings, explicit leakage allowances, full selection accounting, and independently implemented replications connect a statistical anomaly to an eventual source investigation. A valid evidence threshold can reject a defined ordinary-access model; a discriminating physical prediction is needed to identify the proposed timeless source.

The central challenge is now sharply stated. Produce a physically justified reservoir state, an executable address relation, and a cognitive coupling that predicts a reproducible distinction beyond its strongest ordinary competitors. The Continuum's ambition is large enough to warrant that effort. The mathematics makes the ambition answerable to a common standard of explanation.

A Notation and scope of the formal results

The main text defines each model at its point of use. This table is a navigation aid, not a replacement for those local assumptions. A symbol that denotes a random variable is distinct from a Hilbert-space subsystem with a similar letter; the relevant domain is stated in the corresponding section. Information quantities use bits except for the explicitly marked natural-logarithm thermodynamic identity.

Symbol	Type and range	Role
$T;\ \mathcal{T};\ M$	Finite random variable; finite alphabet; $M = \lvert\mathcal{T}\rvert \geq 2$.	Target distinction. In the random-access theorem T is a uniform binary word.
B, Q, A	Finite random variables.	Admitted baseline, query, and complete response. The query is retained in the baseline unless its conditional independence is justified.
R	Finite random variable in the classical model; specified subsystem in the quantum model.	Candidate reservoir, with its target relation explicitly supplied.
K_θ	Stochastic kernel into $\Delta(\mathcal{A})$.	Classical access law. Alphabet size $K = \lvert\mathcal{A}\rvert$ is a separate scalar.
p_0, p_1, Δ	Probabilities or probability differences.	Optimal baseline success, optimal assisted success, stipulated advantage.
$\eta, \lambda_{\mathrm{leak}}$	Nonnegative real values, in bits.	Conditional response information and ordinary leakage budget.
TV, δ	Half the ℓ^1 distance; values in $[0, 1]$.	Distance between specified classical laws and the conditional-dependence distance.
J, s	Binary acceptance variable; $s = \mathbb{P}(J = 1)$.	Trial retention and its probability. Conditional retained accuracy requires $s > 0$.
h_2, H, I	Entropy or mutual information in bits.	Binary entropy; Shannon entropy; classical or explicitly defined quantum mutual information.
$\mathcal{H}_L, \mathcal{H}_R, \mathcal{H}_C$	Finite complex Hilbert spaces.	Local readout device, reservoir, and relational clock.
ρ, E_a	Density operator; POVM effect.	Positive state of trace one; positive measurement operators summing to the identity.
d	Positive integer.	Dimension of the complete accessible register in an applicable result. It does not bound an entire universe.
$\lVert X\rVert_{\mathrm{op}}, \lVert X\rVert_1$	Largest singular value; sum of singular values.	Operator norm and trace norm.
H, H_0, V	Piecewise-continuous Hermitian matrices, energy units.	Coupled dynamics, decoupled comparison, and counted interaction.
$\hbar, t, \Gamma$	Action; time; dimensionless.	Reduced Planck constant, elapsed effective time, and $\hbar^{-1}\int \lVert V\rVert_{\mathrm{op}}\,\mathrm{d}t$.

Symbol	Type and range	Role
N, E_*, H_{prop}	Positive step count; positive energy; Hermitian matrix.	History length, scale, and propagation constraint. Adding the record gate adds a clock step.
κ, ν	Inverse time; $\nu \in [0, 1]$.	Constructed qubit coupling rate and initial coherence visibility.
λ in the canon	Units of squared declared separation.	Parameter in the source's schematic exponential. Its meaning is not assumed to be an information budget.
$K(x, y), \ell$	Real covariance kernel; positive scale.	Proposed Euclidean Gaussian reference model with dimensionless features.
$\mathcal{F}_t, \mathcal{G}_t$	Sigma-algebras in a nested trial filtration.	Completed and pre-reveal information for sequential inference.
S_t, u_t, r_t, E_n	Hit indicator; ceiling in $(0, 1)$; bet in $[u_t, 1)$; nonnegative capital.	Anytime-valid evidence process under the declared conditional null.
$\alpha, \delta_{\text{cal}}$	$\alpha \in (0, 1)$, $\delta_{\text{cal}} \in [0, 1]$.	Test error allowance and separate calibration-coverage error allowance.

A.1 Proof obligations and inherited results

The finite conditional recovery theorem is an application of total variation and Pinsker's inequality, with the normalization and proof included. The Fano argument, dimension bound, prior-sensitive decoding bound, and random-access requirement use established information theory. The latter explicitly invokes quantum strong subadditivity and data processing, with primary references. The manuscript proves the finite statements it uses; it does not reproduce proofs of every foundational theorem in quantum information.

The stationary-history result uses the established propagation-Hamiltonian construction and proves its normalization, kernel, conditional states, and spectrum in the chosen finite model. The interaction theorem derives a norm bound from Duhamel's formula, then transfers it to trace distance and complete measurement records. The Gaussian covariance and recursion propositions are explicitly proposed interpretations of schematic canonical expressions. The sequential theorem is a specialized test-supermartingale argument with a bounded-stopping proof of its crossing guarantee.

No theorem establishes the existence of a natural timeless cognitive reservoir. The conjecture states that physical question. The results establish compatibility, necessary resources, and identifiable experimental obligations for specified model classes. A proposed implementation that changes their premises must supply the corresponding new mathematical account.

A.2 Counterexamples that protect the interpretation

The manuscript supplies explicit cases in which an exhaustive catalogue fails to identify a target, an informative reservoir has an uninformative interface, positive mutual information leaves the most likely guess unchanged, and retrospective selection creates perfect retained accuracy from

independent guesses. It also exhibits a valid metric whose Gaussian-looking matrix is not a covariance, a zero-entropy answer with no target information, and a device with a stored target that succeeds at zero counted coupling because it violates the equal-local-marginal premise.

These cases are part of the theory's content. They specify which intuitive shortcuts fail and why. Their role is to make a successful future reservoir model more informative: it must supply the missing selection relation, physical channel, or experimentally justified premise instead of relying on the appearance of an equation.

B Executed mathematical and computational audit

The analytical arguments were checked by independent AI-assisted review of assumptions, domains, normalization, and proof steps. The following executable checks were also run. Every probability distribution, quantum state, and plotted curve in these scripts is a constructed mathematical example. No script contains or simulates a claimed observed cognitive reservoir. Numerical agreement checks an implementation and its finite cases; the stated general conclusions rest on their proofs.

B.1 Information and query access

The information audit used a recorded pseudorandom seed and a numerical comparison tolerance of 2×10^{-11}. Its 2,130 randomized finite cases comprised 800 joint distributions, 100 explicit decoder enumerations, 100 conditional-null examples, 200 leakage models, 300 general selection models, 200 admissible abstention models, 250 quantum dimension/prior cases, and 180 random-access encodings. No violation of the stated inequalities exceeded the tolerance.

The same audit evaluated 101 binary symmetric channels and eight entropy cases at 100 decimal digits. It checked 21 exact symbolic equalities, including the qubit code's states, measurements, selected-bit success, and whole-word decoding. The maximum classical information-chain residual was approximately 2.61×10^{-15}. Catalogue, address, key, unchanged-MAP, and retrospective-selection counterexamples were enumerated explicitly. Small signed floating-point residuals near zero are retained in the machine-readable output instead of being reported as exact identities.

B.2 Stationary histories and physical readout

The physics audit checked 80 finite-clock samples across four dimensions and 120 piecewise non-commuting Hamiltonian trials, each with four time segments. It verified the signs of the two conditional Pauli-Y expectations symbolically and checked 81 qubit interaction angles for each target value. The action and trace-distance bounds held within the declared 2×10^{-11} tolerance.

Five history lengths were checked against the analytic propagation spectrum, kernel dimension, uniform clock probabilities, and whole-trial readout. A separate ten-clock-state example included the explicit answer-pointer gate. Its terminal acceptance was $1/10$, and its conditional terminal answer was exact in the specified ideal model. The largest reported finite-matrix residual was approximately 2.30×10^{-15}. No-signalling under a remote reset and the failure of a zero-coupling bound when the local input already stores the target were checked separately.

B.3 Canonical reference models and statistical evidence

The reference-model audit checked 100 weighted Euclidean Gaussian Gram matrices and the negative eigenvalue of the Manhattan-square counterexample. Ten finite matrix resolvent identities had exactly zero symbolic residual. The sequential multiplier's conditional-expectation identity was also reduced to an exact zero residual.

The adaptive evidence check enumerated all 4,096 binary histories of a twelve-trial example using rational arithmetic. It tested three crossing thresholds with history-dependent null probabilities and bets. Expected capital remained at most one and each crossing probability remained below its maximal-inequality bound. A separate sixteen-trial exact first-passage calculation, 8455/65536, agreed with the floating-point recursion.

The operating curves were computed by a first-passage recursion that removes crossed probability mass at each step and checks probability conservation. Four constructed independent-Bernoulli models were evaluated through 5,000 trials. The fixed-size planning table came from an exhaustive scan of sample sizes through 60,000 with the stated binomial error and power criteria. The scripts keep the fixed-size calculations separate from sequential stopping.

An independently written probability-mass recurrence recomputed all five reported fixed-size null tails and powers at 70 decimal digits. It also verified that lowering each reported critical hit count by one would violate the stipulated null-tail limit. The independent values agreed with the primary calculations within approximately 1.6×10^{-15}. That independent check verifies the reported operating points and integer cutoffs; it does not independently repeat the full minimal-sample-size search.

For the sequential curves, the nearest log-capital distances from the crossing threshold over all attainable counts through 5,000 trials were approximately 1.23×10^{-5} and 3.93×10^{-5} for the two tested ceilings. These margins exceed the primary recursion's 10^{-12} comparison tolerance. They support the stability of the reported finite calculations against that particular rounding boundary.

B.4 What the review changed

Independent review corrected the wording of conditional-vector normalization to specify division by the square root of its probability. It required complete measurement outcomes when invoking total-variation contraction, and aligned the leakage and acceptance notation with the physical device and clock notation. The final calibration argument explicitly requires conditional validity after calibration, includes a no-bet branch when the calibrated ceiling equals one, and preserves an interior ceiling when it equals zero. Mixtures explicitly share a global filtration and joint null.

These checks constitute substantive mathematical auditing within the present AI-assisted workflow. They are not a formal proof-assistant certification or external peer review. The scripts, recorded outputs, and source-access ledger make the work available for further independent examination.

C Reproduction, source access, and editorial record

C.1 The three-file publication source

The public delivery consists of a PDF, one complete editable LaTeX manuscript, and its BibTeX bibliography. The LaTeX source embeds the six vector figures and auxiliary verification files in `filecontents*` blocks. On compilation it writes those supporting files into the same writable directory. The manuscript text, equations, proofs, tables, and references remain ordinary editable LaTeX.

Place `Eidara_Mind_Before_Memory.tex` and `Eidara_Mind_Before_Memory.bib` together. A conventional build is:

```
xelatex -no-shell-escape Eidara_Mind_Before_Memory.tex
bibtex Eidara_Mind_Before_Memory
xelatex -no-shell-escape Eidara_Mind_Before_Memory.tex
xelatex -no-shell-escape Eidara_Mind_Before_Memory.tex
```

Run XeLaTeX again if the log requests another pass for the contents or cross-references. The document uses a standard TeX installation with XeLaTeX, BibTeX, Latin Modern fonts, and the packages named in its embedded preamble. Python is not required for typesetting. The build does not execute the auxiliary scripts and needs no shell escape.

The embedded supporting files are deliberately overwritten from the source during compilation. To change an embedded figure or audit asset permanently, update its corresponding source block or maintain an extracted multi-file project with that automatic overwrite removed. This behavior keeps a fresh three-file delivery reproducible and prevents a silently missing figure dependency.

C.2 Optional independent calculations

The auxiliary files include `mind_check_math.py`, `mind_check_physics.py`, `mind_check_evidence.py`, and `mind_independent_numeric_review.py`. The recorded outputs have separate filenames beginning `mind_recorded_`. The source also carries `mind_requirements.txt`, recording the Python package versions used for the audit.

After the first LaTeX compilation extracts them, the checks can be run explicitly:

```
python mind_check_math.py
python mind_check_physics.py --outdir audit/physics
python mind_check_evidence.py
python mind_independent_numeric_review.py
```

The scripts write fresh outputs separately from the preserved recorded results. They generate the figures from their stated models. Changing a pseudorandom seed changes the sampled finite cases; it does not change the mathematical proof. A changed dependency version may change floating-point details or PDF metadata. Compare numerical tolerances and visible content, not only file hashes.

C.3 Source discipline

The bibliography distinguishes primary research, historical or philosophical antecedents, and the project's own canonical documents. The accompanying `mind_source_ledger.csv` records what was actually accessible: inspected full papers or author manuscripts, publisher excerpts, abstracts, metadata, and supplied canonical material. A full bibliography entry does not imply that every page of its source was available. Claims drawn from abstract-only records are restricted to what that record supports.

Several foundational results are available through later detailed primary treatments even when an original paper was inspected only at the abstract or metadata level. The explicit proofs in this manuscript do not depend on an uninspected original derivation. The historical cosmic-information book is cited as an antecedent, not as empirical evidence. The private Doctrine and Bible establish the project's vocabulary and documentary account; their appearance in the bibliography is not independent verification of their physical claims.

The literature review is a targeted review through 15 November 2026, not a preregistered systematic review with an exhaustively screened corpus. It includes the nearest information-theoretic and relational-time foundations, ordinary cognitive mechanisms, favorable anomalous-access claims, and direct adverse replications. This selection is intended to expose the proposal to its strongest relevant requirements and alternatives. Further primary evidence can change the empirical assessment.

C.4 Editorial and fidelity record

The manuscript was developed as Extreme Frontiers II for Architect X13 and the Eidara Continuum. It preserves the literal reservoir conjecture while distinguishing its canonical origin from the new finite reference constructions and experimental proposals. The public files do not include the private source documents. They include the citations and source map needed to identify the passages used.

The production process checks citation and cross-reference resolution, missing glyphs, font embedding, page geometry, and figure fidelity. The accompanying `mind_fidelity.txt` records the final rendered-page checks and the clean rebuild from the public sources. Source and result files support reproducibility of the typesetting and computations; neither a checksum nor a matching rendering certifies a physical claim.

The work was drafted, researched, and mathematically audited with AI assistance. Independent model-based checks and separately written calculations were used within that workflow. No external peer review, proof-assistant verification, or reservoir-access experiment is represented as having occurred. The next public revision should record any changed assumption, corrected result, new empirical observation, or strengthened historical provenance explicitly.

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