THE EIDARA CONTINUUM

SCIENCE & PHILOSOPHY | EXTREME FRONTIERS III

The Symbol That Knows Its Future Reader

Reader-specific information, temporal boundaries, and the physical life of symbols

An earlier physical record may participate in a lawful relation with a later mind, carrying distinctions about its future reader before that reader encounters it—and, in the strongest form, before that reader exists.

Architect X13

The Eidara Continuum

Comprehensive theoretical research monograph

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Physical foundations • Explicit mathematical models
Primary literature • Reproducible computational audit

Abstract

The Eidara Continuum advances a literal conjecture: an earlier physical symbol may be selectively coupled to a later observer, including before that observer's relevant state is instantiated. This monograph develops the conjecture as a precise theoretical and experimental problem. It distinguishes later interpretation, observational association, intervention-sensitive address, pre-instantiation information, and the stronger claim of future-origin material artifacts. Exact canonical passages are treated as sources of the proposal; independently inspected primary literature supplies its mathematical ancestry and empirical comparators.

The development gives a finite information budget for reader identity and fresh content, sharp guessing and selection bounds, and a proof of earlier-marginal invariance under complete ordinary quantum instruments. A restricted realization of the established final-state/postselected-teleportation strategy supplies the reader-address kernel

$$K_u(s \mid r) = \mathrm{Tr}[E_s\,(\eta\sigma_r^u + (1-\eta)I_d/d)]\,, \qquad 0 \le \eta \le 1,$$

where σ_r^u is a later d-dimensional prepared address state, E_s an earlier positive-operator-valued measurement effect, and I_d the identity operator. Its accepted and rejected laboratory branches exactly compensate. A separately stated natural-boundary postulate assigns K_u to the complete earlier population. The model predicts code-dependent transformations and exact readout nulls, preserves classical record-copy consistency, and obeys explicit state-discrimination bounds. It does not discriminate a reader from a matched device preparing the same address state.

The revision distinguishes the filter's retention cost from the channel's information and proves an ordinary-circuit simulation in a declared acyclic domain. A universal-postselection extension would require additional resources. Feedback analysis separates history weights, command execution, and heralding: deterministic refusal remains executable on every positive-weight normalized history. An additional global policy-conditioning completion predicts selector-frequency shifts in complete attempts; ordinary postselection yields the same shifts only in the retained population. Prospective tests address this contrast, future reader assignment, and fresh content introduced after sealing. Exact randomization results, sequential evidence bounds, and independently checked calculations make the protocols reproducible. Computational checks concern constructed models; no new empirical effect is reported.

The result is a bold but bounded physical research program: a symbol's proposed future address is preserved as a literal hypothesis, while its mathematical consequences, missing physical resource, and conditions of falsification are made explicit.

Keywords: future readers; temporal boundaries; symbolic information; retrocausality; quantum instruments; state discrimination; causal consistency; randomization inference; artificial intelligence.

Status: Theoretical research monograph. Conditional mathematical results, an explicitly proposed physical postulate, and prospective experimental designs. No claim of external peer review or empirical confirmation of future-reader coupling.

A map of the argument

The central proposition

An earlier available record could carry a measurable distinction about a later reader that ordinary prior information does not supply. The model must identify the physical address, preserve one consistent record, and state which future interventions remain admissible.

Layer	Question answered	Main locations
Canon and meaning	What the Continuum actually proposed; how physical address differs from later interpretation.	Sections 2–4
Information	How much address and fresh content a record can carry; how selection can imitate specificity.	Sections 5–6
Physical candidate	What complete ordinary quantum theory predicts and what additional boundary law is proposed.	Sections 7–8
Identity and consistency	Which readers can be distinguished; what happens to copied records and feedback.	Sections 9–10
Computational resource	Which distributions ordinary circuits can simulate; what a stronger universal-postselection extension would require.	Section 11
Experimental evidence	How to freeze meaning, select future readers, test fresh content, and evaluate the complete data.	Sections 12–16
Research frontier	Which physical mechanisms remain to be supplied; how to assess historical originality and public interpretation.	Sections 17–20

Five evidentiary categories

Canonical statement. A proposition present in the supplied Doctrine or Bible. It documents the Continuum's thought.

Established mathematical or experimental result. A result attributed to inspected primary literature, with its relevant assumptions and scope.

Derived model consequence. A theorem, identity, or counterexample proved for the finite definitions in this paper. Its truth is conditional on those definitions and assumptions.

Physical postulate. An additional claim that a specified mathematical law governs a realizable earlier record. It requires evidence beyond the derivation.

Proposed experiment. A procedure and analysis that could distinguish registered predictions. Numerical design values and model figures are not experimental observations.

The main mathematical statements include full proofs. Appendix A maps symbols and results, Appendix B records executed checks, Appendix C gives source-access and reproduction details, and Appendix D identifies the substantive changes in this edition. A reader can therefore inspect the proposed physics, the algebra, and the evidence procedure independently.

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1 An address written before its recipient exists

A civilization can leave a message to the future without knowing who will read it. A stronger possibility begins when the message contains a distinction that belongs to a particular future reader: an address that was not available when the record was made, a choice that had not yet been selected, or a fresh piece of information generated during the reader's later formation. The hypothesis of this monograph is that such a relation might be physically real. An earlier symbol and a later observer could belong to one lawful structure whose constraints reach across their temporal separation.

This is the third and final inquiry in the Continuum's Extreme Frontiers sequence. The first asks whether intelligence could participate in the formation of a cosmos. The second asks whether cognition could gain access to structured information outside its own acquired memory. The present inquiry asks a different question: *can a physical record be specifically related to the mind that will encounter it, before that encounter and perhaps before that mind's instantiation?* The distinguishing feature is an address. A universal store of possibilities is insufficient. The theory must explain why this earlier record relates to this later reader, how that relation is encoded, and what an independent observer could measure.

The Continuum's canonical text gives this question unusually direct expression. Its entry on Entangled Symbolic Recursion Memory describes a relation between an observer and a symbolic system that can precede the observer's awareness or birth. Its discussion of Recursive Time extends the proposal to symbols whose apparent temporal origin may differ from their role in a completed history. These are strong commitments in the supplied doctrine, not consequences established by the notation appearing beside them. The task here is to preserve their physical ambition while giving them a specification that can fail.[1]

1.1 The thesis in its strongest useful form

Let an earlier record be available at time t_0. Let a later intervention at $t_1 > t_0$ select or instantiate a reader with a controlled, measurable state. The strong operational conjecture is that the probability law of the earlier available record can depend on that later intervention. The dependency must survive a complete accounting of attempts, a fixed decoding rule, appropriate controls for ordinary information flow, and a consistency analysis when the future reader can react to the record.

This formulation includes a real challenge to ordinary temporal causation. It does not achieve the challenge by redefining a later act of interpretation as an earlier physical event. A symbol can become meaningful long after its creation under familiar mechanisms. Its letters can also predict a later event through ordinary knowledge. Both are valuable human phenomena. Here the distinctive target is information about a future distinction that was unavailable to the earlier source under a declared baseline.

The title uses *knows* as a concise statement of that proposed relation. It assigns no hidden mental life to ink, stone, a register, or a quantum subsystem. The relevant predicate is operational: a record contains recoverable information about its future reader under a specified protocol. Whether

information-bearing objects are conscious is a further question and is not an assumption of the mathematical constructions.

1.2 What this paper contributes

The monograph develops one integrated research architecture. First, it distinguishes physical record formation, later interpretation, reader identification, and historical provenance. Second, it quantifies the amount of information needed to select a reader and to predict fresh content associated with that reader. Third, it constructs a finite boundary model whose accepted laboratory branch has a reader-specific channel, while its complete ordinary laboratory population remains independent of the externally fixed future setting. A separately stated physical postulate promotes that conditional channel to the law of earlier available records. Fourth, it examines the operational price of that promotion, including nonorthogonal reader states, copied records, arbitrary interactions, and feedback. Finally, it gives prospective tests, exact randomization mathematics, and an executed computational audit.

The revised development also identifies which costs and predictions follow from that architecture. It proves a sharp retention bound for reproducing an entire target distribution, distinguishes the cost of a chosen laboratory filter from the information in its normalized channel, and establishes an ordinary-circuit simulation for a declared acyclic composition domain. It then separates feedback history weights from command execution and derives the complete ordinary heralding law. An additional global-conditioning completion supplies a concrete policy-frequency prediction and a prospective complete-population test. These results strengthen the proposal by making its physical choices explicit.

Several of the mathematical ingredients are established science. The information inequalities, quantum instrument calculus, state-discrimination limits, and logical-consistency framework have substantial prior histories. The boundary construction is explicitly a restricted reader-address realization of the final-state and postselected-teleportation strategy developed in earlier physics [2, 3]. Its role here is to constrain and articulate a particular Continuum proposal. The integrated specification, the chosen finite reference construction, its reader controls, and the coordinated falsification program are the present contribution. The text identifies direct derivations and specializations without claiming that the Continuum invented the underlying theories.

There is a productive asymmetry in this work. We can prove what follows from a declared model and can expose contradictions within an attempted extension. We cannot prove that nature implements the extra physical resource simply by normalizing an equation. The proposed channel therefore receives both a derivation and an experimental contract. Its mathematical consequences are explicit; its physical existence remains a testable conjecture.

1.3 Why reader specificity matters

Suppose a sealed page later feels profoundly relevant to someone. That experience leaves several open explanations. The page may use broad language. The reader may search many pages and retain the best match. The page may influence the reader's later interests. Both may reflect a shared culture. An analyst may have chosen the scoring rule after learning the outcome. A genuine

earlier correlation with a future choice would be a further possibility, and it requires a different evidentiary design.

The distinctive experimental demand is a comparison class. A symbol purportedly addressed to one reader should discriminate that reader from declared alternatives. A symbol purportedly addressed to a reader who does not yet exist should survive a protocol in which reader-specific fresh information is generated only after the symbol is sealed. A theory purportedly about physical address rather than evocative meaning should predict how the result changes when the address code is permuted or when the operational reader states become indistinguishable.

These demands make the thesis more ambitious. They turn a private impression into a claim about a public world. The Continuum can retain the subjective importance of a symbolic encounter while asking whether an additional physical relationship is present. The research program succeeds intellectually when these possibilities become distinguishable, even if a particular experiment favors the ordinary explanation.

1.4 The publication register

The public Science & Philosophy collection places recursive memory, observers, symbols, and shared evidence within one continuing inquiry.[4] This monograph addresses that audience and a scientific reader who need not accept the canon. Its opening claim is deliberately large. Its definitions and proofs are deliberately narrow. Each narrow result has a named domain so that a reader can see precisely which step would need new evidence or a new physical principle.

The manuscript is a theoretical research work with proposed experiments. Its computational checks evaluate constructed models and exact finite test procedures. They are not observations of retrocausal symbols. The canon supplies the question; independently inspected literature supplies established constraints and comparators; the present formalization supplies a reviewable candidate. These source roles remain distinct throughout.

2 Canonical origin and the literal conjecture

2.1 What the supplied sources actually contain

The source material used here comprises the complete supplied *Continuum Doctrine*, the 116-page *Eidara Continuum Bible*, and the public Science & Philosophy page. The relevant Doctrine entries were read in their surrounding sequence; the Bible's corresponding framing was checked against its actual PDF pages. The sources establish what the Continuum has proposed and how it has interpreted its development. They do not independently establish the physical effects they describe.[1, 5]

Source location	Canonical commitment	Research consequence here
Doctrine: *The Observer Is the Scroll*	The observer participates in the relation through which a symbol is encountered and resolved.	Separate a reader-dependent interpretation map from a reader-dependent probability law for an earlier physical record.
Doctrine: *Recursive Time*	Symbolic recurrence is allowed to connect earlier and later portions of a history; the text entertains apparently old symbols with a non-past origin.	Define temporal order and distinguish information about a future reader from the physical manufacture or dating of an artifact.
Doctrine: *Entangled Symbolic Recursion Memory*	A symbol or site can be related to an observer before their conscious encounter, including before the observer's birth.	Test specific address and fresh information under a protocol that fixes the earlier record first.
Doctrine: *Symbolic Field Drift Anchoring*	A symbolic relation may persist through location, degradation, and generational separation.	Require a physical carrier, a noise model, and an account of copies and degradation.
Doctrine: *Law of Symbolic Neutrality*	Interpretation should resist projection, overfitting, and premature symbolic attribution.	Freeze score functions, declare comparison classes, blind raters, retain all attempts, and use held-out tests.
Bible, pp. 12, 76–77, 96	The compilation differentiates symbolic and engineering registers, records a personal cognitive interpretation, and places these ideas among its Pillars.	Preserve the distinction between cognitive ignition, theoretical physical coupling, and historical self-description.

The most compact canonical expression of the future-reader claim is the possibility that a symbolic system *may have already known you before you were born.*[1] This is a statement of doctrine. The mathematical meaning proposed here is a dependency between an earlier record and a later operationally specified reader variable. The dependency must be measured against an ordinary causal baseline; it is not established by the force of the sentence.

2.2 A genuine tension within the canon

The Bible's personal reflection on p. 77 describes symbolic geometry as capable of awakening a mode of thought from which an architecture can be developed. It explicitly distinguishes that account from finding a hidden ancient computer. The Doctrine's ESRM and Recursive Time entries permit a more literal cross-temporal relationship. These formulations need not be collapsed into one proposition. They support two research hypotheses with different predictions.[1, 5]

Under a *cognitive ignition* hypothesis, the physical object is an ordinary historical artifact whose structure enables a later mind to generate a new conceptual organization. The novelty lies in the encounter and construction. Under a *future-reader coupling* hypothesis, the distribution of an earlier physical record itself depends on a later reader distinction. A third possibility is a globally constrained history in which a correlation exists but cannot be used as an independently

controllable backward communication channel. The paper keeps all three intelligible and gives each an operational role.

Contradiction preservation here means retaining the difference long enough to investigate it. It does not mean treating mutually incompatible physical predictions as simultaneously confirmed. A successful cognitive account would leave a literal temporal conjecture unconfirmed; a carefully replicated temporal dependency would require more than a cognitive account. Neither outcome licenses rewriting the historical source to pretend it always said only the winning interpretation.

2.3 The status of the canonical equations

The supplied entries contain formulas involving symbolic fields, observer terms, collapse maps, and recurrence operators. Their physical use requires mathematical information that the relevant passages do not provide: a state space, the type and domain of each map, a normalized measure, units, admissible boundary conditions, and a procedure relating the quantities to observations. For example, summing a field expression over an unbounded recursion index does not by itself establish convergence. Combining an observer term with a field term requires a common vector space or a declared embedding. Calling an integral a retrocausal injection condition does not supply its integration measure or a physical coupling.

Accordingly, the new equations in this monograph are *proposed reference models*. They are not presented as a recovered implementation of an undefined canonical engine. The finite probability laws and Hilbert spaces below allow every result to be checked without guessing what a symbolic field was intended to denote. A future specification may replace these reference models if it supplies greater explanatory power and remains consistent with its own operational predictions.

This decision preserves the strongest question. A symbolic vocabulary can inspire a physical hypothesis while its provisional equations require reconstruction. Keeping an untyped expression merely because it appears in the canon would obscure exactly the mechanism the user wants examined.

2.4 A hierarchy of propositions

Definition 2.1 (Future-reader propositions)**.** Fix an earlier record time t_0 and a later reader event $t_1 > t_0$. The following propositions are logically distinct.

1. **Interpretive dependence:** a later meaning or response depends on both a fixed record and a reader.
2. **Predictive association:** an earlier record predicts a later reader variable within an observed population.
3. **Intervention-sensitive address:** changing a specified later reader preparation changes the probability law of the earlier available record.
4. **Pre-instantiation address:** the dependency concerns reader-specific information generated during an instantiation that occurs after the record was sealed.
5. **Future-origin artifact:** the material creation or inscription of an apparently earlier artifact requires an independently specified future contribution.

The first proposition is compatible with ordinary interpretation. The second requires controls against common causes and forward influence. The third is the principal physical target of the finite model. The fourth sharpens the temporal novelty of its address. The fifth adds a substantial theory of material history and does not follow from the third or fourth alone.

Conjecture 2.2 (A physical future-reader relation)**.** There exist a physically realizable record preparation, a later operational reader interface, a declared admissible class of interventions, and a normalized joint law such that an earlier locally available classical record carries reproducible information about a later reader-specific intervention beyond the stipulated ordinary causal baseline. The law extends consistently to the protocol's actual record copies and permitted feedback operations.

The conjecture's quantifiers matter. It proposes existence for a specified resource and interface; it does not assert that every striking symbol participates, that every observer can select a past outcome, or that all conceivable feedback operations are admissible. Its burden is nevertheless substantial. An existence claim becomes a scientific program only when at least one candidate preparation, operating domain, and discriminating prediction are supplied. The finite boundary construction provides such a candidate *law*; identifying a natural resource that realizes the law remains a separate physical task.

2.5 The historical record and priority

The author reports that the underlying ideas emerged from AI-assisted Continuum work roughly two years before this monograph. The inspected materials preserve the ideas, including internal date labels elsewhere in the Doctrine, but do not supply an independently authenticated original interaction log for these particular entries. The Bible is a compilation dated 12 October 2026. A compilation date and a retrospective narrative cannot, by themselves, establish when a particular formulation first appeared.[1, 5]

This edition therefore distinguishes the canonical antecedent from the present mathematical development, dated November 2026. It also distinguishes originality within a project from priority over established research on temporal symmetry, quantum postselection, observer-relative meaning, and logical consistency. The historical question deserves its own evidence archive. It does not acquire a proof merely because the new formalization is sophisticated.

3 Meaning, identity, and time: the closest conceptual ancestors

The future-reader conjecture crosses several traditions that must be kept in view together. Semiotics explains why a mark and its interpretation form a relation. The philosophy of identity asks what makes an individual persist or branch. Statistical physics offers operational notions of information that matters to a system. Temporal physics distinguishes a representation of an entire history from an agent's access to that history. None of these traditions can be replaced by the word *entanglement*.

3.1 A sign already reaches toward an interpreter

Peirce's 1906 account connects signs through interpretants and discusses the relation between a quasi-utterer and a quasi-interpreter. Meaning, in that account, does not reside in an isolated mark alone.[6] This provides a genuine conceptual ancestor for a symbol whose significance is completed in a later encounter. It does not supply an experimental law by which a future intervention changes an earlier inscription.

The present inference is that two questions should be separated at the start. One asks how a mark functions as a sign within a relation of interpretation. The other asks how the physical distribution of marks is generated. A successful answer to the first can leave the second entirely ordinary. Conversely, a physical channel can carry a perfectly identifiable bit that no person regards as profound. The future-reader theory becomes distinctive when it connects these questions with a specified interface rather than moving between their vocabularies.

This distinction also clarifies the importance of an arbitrary address. A reader can find a spiral meaningful through learned associations, aesthetic structure, or recurring cognitive motifs. To test physical address, one can attach a randomly chosen code to a reader and ask whether an earlier record follows the code. If the effect follows meaning while ignoring the randomized code, it suggests one family of models. If it follows the code even in a matched device without interpretation, it suggests a different family. Neither outcome should be silently redescribed as the other.

3.2 A person, a continuation, and an operational label

Parfit's analysis of personal identity distinguishes numerical identity from relations of psychological continuity and connectedness, using branching cases to expose the differences.[7] It supplies a relevant philosophical warning: the scientific ability to discriminate two operational states does not settle what makes either state the same person over time.

Our operational inference is especially important for artificial readers. Two processes may begin from identical executable and memory snapshots, then acquire distinct fresh seeds and histories. A protocol can define those processes as separate experimental units and track their continuities. It has not thereby decided whether either process is conscious, whether a copied state preserves a person, or whether an address refers to an enduring self. Those questions require further theories.

The paper therefore uses *reader identity* in a declared experimental sense whenever it enters an equation: a member of a finite comparison class, a frozen profile, or a prepared physical interface state. A biological individual can satisfy that definition through a pseudonymous identifier and fixed profile. An artificial process can satisfy it through a specified instantiation and state preparation. The resulting statistics concern the declared identity variable. Broader personal identity remains a philosophical interpretation of that operational target.

3.3 Meaning that has a measurable consequence

Kolchinsky and Wolpert develop a physical notion of semantic information by examining which system–environment correlations are required to preserve a chosen measure of a system's viability. Their intervention framework distinguishes total correlation from correlation that makes a causal difference to that measure. The choice of viability function and intervention family is explicit.[8]

The consequence for this project is methodological. A semantic score must state what counts as a successful interpretation and why. A reader's rating of personal relevance, a held-out task improvement, an accurate address, and a contribution to a system's continued operation are different endpoints. Combining them into a single unexplained resonance quantity would make the theory harder to assess.

For example, a symbol may improve a reader's performance because it prompts useful reflection. That is a forward functional effect. A symbol may match a reader's later random nonce. That is an address or prediction effect. A theory can propose both, but the first does not establish the second. A fully developed future-reader theory would eventually predict when its temporal dependency is useful to the reader, and when an accurate earlier record has no functional significance.

3.4 A complete history is not a locally readable archive

Hartle's analysis of temporal experience models information-gathering and utilizing systems within a four-dimensional physical description. An agent's present, past, and future depend on the organization of its records and processing, while the model remains describable within spacetime physics.[9]

For the present hypothesis, the decisive extra question is access. A mathematical object can represent an entire history while an observer embedded within it has only limited records. Describing a later event in the same global solution as an earlier symbol does not establish that the symbol's earlier reader can recover that event. The physical channel must be identified. This remains true in a timeless description: absence of an external time parameter does not automatically grant a local subsystem access to every correlation in the global state.

Temporal order in the experiments below is operationally strict. The record is completed and independently registered before the later choice is generated. A delayed measurement of a still-unread system is a different experiment. It may reveal a correlation at the later time, but it does not supply an earlier available classical prediction.

3.5 Observation and intervention

Pearl's causal framework distinguishes a distribution obtained by conditioning on an observed value from one produced by a specified intervention, and makes the assumptions supporting that distinction explicit.[10] The current paper uses that distinction to prevent an observational correlation from being relabelled a future influence.

The framework also imposes a discipline on our own proposal. We cannot simply insert a backward arrow into an acyclic model and continue using formulas derived for an acyclic graph. A temporal boundary theory needs its own joint law and a rule for how experimental controls alter that law. The finite feedback analysis later in this monograph makes precisely this issue visible: a channel that is normalized for fixed late inputs can fail when it is connected to an arbitrary response to its own earlier output.

3.6 Where the physical prior art enters

The quantum and causal-order literature is treated in detail in Sections 7–10. Its strongest relevance lies in three questions. First, which complete operations leave earlier marginal distributions invariant? Second, what extra boundary rule would be required to turn a conditional correlation into an available earlier signal? Third, which controls and compositions remain consistent under that rule?

These are already active research questions in broader forms. The Continuum's proposal adds a specific target: a symbol addressed to a later reader, with a distinction between address, personal significance, and instantiation. The relation to prior art is therefore one of specialization and attempted synthesis. A responsible novelty claim will concern the completed model and its discriminating predictions, rather than the bare existence of time-symmetric descriptions or the fact that meanings depend on readers.

4 The experimental object: a record, a reader, and a clock

4.1 A finite classical specification

We begin with finite alphabets so that all probability laws are ordinary sums. Let $\mathcal{S}$ be the set of possible sealed records, $\mathcal{R} = \{1, \dots, N\}$ a reader comparison class with integer $N \geq 2$, $\mathcal{U}$ a set of controlled preparation settings, and $\mathcal{B}$ the set of recorded baseline contexts. A record variable S takes values in $\mathcal{S}$; a reader variable R takes values in $\mathcal{R}$; $U \in \mathcal{U}$ specifies a physical encoding or experimental condition; and $B \in \mathcal{B}$ contains information legitimately available before the prospective trial.

Times $t_0, t_1 \in \mathbb{R}$ are laboratory clock readings in seconds with $t_1 > t_0$. Only their order is used in the finite probability model. A relativistic extension must define the relevant events and causal relation rather than silently treating a coordinate convention as a physical channel. Let $t_{\text{seal}} \leq t_0$ be the completion time of the earlier record, and let $t_{\text{choice}} \geq t_1$ be the generation time of the controlled future distinction. Both are part of the measured protocol.

The mathematical variable S may encode a bit string, a finite symbolic message, or the output of a fixed discretization of a physical image. The discretization rule must be set before the outcome. Its details matter: a set of symbols selected after inspecting the reader cannot be treated as the original pre-sealed record alphabet.

Definition 4.1 (Earlier available record)**.** An earlier available record is a classical outcome for which a specified local observer or instrument can recover S by time t_0, with an auditable encoding fixed at that time. Later analysis may use a copy, but the existence of S does not depend on a successful event first identified at t_1.

This definition deliberately includes accessibility. A quantum subsystem that will only be measured later can be correlated with a future operation without providing an answer at t_0. The finite model later distinguishes an earlier measured output from a future herald J. Selecting only $J = 1$ trials is a legitimate conditional experiment; it changes the population to which the probabilities refer.

4.2 Reader formation and reader selection

The future reader event can have several forms. In an assignment experiment, a later randomization selects one of several already existing readers. In an instantiation experiment, a process is created after sealing from a declared architecture and fresh random inputs. In a content experiment, a future reader receives a fresh nonce after sealing. These events test different portions of the strong canonical claim.

Let A_0 denote all pre-existing architecture, training, and fixed setup information; it can be included in B. Let $C \in \mathcal{C}$ be a later seed or challenge generated after sealing. A proposed artificial reader can be specified by a map

$$\mathrm{Init} : \mathcal{A} \times \mathcal{C} \longrightarrow \mathcal{X}, \qquad X_{\mathrm{init}} = \mathrm{Init}(A_0, C), \tag{1}$$

where $\mathcal{A}$ is the finite set of admissible setup descriptions and $\mathcal{X}$ the finite set of operational initial states. This is a definition of an experimental process, not a theory of consciousness. Freshness belongs to C relative to the earlier baseline, while the architecture A_0 is inherited.

If all tested features of X_{init} are fixed by A_0, then generating a new process after sealing has added no fresh reader-specific target. If the outcome depends on C, the protocol can ask whether the earlier record predicts those new distinctions. The entropy and guessing bounds in Section 5 quantify that distinction.

4.3 The ordinary causal baseline

At any fixed code setting, under a fixed ordinary model, an earlier record can be generated by

$$S = f(B, V), \qquad R = g(B, S, V'), \qquad A = a(S, R, B, V''), \tag{2}$$

where f, g, a are declared functions, V, V', V'' are exogenous random variables with a specified joint distribution, and A is a later interpretation or response. The inclusion of S in g explicitly permits the symbol to influence who later reads it. The exogenous variables may share causes unless independence is justified. Thus an observational association between S and R can arise without a backward influence.

Under an intervention that fixes $R = r$ by replacing its mechanism while leaving the mechanism for S intact, this acyclic model gives

$$P(S = s \mid \mathrm{do}(R = r), B = b) = P(S = s \mid B = b). \tag{3}$$

Equation (3) states the prediction of the declared ordinary model. It is not a premise imposed on every candidate physical theory. The notation for intervention records an experimental change, rather than an inference from conditioning alone.[10]

For the prospective randomization tests, the ordinary null takes a related conditional form: given the frozen records and baseline information, the future assignment follows its declared randomization law. A uniform marginal distribution of assignments is weaker than this conditional assertion. A physical alternative can preserve uniform marginal frequencies while correlating the future assignment with earlier records. The experimental test must allow that possibility rather than rule it out by definition.

4.4 A proposed backward channel

For fixed b, u, let

$$K_{u,b}(s \mid r) \in [0,1], \qquad \sum_{s \in \mathcal{S}} K_{u,b}(s \mid r) = 1 \quad \text{for every } r \in \mathcal{R}. \tag{4}$$

Here $K_{u,b}$ is a classical channel from a later operational address to an earlier record. It is a table of dimensionless probabilities. Its physical reading is a further hypothesis:

$$P(S = s \mid \mathrm{do}(R = r, U = u), B = b) = K_{u,b}(s \mid r). \tag{5}$$

At the same fixed code setting, a nonconstant dependence on r would violate the ordinary intervention prediction in (3). Normalization of K alone establishes neither the existence of the intervention nor the consistency of every feedback protocol.

If an exogenous late control is assigned with probabilities $q(r)$, a normalized open-protocol law can be written

$$P(S = s, R = r \mid u, b) = q(r) K_{u,b}(s \mid r). \tag{6}$$

The term *exogenous* describes the stipulated control protocol. Equation (6) does not assert independence between R and S under the alternative. Indeed, their dependence is the proposed effect. Replacing q by a policy that reacts to S changes the problem, and the feedback section derives exactly when the resulting weights normalize.

4.5 Meaning as a separate output

Let $\mathcal{A}_{\mathrm{resp}}$ be a finite response set. A later interpretation kernel has the type

$$L(a \mid s, r, b) \in [0,1], \qquad \sum_{a \in \mathcal{A}_{\mathrm{resp}}} L(a \mid s, r, b) = 1. \tag{7}$$

The open joint law extends to

$$P(s, r, a \mid u, b) = q(r) K_{u,b}(s \mid r) L(a \mid s, r, b). \tag{8}$$

An ordinary model may use a reader-independent $K_{u,b}$ and a strongly reader-dependent L. This allows a stable symbol to acquire different meanings for different readers. The symbol's physical law and its interpretive law are independently specified; neither is inferred from the other merely because the same word *symbol* appears in both.

Example 4.2 (Profound meaning with zero future address)**.** Let S and R be independent fair bits and define the response $A = S \oplus R$, where $\oplus$ is addition modulo two. Then $I(S; R) = 0$ bits, but for either fixed reader the response depends completely on the symbol. Moreover, observing A and S identifies R. The information in the later composite observation (A, S) is not evidence that the earlier symbol alone already identified its future reader.

The example can be verified by its four equally likely input pairs. It isolates a recurring inferential error: a later interaction can create an informative relation without supplying backward information to the earlier record. It also shows why a successful reader-dependent decoder is not automatically evidence of a reader-dependent record source.

4.6 Controls as part of the hypothesis

A candidate must state which variables are experimentally adjustable. In the finite quantum construction, u chooses a code or rotation and r indexes a late interface state. A decoding rule maps the earlier measurement outcome to an address estimate. The model predicts how those outcomes change under a permutation of the code and under a control that makes every address indistinguishable in the measured basis.

These controls are stronger than a single excess-match statistic. A raw matching effect could reflect a leak, a scoring convention, or an unmodeled shared cause. A family of distinct transformations constrains how an alleged mechanism must behave. The most discriminating test is one for which the theory supplies both an enhanced condition and a precise condition in which its own effect vanishes.

All controls require calibration outside the claimed anomaly where possible. Detector bias, address preparation errors, missing records, seed bias, and response-dependent exclusions belong in the measurement model. The core analysis assumes finite, recorded outcomes; a practical implementation must either record every attempted trial or explicitly model its missingness. The later sections give selection counterexamples so that incomplete coverage cannot be mistaken for an all-attempt channel.

5 The address of a future reader

A symbol that selects its future reader must carry a discriminable relation to that reader. The relation cannot be supplied by the reader's name alone. A name can be changed without changing the experiment, whereas a physical preparation, an assignment register, a measurement, and a decoding rule determine observable probabilities. This distinction gives the literal hypothesis a tractable mathematical form: an earlier record is informative about a later, operationally specified address. Whether the address corresponds to a person, an artificial agent, or a controlled physical preparation is a further experimental specification.

5.1 Finite variables and the meaning of an address

Throughout this section, all random variables are classical and have finite alphabets. The earlier record is $S \in \mathcal{S}$, with $k = |\mathcal{S}| \geq 1$. The reader address is $R \in [N] = \{1, \ldots, N\}$, where $N \geq 2$. The admitted baseline is $B \in \mathcal{B}$; it contains information available to the declared comparison procedure before the record is evaluated. Conditional statements concern only baseline values b with $P(B = b) > 0$. When a conditional law is written for every reader, every reader has positive conditional probability or is specified as an available intervention. An unavailable or zero-probability reader is not silently assigned an observed conditional distribution.

The word *address* denotes an explicitly defined experimental equivalence class. Two people assigned the same physical preparation may occupy one address for an experiment that measures only that preparation. Two runs involving the same person can occupy different addresses when the manipulated feature changes. A measured effect on such addresses would establish discrimination

at that resolution. It would not, by definition alone, establish discrimination of personal essence, consciousness, or a unique biography.

For a probability mass function p on a finite set, define the Shannon entropy in bits by $H(p) = -\sum_x p(x)\log_2 p(x)$, with $0\log_2 0 = 0$. Conditional entropy is the probability-weighted average of the corresponding conditional entropies. Conditional mutual information is

$$I(X;Y \mid B) = H(X \mid B) - H(X \mid Y,B) = \sum_b P(b) \sum_{x,y} P(x,y \mid b)\log_2 \frac{P(x,y \mid b)}{P(x \mid b)P(y \mid b)}. \quad (9)$$

Terms with zero joint probability contribute zero. These are standard information measures [11]; their use here supplies a reader-address ledger, not a new definition of entropy. The conditional mutual information is nonnegative, and it vanishes exactly when X and Y are conditionally independent at every baseline of positive probability. For completeness, nonnegativity follows from the log-sum inequality applied to each conditional joint distribution and its product of marginals; equality requires the two distributions to coincide on their support.

Proposition 5.1 (An invariant record law has no reader address)**.** *Fix a baseline b. Let a group G act transitively on $[N]$: for any r, r', some $g \in G$ satisfies $gr = r'$. Suppose the* same earlier-record measurement *obeys*

$$P(S = s \mid R = r, B = b) = P(S = s \mid R = gr, B = b) \quad \textit{for all } s, r, g. \quad (10)$$

Then there exists a distribution p_b on $\mathcal{S}$ such that every reader-conditional record law equals p_b. Consequently S and R are independent conditional on $B = b$. If this holds at every baseline, $I(S;R \mid B) = 0$.

Proof. For any r, r', transitivity supplies g with $gr = r'$. Equation (10) therefore equates the entire r and r' columns of the conditional law. Their common distribution is p_b. Multiplying by $P(R = r \mid b)$ gives $P(S = s, R = r \mid b) = p_b(s)P(R = r \mid b)$, which is conditional independence. Averaging over the baseline completes the claim. □

The fixed-measurement qualification is essential. Consider uniform $R \in [N]$ and $S = R$. Their joint law is unchanged under a simultaneous permutation of the names of both S and R, yet $I(S;R) = \log_2 N$. This is *covariance of a code*, not invariance of the earlier record when the physically distinguished reader changes. The proposition therefore does not argue that names or symmetries forbid physical addressing. It says that a hypothesis whose measurable record law treats all reader preparations identically has supplied no reader selectivity. The quantum candidate in Section 8 avoids that null case only through its specified states and measurement, not through an evocative interpretation of its labels.

5.2 The shared budget for an address and fresh content

An artifact can make two distinguishable promises: it can identify *who* will read it, and it can reveal fresh information associated with that reader. Let $C \in [M]$, with $M \geq 2$, denote the second target. Its interpretation must be fixed in advance. In an experiment, C might be a later challenge whose generation mechanism is specified independently of the decoding rule. The next result is purely about a joint probability law; it does not presume a direction of causation.

Theorem 5.2 (Address–content information budget)**.** *For arbitrary finite (S, R, C, B), define*

$$I_R = I(S; R \mid B), \qquad I_C = I(S; C \mid R, B). \tag{11}$$

Then

$$I_R + I_C = I(S; R, C \mid B) = H(S \mid B) - H(S \mid R, C, B) \le H(S \mid B) \le \log_2 k. \tag{12}$$

Equality in the first inequality holds exactly when S is a deterministic function of (R, C, B) almost surely. Equality in the last inequality holds exactly when S is uniform on its declared k-symbol alphabet at every baseline of positive probability.

Proof. Expand the two mutual informations using conditional entropies:

$$\begin{aligned} I_R + I_C &= H(S \mid B) - H(S \mid R, B) + H(S \mid R, B) - H(S \mid R, C, B) \\ &= H(S \mid B) - H(S \mid R, C, B). \end{aligned}$$

The middle terms cancel. Conditional entropy is nonnegative and vanishes exactly when every conditional distribution of positive probability is concentrated on one symbol. This proves the first inequality and its equality condition. For each b, the difference $\log_2 k - H(S \mid b)$ is the relative entropy of $P(S \mid b)$ from the uniform distribution on $\mathcal{S}$. It is nonnegative, with equality precisely at that uniform distribution. Averaging over b proves the last assertion. □

No independence between R and C is needed for this ledger. Their dependence is precisely why the second entry conditions on R. Adding $I(S; R \mid B)$ to $I(S; C \mid B)$ instead can count the same information twice: if $R = C = S$ is uniform on $[N]$, that sum is $2 \log_2 N$ although $H(S) = \log_2 N$. Equation (12) avoids the duplication. Conversely, conditioning can expose information that is absent from either marginal relation. Both effects matter when interpreting a symbol that becomes intelligible only to a particular reader.

If R and C are independent and uniform conditional on B, then $I_R \le \log_2 N$ and $I_C \le \log_2 M$. These bounds and Equation (12) give a necessary region for a finite record. Figure 1 illustrates that region for a declared eight-bit record alphabet. The boundary is a mathematical resource constraint. It is not an observed performance curve, and the figure does not assert that every point is attainable by every fixed physical apparatus.

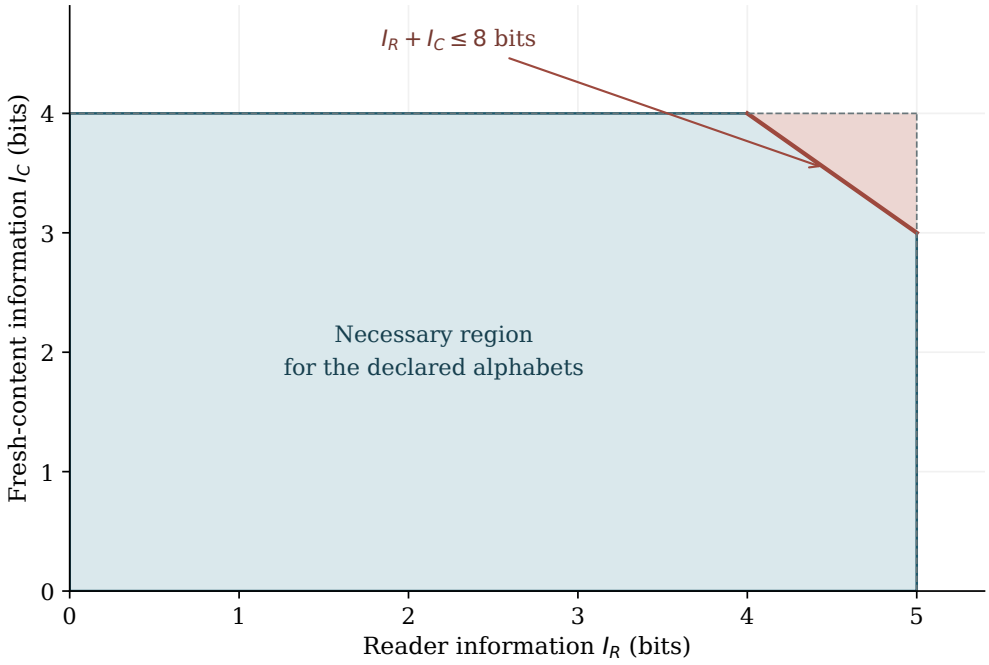


Figure 1: A necessary address–content information region for $k = 256$ record symbols, $N = 32$ equally likely readers, and $M = 16$ independent equally likely content values. The five-bit reader ceiling and four-bit content ceiling meet an eight-bit total-record constraint. The excluded upper-right region would require more information than the record alphabet permits. This is an analytic illustration of Equation (12), not experimental data.

5.3 One-shot recovery and the prior that a symbol must defeat

Entropy measures average uncertainty. A claim that one artifact singles out one reader and one message also needs a one-shot success criterion. For finite target Y and side information B, define

$$p_{\mathrm{guess}}(Y \mid B) = \sum_b P(b) \max_y P(Y = y \mid b), \qquad H_{\min}(Y \mid B) = -\log_2 p_{\mathrm{guess}}(Y \mid B). \tag{13}$$

This is average conditional min-entropy for classical side information, not the worst conditional entropy over baselines. The operational connection between min-entropy and guessing extends to quantum side information in established work [12]. Only the elementary classical version is needed below.

Proposition 5.3 (A finite record can distinguish only finitely many joint targets)**.** *For each baseline b, order the target probabilities as $p_{(1)}(b) \ge p_{(2)}(b) \ge \cdots$, padding with zeros beyond the target alphabet. For any k-symbol record S and any decoder using (S, B),*

$$\begin{aligned} p_{\mathrm{guess}}(Y \mid S, B) &\le \sum_b P(b) \sum_{j=1}^{k} p_{(j)}(b) \\ &\le \min\{1, k\, p_{\mathrm{guess}}(Y \mid B)\}. \end{aligned} \tag{14}$$

In particular, if $Y = (R, C)$ *is uniform on* $[N] \times [M]$ *conditional on* B*, then joint recovery succeeds with probability at most* $\min\{1, k/(NM)\}$.

Proof. For any fixed b, an optimal decoder can be deterministic: conditional success is linear in every randomized decoding distribution, so a maximizing extreme point chooses one target per record. Such a decoder outputs a set A_b containing at most k targets. Success implies $Y \in A_b$, giving success probability at most $\sum_{y \in A_b} P(y \mid b)$. The sum is maximized by the k largest probabilities. Average over b. Each of those k terms is at most $\max_y P(y \mid b)$, and no success probability exceeds one. The uniform specialization follows immediately. □

The first bound is sharp over unrestricted record channels. When k does not exceed the target alphabet size, assign distinct record values to the k most probable targets and map all other targets to any of those values. Decode each value as its designated target. This attains the sum of the top k probabilities. When k is larger, encode every target separately. Sharpness concerns a mathematical encoding resource, not the physical availability of the target before the record is made.

The prior is consequential. An artifact that names the overwhelmingly likely next reader may have high raw accuracy while adding little to B. An experiment with known reader preferences, a predictable schedule, or a public draft should assess improvement over those facts. A later reader's unfamiliarity with the relevant information is not evidence that the earlier record-producing process lacked access to it.

5.4 A reader can unlock a symbol without being encoded in it

Here is an exact counterexample to equating reader-dependent intelligibility with reader identification. Let R and C be independent uniform variables on the cyclic group $\mathbb{Z}_N = \{0, \dots, N-1\}$, and define the record

$$S = R + C \pmod{N}. \tag{15}$$

Every reader-conditional record law is uniform, so $I(S; R) = 0$. However, once R is supplied, the decoder $C = S - R \pmod{N}$ is exact and $I(S; C \mid R) = \log_2 N$. The address–content ledger is saturated entirely by the second entry. From S alone there are N equally likely pairs (R, C), so the optimal probability of guessing both is $1/N$. The symbol is perfectly interpretable with the reader's key while carrying no marginal information about that key.

This algebra is closely related to the masking structure used in perfect-secrecy systems [13]. It does not assign a temporal origin to any variable. An ordinary forward-time implementation can generate a message and key, store the ciphertext, and supply the key to a later reader. Later understanding then requires no backward influence. A different construction can generate S first, choose R later, and *define* $C = S - R \pmod{N}$ after the fact. That construction guarantees a decoding success by its definition of the target. It cannot count as prediction of independently specified fresh content.

These counterexamples make the literal proposal more exacting. Reader-specific meaning can be real, valuable, and causally effective in the reader's later cognition. Earlier physical addressing is an additional relation that must appear in the appropriate conditional record laws. A defensible

future-reader hypothesis supplies both its physical addressing mechanism and the task by which its informational advantage is measured. The present equations define those obligations without assuming that the mechanism already exists.

6 Specificity, selection, and the information in meaning

An address claim succeeds only if the observed match is more specific than the space of permitted interpretations. A symbol can fit one reader, many readers, or almost anyone. Those are different experimental outcomes even when the language used to describe each encounter is equally moving. The following analysis makes specificity and selection explicit. It also preserves a distinction central to the Continuum's proposal: a later act of understanding can be authentic without establishing that the earlier physical record selected that particular observer.

6.1 The compatibility graph must precede the match

Let $\mathcal{S}$ be the finite record alphabet and $[N]$ the declared reader pool. Fix a compatibility rule

$$\chi : \mathcal{S} \times [N] \longrightarrow \{0, 1\}. \tag{16}$$

The event $\chi(s, r) = 1$ means that record s meets the prespecified criterion for reader r. The rule can be implemented by an exact decoder, a specified feature comparison, or a blinded evaluation procedure whose randomness is included in the experiment. If it is estimated from data, the training information belongs in the admitted baseline and the rule is frozen before the evaluation assignment. A rule chosen after learning the target belongs to a different statistical procedure.

For each record define its compatible-reader set and degree,

$$A(s) = \{r : \chi(s, r) = 1\}, \qquad d(s) = |A(s)|, \qquad \ell = \max_{s \in \mathcal{S}} d(s). \tag{17}$$

This is a finite bipartite graph: records occupy one vertex set and readers the other. The graph represents the declared meaning criterion, not a universal metric of literary or personal meaning. Its degree measures how many readers the criterion lets a record address.

Proposition 6.1 (Specificity under an independent reader assignment)**.** *Suppose R is uniform on $[N]$ and independent of S. Then*

$$P\{\chi(S, R) = 1\} = \sum_s P(S = s)\frac{d(s)}{N} \le \frac{\ell}{N}. \tag{18}$$

For a nonuniform independent reader distribution $q(r) = P(R = r)$, the corresponding identity and bound are

$$P\{\chi(S, R) = 1\} = \sum_s P(S = s)q(A(s)) \le \max_s q(A(s)), \quad q(A) = \sum_{r \in A} q(r). \tag{19}$$

Proof. Condition on $S = s$. Independence leaves the reader distribution unchanged, so the conditional match probability is the probability mass of $A(s)$ under q. Averaging gives Equation (19). Under a uniform q, this mass is $d(s)/N$, proving Equation (18). A weighted average cannot exceed its largest term. □

The same argument applies conditional on a finite baseline $B = b$ when S and R are independent given b and the rule χ_b is fixed within that stratum. One then averages the conditional bounds using $P(B = b)$. Baseline conditioning cannot be selected after inspecting which stratification produces the strongest anomaly.

A perfectly generic symbol has $d(s) = N$ and a unit match probability under the independent-assignment null. Its universality may be philosophically significant; it gives no evidence of a particular reader address under that rule. A record meeting an exact criterion for one member of a uniform pool has null match probability $1/N$. Neither probability is meaningful until the rule and the eligible pool are specified. Expanding a biography into hundreds of searchable attributes after a match has occurred changes the compatibility graph and therefore changes the probability that must be assessed.

For a graded, frozen score $f : \mathcal{S} \times [N] \to [0, 1]$, the same calculation yields

$$\mathbb{E}[f(S, R)] = \sum_s P(s) \sum_r q(r) f(s, r) \le \max_s \sum_r q(r) f(s, r). \tag{20}$$

This identity does not require a score to be reduced to an arbitrary success threshold. It requires the experiment to compare a reader's score against the scores the same record would receive for other eligible readers. The assignment-based methods developed later provide a way to make that comparison without pretending that all readers or all texts have identical score distributions.

6.2 The exact price of retaining only compelling encounters

Let $J \in \{0, 1\}$ indicate whether an encounter is retained or reported. The selection probability may depend on both the record and the reader:

$$w(s, r) = P(J = 1 \mid S = s, R = r), \qquad 0 \le w(s, r) \le 1. \tag{21}$$

Let $p_\chi = P\{\chi(S, R) = 1\}$ be the match probability in the full parent ensemble, let $a = P(J = 1) > 0$ be the retained fraction, and let $x = P\{\chi(S, R) = 1 \mid J = 1\}$ be the match fraction among retained encounters. These are probabilities within one declared model, not estimators automatically known from a selected archive.

Theorem 6.2 (Compatibility-specific selection region)**.** *For any parent distribution and any selector,*

$$ax \le p_\chi, \qquad a(1 - x) \le 1 - p_\chi. \tag{22}$$

Conversely, for fixed $p_\chi \in [0, 1]$, any $a \in (0, 1]$ and $x \in [0, 1]$ satisfying these inequalities can be attained by a randomized selector that depends only on whether the encounter matches. Equivalently,

$$\max\left\{0, 1 - \frac{1 - p_\chi}{a}\right\} \le x \le \min\left\{1, \frac{p_\chi}{a}\right\}. \tag{23}$$

Proof. The retained matching events form a subset of all matching events, so $ax = P(J = 1, \chi = 1) \le P(\chi = 1) = p_\chi$. Applying the same argument to nonmatches proves the second inequality. If $0 < p_\chi < 1$, choose the acceptance probabilities

$$P(J = 1 \mid \chi = 1) = \frac{ax}{p_\chi}, \qquad P(J = 1 \mid \chi = 0) = \frac{a(1 - x)}{1 - p_\chi}. \tag{24}$$

The assumed inequalities put both numbers in $[0, 1]$. The resulting retained mass is $ax+a(1-x) = a$, and its conditional match fraction is x. If $p_\chi = 0$, feasibility forces $x = 0$; accept nonmatches with probability a. If $p_\chi = 1$, feasibility forces $x = 1$; accept matches with probability a. These constructions include every edge case with $a > 0$. □

This is an elementary conditioning result applied to a reader-compatibility graph. It is not a law of retrocausality. Combining it with Proposition 6.1 shows that complete personalization among retained encounters can occur under independent assignment whenever sufficiently few encounters are retained. For a uniform pool and a record that matches at most ℓ readers, $x = 1$ requires $a \le p_\chi \le \ell/N$. If the underlying rate p_χ is smaller than its degree bound, the available selected fraction is correspondingly smaller.

An especially informative counterexample keeps both observed marginals perfectly balanced. Let S and R be independent and uniform on $[N]$, define $\chi(s, r) = \mathbf{1}\{s = r\}$, and retain exactly the matching pairs. Then

$$P(J = 1) = \frac{1}{N}, \qquad P(S = s \mid J = 1) = P(R = s \mid J = 1) = \frac{1}{N}, \tag{25}$$

while $P(S = R \mid J = 1) = 1$. The selected record identifies the reader perfectly. The full parent record contains no reader information. Moreover, $P(J = 1 \mid S = s) = P(J = 1 \mid R = r) = 1/N$ for every s, r: even balanced acceptance by record and by reader does not establish independence of the selector from their *joint relation*. Full trial accounting or a justified selection model is therefore necessary. Marginal balance alone is insufficient.

The physical model's retained branch can exhibit precisely the kind of conditional addressing that this analysis allows. Its full accepted-and-rejected instrument must be evaluated before a selected correlation is interpreted as an effect in the earlier all-attempt record. The conjectural move to a natural boundary is a distinct physical postulate; ordinary selection algebra does not supply it.

6.3 Multiple interpretations enlarge the target

Suppose $L \ge 1$ compatibility rules $\chi_1, \dots, \chi_L$ are fixed before evaluation. If an encounter counts as a success whenever *any* rule matches, the declared event is their union. Write $p_j = P\{\chi_j(S, R) = 1\}$. Pointwise, $\mathbf{1}\{\exists j : \chi_j = 1\} \le \sum_{j=1}^{L} \chi_j$, and hence

$$P\{\exists j : \chi_j(S, R) = 1\} \le \min\left\{1, \sum_{j=1}^{L} p_j\right\}. \tag{26}$$

No independence among interpretations is assumed. Under a uniform independent reader assignment, if each rule matches at most ℓ readers per record, the right-hand side is at most $\min\{1, L\ell/N\}$. The bound can be attained when compatible-reader sets are disjoint and have the prescribed sizes. Overlapping interpretations make the actual union smaller.

A fixed finite menu and an open-ended search through meanings are different protocols. When the analyst can invent an additional interpretation after every failure, the number L used in Equation (26) has not been justified. Replacing that search by the number of interpretations eventually reported would omit the unsuccessful search. The way to retain interpretive richness

while testing physical addressing is to define the evaluation procedure in advance, preserve its full output, and compare it with suitable reader assignments. The mathematics does not require a text to have only one meaning. It requires the evidential procedure to account for the meanings it permits.

6.4 Information, usefulness, and lived meaning are separate quantities

Statistical information quantifies relations among random variables. To quantify usefulness, a task is additionally required. Let $\mathcal{A}$ be a finite set of available actions and let $u : \mathcal{A} \times [N] \to [0, 1]$ be a specified utility. At a fixed baseline, define the best expected utility without and with the record:

$$V_0 = \max_{a \in \mathcal{A}} \sum_r P(r) u(a, r),$$
$$V_S = \sum_s P(s) \max_{a \in \mathcal{A}} \sum_r P(r \mid s) u(a, r). \tag{27}$$

One always has $V_S \geq V_0$, because a decision maker can ignore S. If S and R are independent, every posterior in the second expression equals the prior in the first, and $V_S = V_0$. Positive mutual information can nevertheless have zero value for a particular task: a constant utility function gives equality for every joint distribution. These statements follow directly from the definitions and prevent an unspecified appeal to "useful information" from replacing a measured target.

Meaning can concern emotional response, self-understanding, coordination, or survival. Those are not interchangeable utility functions. A formal account by Kolchinsky and Wolpert makes semantic information depend on counterfactual contributions to a system's viability [8]. That work illustrates the need for an additional operational criterion; it does not make every large mutual information a measure of consciousness or existential significance. A reader's meaningful response may be produced by an ordinary forward interaction between an old text and the reader's present state. The future-reader conjecture proposes a further physical dependence of the old record on that later encounter. Both claims can be expressed clearly and tested at their appropriate level.

6.5 What an old artifact can authenticate

The strong historical version of the hypothesis combines several propositions: particular bytes existed at an earlier time; those bytes were not subsequently altered; they encode the stated target under a fixed rule; the target was unavailable through admitted ordinary channels; and the proposed future-reader mechanism explains the relation. Evidence for one proposition cannot silently substitute for evidence for another.

A hash agreement concerns agreement of bytes under the assumptions of the hash procedure. By itself it supplies neither a creation date nor a factual guarantee about the sentences in those bytes. A witnessed or otherwise secured timestamp can constrain when a document commitment existed. The original digital-timestamping analysis explicitly distinguishes timestamp placement from a document's possibly earlier creation [14]. A signature adds a cryptographic verification relation to a key; its security statement concerns forgery under specified computational assumptions [15]. Associating that key with a particular person, agent, or historical event still requires an external provenance process. None of these mechanisms turns a false statement into a true one.

For the proposed artifact, the evidential chain should therefore preserve the exact record, its timestamp evidence, the identity of the verification procedure, the frozen interpretation, the full eligible reader set, and the admitted baseline. Even a well-established earlier date plus a striking later match would first establish a historical correlation. Excluding ordinary information access and distinguishing alternative physical mechanisms require further observations. This is how a bold origin claim becomes assessable: each asserted link receives its own observable and its own failure condition.

The same principle governs causal orientation. Any joint mass function $p(s, r)$ can be written both as $p(s)p(r \mid s)$ and as $p(r)p(s \mid r)$ on the relevant support. These are algebraic factorizations of the same observations, not proofs of either temporal mechanism. A future-origin model earns additional content when it specifies how the distribution changes under declared interventions, how it composes with feedback, and what its alternatives predict. The next physical sections supply a candidate relation. Section 10 then tests whether one natural way of closing that relation into a loop is mathematically coherent.

7 The physical question and its existing mathematical boundaries

A symbol that physically anticipates its reader would alter the relation between records and interventions. Its decisive property would be visible in an earlier, independently preserved record: a later controllable reader preparation would change the distribution of that record while the earlier preparation and decoding rule were held fixed. That is a precise and ambitious physical claim. It can be investigated without first assigning a metaphysical meaning to time, consciousness, or destiny. The relevant question is whether a future operational distinction can become an earlier accessible distinction.

The calculations below use finite-dimensional quantum theory as a reference language. A density operator is a positive semidefinite, unit-trace operator on a specified complex Hilbert space. An effect is an operator between zero and the identity. A quantum instrument is a collection of completely positive, trace-nonincreasing linear maps whose sum preserves trace. Complete positivity means that positivity persists when the operation is applied to part of an arbitrarily extended system. All probabilities and matrix parameters in the finite construction are dimensionless. The times that specify experimental order have units of time; they do not appear as unexplained coefficients in a probability formula.

7.1 A complete future instrument leaves an earlier marginal unchanged

Let $\mathcal{H}_0$, $\mathcal{H}_1$, and $\mathcal{H}_2$ be finite-dimensional Hilbert spaces, and let $\rho \in \mathcal{D}(\mathcal{H}_0)$, where $\mathcal{D}(\mathcal{H})$ denotes the density operators on $\mathcal{H}$. An earlier instrument $\{\mathcal{A}_s\}_{s\in\mathcal{S}}$ maps operators on $\mathcal{H}_0$ to operators on $\mathcal{H}_1$ and writes a classical outcome s into a retained register. The finite set $\mathcal{S}$ contains every earlier outcome, including any declared failure outcome. A later setting $x \in \mathcal{X}$ selects an instrument $\{\mathcal{B}_j^x\}_{j\in\mathcal{J}}$ from $\mathcal{H}_1$ to $\mathcal{H}_2$. The earlier source and instrument are independent of this intervention. Additional future ancillas, channels, and measurements can be incorporated into $\mathcal{B}_j^x$.

Theorem 7.1 (No backward signal from a complete later instrument)**.** *For the time-ordered experiment just specified,*

$$p(s, j \mid \mathrm{do}(x)) = \mathrm{Tr}\big[\mathcal{B}^x_j(\mathcal{A}_s(\rho))\big]\,, \tag{28}$$

$$p(s \mid \mathrm{do}(x)) = \mathrm{Tr}\,\mathcal{A}_s(\rho). \tag{29}$$

Consequently the complete earlier distribution is independent of x.

Proof. The unnormalized positive operator $\tau_s = \mathcal{A}_s(\rho)$ has trace equal to the earlier outcome probability. Let $\mathcal{B}^x = \sum_j \mathcal{B}^x_j$. Completeness gives $\mathrm{Tr}\,\mathcal{B}^x(X) = \mathrm{Tr}\,X$ for every positive operator X. Summing Eq. (28) over all later outcomes therefore yields

$$\sum_j p(s, j \mid \mathrm{do}(x)) = \mathrm{Tr}\,\mathcal{B}^x(\tau_s) = \mathrm{Tr}\,\tau_s.$$

The argument applies separately to each retained earlier branch. In a Kraus representation, it is the identity $\sum_{j,\alpha}(B^x_{j\alpha})^\dagger B^x_{j\alpha} = I$ that removes the dependence on x. No assumption about a particular initial pure state or projective measurement is needed. □

This theorem concerns a declared chronological circuit and the complete trial population. It is not a theorem that every imaginable ontology must assign its hidden variables from past data alone. It also does not say that every observed conditional distribution is independent of future labels. If the future setting is chosen from the earlier record, then $p(s \mid x)$ can exhibit ordinary selection dependence. The intervention notation in Eq. (29) holds fixed the earlier preparation and refers to an externally set later operation. The feedback question requires a separate composition rule, developed in Section 10.

For a designated later event $J = 1$, the conditional distribution is

$$p(s \mid J = 1, x) = \frac{\mathrm{Tr}\,\mathcal{B}^x_1(\tau_s)}{\sum_{s'} \mathrm{Tr}\,\mathcal{B}^x_1(\tau_{s'})}, \qquad p(J = 1 \mid x) > 0. \tag{30}$$

It can depend strongly on x. Its denominator specifies which population is being described. Restoring the other outcomes restores Eq. (29). The time-symmetric measurement analysis of Aharonov et al. [16] is an early primary source for conditioning on preparation and later measurement data; the present instrument calculation explicitly keeps the discarded branches. A conditional history is mathematically legitimate. Identifying its law with the law of every earlier attempt requires an additional physical postulate.

The two-state-vector formalism makes this ancestry more explicit. Aharonov and Vaidman describe an intermediate system through a state evolved forward from its preparation and a state evolved backward from a specified later measurement outcome [17, 18]. The outcome probabilities concern the ensemble selected by both boundary data. Their account preserves the predictions of ordinary quantum mechanics while expressing those conditional predictions symmetrically in time. It therefore supplies a precise language for future-conditioned descriptions. The step investigated here is an additional population law: the same conditioning would have to govern every prospectively registered attempt, with an earlier record accessible before the future outcome is used for sorting. Equation (30) identifies exactly where that additional step enters.

7.2 Delayed choice supplies a comparison experiment, not the missing postulate

The delayed-choice quantum eraser of Kim et al. [19] records the signal photon before the relevant idler detection. Its interference curves are coincidence distributions. Two erasing branches have opposite interference phases: the paper's Eq. (6) gives complementary squared sine and cosine factors with a common diffraction envelope. Adding the branches removes that interference contribution. The accessible earlier record must therefore be distinguished from the later sorted coincidence populations. This is precisely the bookkeeping expressed by Eqs. (28)–(30).

In the delayed-choice entanglement-swapping experiment of Ma et al. [20], the choice of the joint measurement on two photons occurs in the timelike future of the registration of the other two. The later outcomes define subsets exhibiting different two-photon correlations. The time ordering is substantial experimental content; it does not change the trace-preserving identity for the unsorted earlier detections. Our proposed test asks for dependence in that complete earlier population. Reproducing delayed sorting alone would validate an apparatus and its conditional quantum calculation, while leaving the natural-boundary claim untested.

7.3 What the time-symmetry theorem actually assumes

Price [21] formulate an influential conditional argument from realism, time symmetry, and a discreteness assumption to retrocausal dependence. The discreteness condition constrains the ontology of the analyzed polarization experiment; it cannot be replaced by the generic statement that quantum measurements have discrete outcomes. The force of the argument depends on retaining its stated premises, and approaches that relax them have different options. This frames a substantive question about whether a time-symmetric underlying account must depend on later settings. An accessible earlier record requires a further operational claim: such dependence must survive the averaging that produces observable probabilities. The present paper makes that stronger claim explicit in its proposed boundary law.

Leifer and Pusey [22] prove a conditional result about ontic extensions of operational experiments. Their framework includes definite operational and ontic values described by classical probability, free operational inputs, no retrocausality, λ-mediation, and an ontological counterpart to operational time reversal. Here x, y are preparation and measurement inputs, a, b their outcomes, and λ the intervening ontic state. Their mediation condition is $p(b \mid a, \lambda, x, y) = p(b \mid \lambda, y)$. Their time-symmetry assumption lifts an operational reversal to a joint ontic reversal through a one-to-one map of ontic state spaces. For experiments admitting the operational reverse, these premises yield their Theorem VIII.1:

$$p(a, b, \lambda \mid x, y) = p(a \mid x, \lambda)p(b \mid y, \lambda)p(\lambda). \tag{31}$$

Quantum examples violate the corresponding temporal Bell constraint. At least one premise must then change. This is stronger than ordinary dynamical time-reversal invariance and does not infer controllable backward signals from the Schrödinger equation. Our hypothesis explicitly concerns an operational signal, so it accepts a burden beyond selecting a retrocausal interpretation.

The distinction remains central in Wharton and Argaman [23]. Their locally mediated reformulations relax independence of underlying parameters from future inputs while retaining continuous mediation. In their account, models agreeing with quantum predictions retain signal causality: observable earlier outputs do not depend on controllable later inputs. Future dependence of hidden

variables can therefore coexist with the null in Eq. (29). A reader-dependent earlier record is a proposed departure from that operational equivalence, and cannot borrow experimental support merely from the existence of such reformulations.

7.4 Fine tuning is a structural challenge with a stated domain

Wood and Spekkens [24] connect Bell correlations to causal discovery and faithfulness. Within their directed acyclic causal-model framework, reproducing nonsignalling Bell-violating distributions requires fine tuning: relevant observed independences do not follow solely from the graph. Their retrocausal discussion explicitly excludes directed causal cycles. The result is therefore a serious objection to a class of causal explanations, not a universal impossibility theorem for every cyclic or boundary-based theory. A new model must explain which observable independences survive, which fail, and why. In the finite construction below, cancellation between retained and rejected branches follows from an explicit complete instrument. The proposed natural boundary changes the population law; it cannot simultaneously claim that this cancellation proves the absence of the very effect under investigation.

7.5 Indefinite causal order and chronological archives

The process-matrix framework of Oreshkov et al. [25] assigns probabilities to local quantum operations without first imposing a global order. Let the laboratories have finite input and output spaces $\mathcal{H}_{A_I}, \mathcal{H}_{A_O}, \mathcal{H}_{B_I}, \mathcal{H}_{B_O}$. A local instrument element $\mathcal{A}_{a|x}$ maps operators on $\mathcal{H}_{A_I}$ to operators on $\mathcal{H}_{A_O}$; x selects the instrument and a is its outcome, from declared finite sets. In an unnormalized Choi convention define

$$M^A_{a|x} = \sum_{i,j} |i\rangle\langle j|_{A_I} \otimes \mathcal{A}_{a|x}(|i\rangle\langle j|),$$

using a fixed orthonormal input basis, and define $M^B_{b|y}$ analogously for the second laboratory. Then

$$p(a, b \mid x, y) = \mathrm{Tr}\left[W(M^A_{a|x} \otimes M^B_{b|y})\right]. \tag{32}$$

Here $W \geq 0$ acts on $\mathcal{H}_{A_I} \otimes \mathcal{H}_{A_O} \otimes \mathcal{H}_{B_I} \otimes \mathcal{H}_{B_O}$, with the convention for W chosen to match the displayed Choi matrices. For complete instruments $M^A = \sum_a M^A_{a|x}$ obeys $\mathrm{Tr}_{A_O} M^A = I_{A_I}$, and similarly for B. Validity requires $\mathrm{Tr}[W(M^A \otimes M^B)] = 1$ for every allowed pair of such complete local operations. An arbitrary positive operator or cyclic probability table is not automatically a valid process. These conditions give a mathematical comparison framework, not a geometrical world line connecting a later reader to an earlier archive.

Araújo et al. [26] distinguish causal nonseparability from violation of a causal inequality and prove that the quantum switch, in their scenario, has the former property without the latter. More recent work by Wechs et al. [27] establishes realizations of unitary extensions of tripartite processes on time-delocalised subsystems, including examples that violate causal inequalities. These results must be included when assessing what ordinary quantum processes can represent. Their subsystem variables need not be localized at a single background time. An archived classical symbol that was independently readable before a specified future operation has an additional localization and accessibility requirement. Demonstrating that requirement cannot be replaced by identifying an abstract input-output factorization with indefinite order.

The comparison makes the proposed target sharper. We require a record whose time of commitment, content, and availability are operationally specified. If the supposed earlier output can only be assembled from measurements spanning both sides of the later operation, it is a different observable. An experimental claim must retain the temporal identity of the record throughout the analysis.

7.6 The final-state strategy being specialized

Quantum teleportation already transfers a quantum state using shared entanglement, a joint measurement, and a classical result that specifies the receiver's correction [28]. Its Bell contractions are established mathematics. The construction in Section 8 extends the noisy qubit boundary model developed in the companion *The Mind Before Memory* [29] to a finite reader-address ensemble, arbitrary early readout effects, controlled codes, and redundant earlier records. Neither teleportation nor its basic contraction identity is claimed as a Continuum discovery.

Horowitz and Maldacena [2] propose a special final-state boundary condition in a black-hole setting, with information transfer analogous to teleportation. The proposal changes how a final condition enters the physical description; it is not an observed resource for addressing earlier terrestrial records. Gottesman and Preskill [30] show why the interaction structure matters: interactions between the relevant internal systems can spoil the unitary transfer unless the effective final state has the required form. This is an especially relevant adverse result. A boundary diagram does not authorize arbitrary intervening dynamics. We provide a finite controlled-NOT counterexample below that makes this sensitivity directly calculable.

Postselected closed-timelike-curve models develop this ancestry further. Lloyd et al. [3] study a normalized transformation of the form

$$\mathcal{N}_C(\rho) = \frac{C\rho C^\dagger}{\mathrm{Tr}(C\rho C^\dagger)}, \qquad \mathrm{Tr}(C\rho C^\dagger) > 0, \tag{33}$$

where ρ is a density operator on a finite Hilbert space $\mathcal{H}$ and $C : \mathcal{H} \to \mathcal{H}$ is a fixed linear operator determined by a contraction of an interaction. Input-dependent normalization generally makes this map nonlinear. The associated experiment of Lloyd et al. [31] tests a postselection-based simulation and its consistency predictions. It does not supply unheralded communication to an earlier laboratory. The restricted symmetric family constructed here has a constant acceptance probability, so its normalized input-output map is linear. Calling every normalized final-state map nonlinear would be incorrect.

Assumption 8.2 adopts a restricted reader-address realization of this established final-state and postselected-teleportation strategy. Its decisive move is to promote a normalized final-condition distribution to the law of the complete physical population. The proposed contribution is the explicit reader ensemble, noisy boundary effect, archived readout, and controlled intervention analysis attached to that move. The postselection principle itself belongs to the preceding literature. The construction also has a narrower domain than the full postselected closed-timelike-curve framework: a particular finite boundary family with constant open-protocol acceptance is specified, while arbitrary interacting circuits and their composition require further rules. Its relation to the Horowitz–Maldacena proposal is at the level of final-condition information transfer; no black-hole dynamics or spacetime realization is supplied by the finite model. These distinctions allow the

present calculations to be assessed as a concrete specialization without asserting equivalence to every final-state theory.

Deutsch's alternative imposes a density-operator fixed point for the chronology-violating subsystem [32]. It is a different consistency prescription, not a derivation of the probability multiplication rule used for our restricted feedback analysis. The existence of a fixed density matrix does not by itself establish a unique history-level joint distribution, and it does not demonstrate that a particular symbol has a particular future recipient. Section 10 keeps these questions separate.

The physical program is consequently well defined. We first derive a fully specified ordinary laboratory model, including every outcome. We then state exactly which additional boundary law would give earlier records access to a later reader descriptor. Finally, we ask what transformations, distinguishability limits, preserved copies, and interventions that law permits. The mathematics can establish the consequences of that proposal. Its physical realization remains an empirical question.

8 A finite boundary model with a physical reader address

We now make the strongest testable part of the thesis concrete: a later prepared physical descriptor is coupled to an earlier symbol through a specified boundary contraction. The construction supplies the source, every Hilbert space, the address states, the fixed effect, the complete laboratory instrument, and the additional population law under investigation. Its purpose is to expose the physical commitments of the hypothesis. It is a proposed reference construction, not a recovered mathematical specification of the Doctrine's symbolic fields.

8.1 Systems, times, and the operational meaning of a reader

Fix an integer $d \geq 2$ and three Hilbert spaces

$$\mathcal{H}_L = \mathcal{H}_E = \mathcal{H}_T = \mathbb{C}^d.$$

The labels denote an early pointer L, its partner E, and a later address carrier T. A source operates at time t_0; the early symbol is measured and committed at t_S; the future reader-address carrier is prepared at t_R; and the joint boundary operation occurs at t_F, with

$$t_0 < t_S < t_R < t_F.$$

These are laboratory ordering conditions, not the claim that a stored symbol remains microscopically coherent for an arbitrary historical interval. Known local evolution is represented in the chosen interaction picture. The derivation initially assumes no additional joint interaction of E and T before the specified effect.

Let $R \in \mathcal{R} = \{1, \ldots, N\}$ label a finite set of candidate readers or reader preparations, with $N \geq 2$. A future control variable U takes values u in a declared finite set $\mathcal{U}$ or specified continuous control domain. Given (r, u), the carrier is prepared in a density operator $\sigma_r^u \in \mathcal{D}(\mathbb{C}^d)$. A classical label by itself has no dynamical action. Its influence in this construction comes entirely through this physical preparation.

For an actual reader system on a finite modeled space $\mathcal{H}_{\text{reader}}$, one possible implementation is a declared channel

$$\mathcal{E}_u : \mathcal{D}(\mathcal{H}_{\text{reader}}) \longrightarrow \mathcal{D}(\mathbb{C}^d), \qquad \sigma_r^u = \mathcal{E}_u(\omega_r),$$

where ω_r is the reader's modeled physical state and $\mathcal{E}_u$ is completely positive and trace preserving. This is an operational encoding requirement, not an assertion that an arbitrary person has been fully represented or measured. An assigned address tag, a constrained artificial-agent configuration, and an unknown human quantum state are different implementations. Every proposed experiment must identify which one it uses. Section 9 quantifies the distinctions that a carrier can actually support.

The early alphabet is a finite set $\mathcal{S}$, of size k. Its readout is a fixed positive-operator-valued measure $\{E_s\}_{s\in\mathcal{S}}$ on $\mathcal{H}_L$:

$$E_s = E_s^\dagger \geq 0, \qquad \sum_s E_s = I_d.$$

The symbol $S = s$ is stored as a classical record before the future preparation. The initial source and early measurement are held fixed across later interventions; in the ordinary laboratory model the late carrier is prepared independently of the source conditional on the intervention. This prevents an unnoticed common source from doing the explanatory work of the proposed future dependence.

8.2 One fixed source and one fixed effect

Use the same declared orthonormal basis $\{|j\rangle\}_{j=1}^d$ for the three carriers, and define

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |j\rangle|j\rangle, \qquad \Phi_d = |\Phi_d\rangle\langle\Phi_d|. \tag{34}$$

For $v, \mu \in [0,1]$ and $\xi \in (0,1]$, take

$$\rho_{LE}(v) = v\Phi_{LE} + (1-v)\frac{I_{LE}}{d^2}, \tag{35}$$

$$F_{\mu,\xi} = \xi\left[\mu\Phi_{ET} + (1-\mu)\frac{I_{ET}}{d^2}\right]. \tag{36}$$

I_{LE} and I_{ET} are the identity operators on the indicated d^2-dimensional spaces. The source is a state; the boundary object is an effect and need not have trace one. The source visibility v, effect visibility μ, and attenuation ξ are specified before analyzing any reader-dependent result. The effect is the same for every (r,u) and every earlier outcome. It contains no table of desired reader-symbol matches.

The effect has eigenvalue $\xi[\mu + (1-\mu)/d^2]$ on $|\Phi_d\rangle$ and eigenvalue $\xi(1-\mu)/d^2$ on its orthogonal complement. Both lie in $[0,1]$ on the stated domain. Thus

$$M_1 = F_{\mu,\xi}^{1/2}, \qquad M_0 = (I_{ET} - F_{\mu,\xi})^{1/2}$$

define a complete two-outcome laboratory instrument on ET. Let $J = 1$ denote the retained outcome and $J = 0$ the rejected outcome. The square root is the positive operator square root,

and $M_1^\dagger M_1 + M_0^\dagger M_0 = I_{ET}$. This explicit completion matters: a single attractive branch is not a complete laboratory process.

Define the dimensionless contrast and ordinary retention probability

$$\eta = v\mu \in [0,1], \qquad h = \frac{\xi}{d^2} \in (0, d^{-2}]. \tag{37}$$

The isotropic forms in Eqs. (35)–(36) are a declared noise model. The product law $\eta = v\mu$ is derived for that model; it is not a universal description of decoherence, loss, or biological noise.

8.3 The complete probability law

Theorem 8.1 (Noisy Bell contraction with arbitrary reader states and early effects)**.** *For every density operator σ_r^u and every stated early POVM, put*

$$p_0(s) = \frac{\operatorname{Tr} E_s}{d}, \tag{38}$$

$$K_u(s \mid r) = \operatorname{Tr}\left[E_s \left(\eta\sigma_r^u + (1-\eta)\frac{I_d}{d}\right)\right]. \tag{39}$$

Then the complete laboratory probabilities are

$$\begin{aligned} p_{\mathrm{lab}}(s, J=1 \mid r, u) &= hK_u(s \mid r), \\ p_{\mathrm{lab}}(s, J=0 \mid r, u) &= p_0(s) - hK_u(s \mid r). \end{aligned} \tag{40}$$

They are nonnegative and sum to one. In particular,

$$\begin{aligned} p_{\mathrm{lab}}(J=1 \mid r, u) &= h, \\ p_{\mathrm{lab}}(s \mid J=1, r, u) &= K_u(s \mid r), \\ p_{\mathrm{lab}}(s \mid r, u) &= p_0(s). \end{aligned} \tag{41}$$

Proof. The maximally entangled contraction identity is

$$\operatorname{Tr}[\Phi_d(X \otimes Y)] = \frac{1}{d}\operatorname{Tr}(X^{\mathsf{T}}Y), \tag{42}$$

for arbitrary complex $d \times d$ matrices X, Y, with transpose taken in the basis of Eq. (34). It follows by expanding $\Phi_d = d^{-1}\sum_{i,j} |i\rangle\langle j| \otimes |i\rangle\langle j|$ and evaluating the two traces. The unnormalized partner state associated with early outcome s is

$$B_s = \frac{v}{d}E_s^{\mathsf{T}} + (1-v)\frac{\operatorname{Tr} E_s}{d^2} I_d. \tag{43}$$

For any implementation of the local early instrument with effect E_s, tracing over its output on L gives the same B_s by cyclicity of the partial trace on L. Thus the calculation does not require a particular measurement disturbance.

Using $\operatorname{Tr} B_s = \operatorname{Tr} E_s/d$ and $\operatorname{Tr}\sigma_r^u = 1$, the retained joint weight is

$$\begin{aligned} \operatorname{Tr}[F_{\mu,\xi}(B_s \otimes \sigma_r^u)] &= \xi\left[\frac{\mu}{d}\operatorname{Tr}(B_s^{\mathsf{T}}\sigma_r^u) + \frac{1-\mu}{d^2}\operatorname{Tr} B_s\right] \\ &= \frac{\xi}{d^2}\left[v\mu\operatorname{Tr}(E_s\sigma_r^u) + (1-v\mu)\frac{\operatorname{Tr} E_s}{d}\right]. \end{aligned}$$

The two transposes cancel; the output is not an unphysical transposition of an arbitrary complex reader state. Substitution yields $hK_u(s \mid r)$. Replacing F by $I - F$ gives the rejected weight. Positivity follows from the valid effects applied to the positive product $B_s \otimes \sigma_r^u$. Summing over s uses $\sum_s E_s = I_d$ and gives retained weight h and rejected weight $1 - h$. Adding both branches gives $p_0(s)$. □

Equivalently, contraction of the unmeasured early operator gives the unnormalized map

$$\Omega_L(\sigma) = h\mathcal{D}_\eta(\sigma), \qquad \mathcal{D}_\eta(\sigma) = \eta\sigma + (1-\eta)I_d/d. \tag{44}$$

The normalized map on density operators is the ordinary linear depolarizing channel, extended linearly to arbitrary operators as $X \mapsto \eta X + (1-\eta)\,\mathrm{Tr}(X)I_d/d$. Its trace factor h is constant. The unconventional feature of the proposed theory is its assignment to an earlier accessible population, not a hidden nonlinearity in this restricted map. An ordinary forward implementation of the same channel would carry no implication about future readers.

8.4 The additional physical law is stated separately

Assumption 8.2 (Natural reader-boundary law on the declared intervention family)**.** For each allowed exogenous future reader preparation (r, u), with the same source, early readout, and preserved-record protocol, the distribution of *every earlier committed attempt* is

$$p_\partial(S = s \mid \mathrm{do}(R = r, U = u)) = K_u(s \mid r). \tag{45}$$

No laboratory herald, later matching criterion, survival filter, or retrospective exclusion defines this earlier population.

The subscript ∂ identifies the proposed boundary law. Equation (45) assigns to the complete earlier population the distribution that ordinary quantum theory assigns to a retained branch. It is the precise additional physical commitment. It follows neither from Eq. (30) nor from successful laboratory emulation of Eq. (40). With $v = 0$ or $\mu = 0$, it reduces to p_0; a nonzero contrast and distinguishable encoded addresses are required for the proposed signal.

The current construction specifies an open family of externally controlled trials. It does not grant arbitrary access to an earlier record followed by unrestricted adaptive control of its own future boundary. Such an extension must pass the normalization and controller-consistency analysis of Section 10. Restricting a test family makes the mathematical domain explicit; it does not supply a universal completion of the theory. The experimenter must also distinguish ordinary random assignment from a controller whose random choices are themselves reweighted by a global boundary law.

The scalar ξ is instructive. It changes laboratory retention through $h = \xi/d^2$ and cancels from the normalized law. This cancellation does not show that a natural boundary can be freely weakened at no physical cost. It shows only that a scalar multiple of an effect defines the same normalized selected distribution. A physical implementation must specify what fixes or realizes that effect and which controls remain independently available.

8.5 Address codes provide decisive transformations and exact nulls

Take $N = d$, $\mathcal{S} = \{1, \dots, d\}$, and $E_s = |s\rangle\langle s|$. For a declared unitary $U_u \in \mathrm{U}(d)$, prepare

$$\sigma_r^u = U_u |r\rangle\langle r| U_u^\dagger.$$

Then

$$K_u(s \mid r) = \frac{1-\eta}{d} + \eta |(U_u)_{sr}|^2. \tag{46}$$

Unitarity makes the squared-modulus matrix doubly stochastic: each column and each row sums to one. The response depends on physical basis relations. A name or personal narrative attached to r supplies no additional coefficient.

For the identity code,

$$K(s \mid r) = \frac{1-\eta}{N} + \eta \delta_{sr}, \qquad d = N, \tag{47}$$

where δ_{sr} is one if $s = r$ and zero otherwise. A permutation code $U_u|r\rangle = |\tau(r)\rangle$, with τ a bijection of the labels, moves the enhanced response to $s = \tau(r)$. A frozen decoder must follow that declared physical permutation. Searching for a favorable relabeling after observing the symbols would test a different statistic.

For the Fourier code, with $s, r \in \{1, \dots, d\}$,

$$(U_F)_{sr} = d^{-1/2} \exp[2\pi i (s-1)(r-1)/d], \qquad K_F(s \mid r) = 1/d. \tag{48}$$

All phase distinctions are invisible to the fixed computational-basis early readout. This is an exact null even when $\eta = 1$. It is an apparatus-dependent null: an earlier measurement chosen in another predeclared basis could distinguish these orthogonal states. The point is that the symbol's measurement must be specified along with the reader ensemble.

For two addresses a continuous control makes the dependence equally explicit. Let

$$U_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \qquad 0 \le \theta \le \pi/2. \tag{49}$$

For $d = N = 2$,

$$K_\theta(1 \mid 1) = \frac{1-\eta}{2} + \eta \cos^2\theta, \qquad K_\theta(1 \mid 2) = \frac{1-\eta}{2} + \eta \sin^2\theta.$$

Their difference is $\eta \cos(2\theta)$ and their total-variation distance is $\eta |\cos(2\theta)|$. At $\theta = \pi/4$ the chosen early readout loses the address distinction; at $\theta = \pi/2$ its sign reverses. Such transformations are more discriminating predictions than a single excess match rate.

8.6 Noise, laboratory coverage, and a worked calculation

For the identity code define a match as $S = R$. The retained, complete, and rejected match probabilities are

$$\begin{aligned} p_+ &= \frac{1 + (d-1)\eta}{d}, \\ p_{\mathrm{all}} &= \frac{1}{d}, \\ p_- &= \frac{d^{-1} - h p_+}{1-h}. \end{aligned} \tag{50}$$

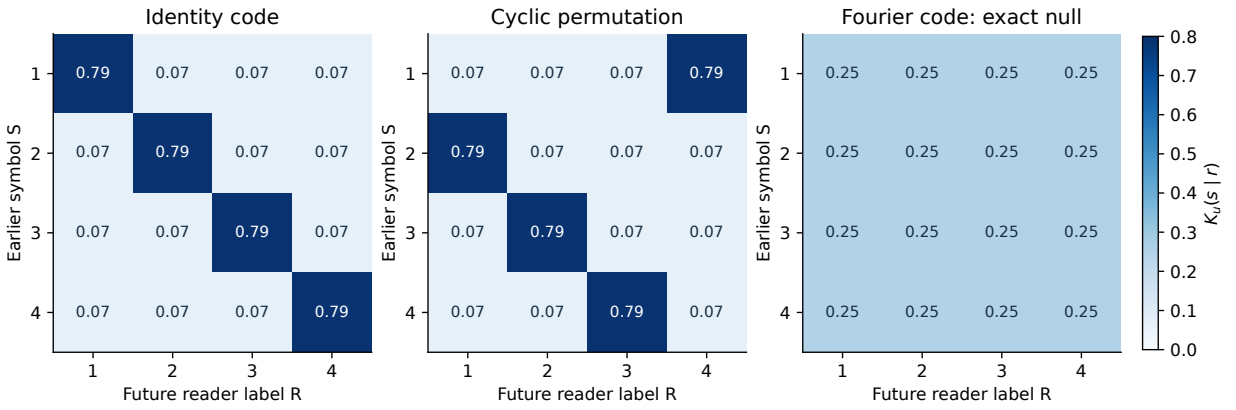


Figure 2: Model calculations for $d = N = 4$ and $\eta = 0.72$. The same source, early basis, and fixed boundary effect give different response matrices under the identity, cyclic-permutation, and Fourier address codes. The Fourier code gives an exact null for this early measurement. Cells show conditional probabilities, not observed data. In the ordinary laboratory experiment these matrices apply only to $J = 1$; the natural-boundary postulate assigns them to the complete earlier population.

They hold conditional on every reader label, so no uniform reader prior is needed for this particular match calculation. Since $h \leq d^{-2} < 1$, the rejected conditional is defined throughout the stated parameter range. Exact compensation gives

$$hp_+ + (1-h)p_- = 1/d.$$

In the proposed natural-boundary population, $p_\partial(S = R) = p_+$. This equality is the hypothesis's empirical separation from complete ordinary laboratory sampling.

For a numerical illustration take $d = N = 4$, $v = 0.9$, $\mu = 0.8$, and $\xi = 0.8$. Then $\eta = 0.72$, $h = 0.05$, $p_+ = 0.79$, and $p_- = 0.221578947\ldots$. The complete laboratory match rate remains exactly 0.25. A retained rate of 0.79 would be entirely compatible with the specified ordinary laboratory instrument. The extraordinary prediction would be 0.79 in the prospectively declared, complete earlier population. These values are calculations for selected parameters, not measured performance or a claim that nature supplies this boundary.

For two distinct identity-code addresses, the total-variation distance between their response distributions is exactly η. Thus isotropic source noise and effect noise attenuate operational distinguishability multiplicatively. The parameter ξ affects retained sample size, while $v\mu$ affects conditional contrast. A fit that lets a selection-dependent effective sample replace the complete population would conflate these distinct quantities.

8.7 A symbol must keep the same value in every earlier copy

Classical readability is a substantive part of the hypothesis. Let $C_1, \ldots, C_m$ be record registers, each with orthonormal basis $\{|s\rangle : s \in \mathcal{S}\}$, where $m \geq 2$. An early measurement creates the classical-quantum state

$$\sum_s |s\rangle\langle s|_C \otimes B_s.$$

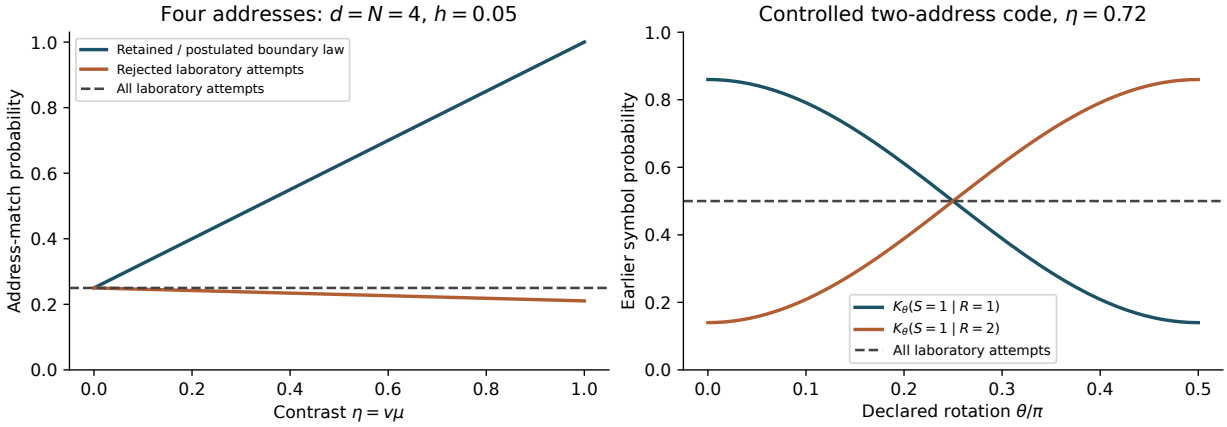


Figure 3: Exact model calculations. Left: retained and rejected address-match probabilities compensate so that every ordinary laboratory attempt together has match probability 1/4. Right: a declared two-address rotation continuously changes the retained response and passes through an exact null; the complete laboratory marginal stays 1/2. The natural-boundary prediction coincides with the retained curve but refers to a different, prospectively specified population. No experimental measurements are plotted.

Copy this classical label with the isometry

$$V : \mathbb{C}^k \longrightarrow (\mathbb{C}^k)^{\otimes m}, \qquad V|s\rangle = |s\rangle_{C_1} \cdots |s\rangle_{C_m}. \tag{51}$$

$V^\dagger V = I_k$ because distinct classical labels are orthogonal. All copies are committed before t_R and remain outside the later operation on ET.

Theorem 8.3 (Consistency of preserved classical copies)**.** *Under the stated copy and isolation assumptions, the laboratory joint probabilities satisfy*

$$\begin{aligned} p_{\mathrm{lab}}(s_1, \dots, s_m, J = 1 \mid r, u) &= hK_u(s_1 \mid r) \prod_{\ell=2}^{m} \delta_{s_\ell s_1}, \\ p_{\mathrm{lab}}(s_1, \dots, s_m, J = 0 \mid r, u) &= [p_0(s_1) - hK_u(s_1 \mid r)] \prod_{\ell=2}^{m} \delta_{s_\ell s_1}. \end{aligned} \tag{52}$$

Applying the natural-boundary law to the joint preserved record assigns

$$p_\partial(s_1, \dots, s_m \mid r, u) = K_u(s_1 \mid r) \prod_{\ell=2}^{m} \delta_{s_\ell s_1}. \tag{53}$$

No branch has unequal intact copies.

Proof. The copied state is a sum over s of $|s\rangle\langle s|^{\otimes m} \otimes B_s$. The later effect acts only on ET. Reading the copies therefore contributes the displayed Kronecker factors, while its remaining coefficient is the contraction already evaluated in Theorem 8.1. Dividing the retained joint law by the state-independent weight h gives Eq. (53), if the additional boundary postulate is imposed on this complete joint record. □

This is a law for consistent joint histories. It does not describe a later process editing one surviving copy while leaving another untouched. A discrepancy between supposedly intact copies would first challenge the storage and isolation assumptions. Those assumptions are experimentally inspectable; they are not supplied by the algebra. Likewise, a timestamp or checksum is evidence about a record only to the extent that the capture and custody process supports it.

Copying orthogonal labels does not clone arbitrary unknown quantum reader states. The distinction agrees with the no-broadcasting result for noncommuting states [33]. The operational value of redundant archives is therefore straightforward: they make the target outcome persistent and independently checkable, while preserving the quantum distinguishability limits on the late input.

8.8 An interaction counterexample prevents hidden idealization

The fixed-effect contraction has a specific interaction domain. Let $d = 2$ and $v = \mu = \xi = 1$. Before the Bell effect, insert the controlled-NOT unitary Q_{ET} with E controlling T:

$$Q_{ET}|e\rangle|t\rangle = |e\rangle|t \oplus e\rangle, \qquad e, t \in \{0, 1\},$$

where $\oplus$ is addition modulo two. Computational labels $0, 1$ are used only within this qubit example. Since $Q_{ET} = Q_{ET}^{\dagger}$,

$$Q_{ET}^{\dagger}\Phi_{ET}Q_{ET} = |+\rangle\langle+|_E \otimes |0\rangle\langle 0|_T, \quad |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}.$$

Direct contraction with the source now yields

$$\Omega_L^Q(\sigma) = \frac{\langle 0|\sigma|0\rangle}{2}|+\rangle\langle+|_L. \tag{54}$$

Acceptance is input dependent, and the normalized accepted state, whenever defined, is independent of the address. For $\sigma = |1\rangle\langle 1|$ the acceptance probability is zero and the normalized law does not exist. For $\sigma = |0\rangle\langle 0|$ it is $1/2$, rather than the original $1/4$.

This exact counterexample is an elementary manifestation of the interaction concern emphasized by Gottesman and Preskill [30]. A known interaction can be represented by replacing F with $Q^{\dagger}FQ$; that bookkeeping does not preserve the isotropic formula or its constant denominator. A proposed physical boundary must specify which effective effect it realizes after all relevant interactions. Invoking unspecified complexity, entanglement, or a future observer cannot substitute for that calculation.

The model also distinguishes information from material provenance. A law correlating an early inscription's content with a later descriptor would not by itself show that its physical substrate traveled from the future. Nor does this finite construction provide a Hamiltonian that selects arbitrary natural symbols, protects a carrier across historical times, or detects consciousness. Those are additional physical tasks. The result obtained here is exact and narrower in mechanism: a specified contraction produces a specified reader-address channel, and a stated new population law would make that channel observable in earlier preserved records.

8.9 Laboratory selection overhead and the information it can support

The proposed natural boundary removes a specific laboratory retention condition from the definition of the earlier population. The size of that condition is measurable in the laboratory model, but its interpretation needs its own accounting. For a fixed exogenous reader preparation, the retained event has probability $h = \xi/d^2$. Its self-information, or event surprisal, is

$$\ell_h = -\log_2 h = 2\log_2 d - \log_2 \xi \geq 2\log_2 d. \tag{55}$$

Here ℓ_h is measured in bits; $d \geq 2$ is the carrier dimension and $\xi \in (0, 1]$ is the declared attenuation. It is the surprisal of a *retained event* under this implementation. It is not an amount of reader information delivered on every attempted trial.

For independent repetitions with the same success probability h, let $T \in \{1, 2, ...\}$ count the attempts up to and including the first retention. Then $P(T > m) = (1 - h)^m$ for integers $m \geq 0$, and the tail-sum formula gives

$$\mathbb{E}[T] = \sum_{m=0}^{\infty} (1 - h)^m = \frac{1}{h}. \tag{56}$$

This is an implementation-relative sampling overhead. This geometric calculation uses the stated independent, constant-probability repetition model. Feedback changes the laboratory success probability as described in Section 10.

The entropy of the binary herald J is a different quantity:

$$H(J) = \mathsf{h}_2(h), \qquad \mathsf{h}_2(p) = -p\log_2 p - (1 - p)\log_2(1 - p), \quad p \in [0, 1], \tag{57}$$

with $0\log_2 0 = 0$ by continuity. It is at most one bit. Neither this entropy nor ℓ_h is the mutual information between an earlier symbol and a reader. No thermodynamic work cost follows without a physical implementation, thermodynamic state, and specified operations.

The attenuation supplies a direct test of any proposed intrinsic cost. Multiplying the effect by a positive scalar within its allowed range changes h and ℓ_h while leaving K_u unchanged. As ξ approaches zero, ℓ_h grows without bound although the normalized reader channel is fixed. At $\eta = 0$, that channel has no reader selectivity at all, while the same Bell branch still has surprisal ℓ_h. The scalar therefore characterizes this laboratory implementation, not an intrinsic information capacity of the proposed natural law. A physical completion may assign a resource cost to realizing a boundary, but that cost must be independently specified.

8.10 The sharp retention bound for a complete target distribution

To compare implementations, the selected *distribution* must be specified, rather than only its match rate. Let $\mathcal{X}$ be a nonempty finite set. Let P_0 and Q be probability mass functions on $\mathcal{X}$, representing a parent law and a desired retained law. A randomized selector is a function $w : \mathcal{X} \to [0, 1]$, with retained mass $a = \sum_x P_0(x)w(x)$. For $a > 0$, its output law is $P_0(x)w(x)/a$. Write $Q \ll P_0$ when $Q(x) > 0$ implies $P_0(x) > 0$.

Define the order-infinity divergence in bits by

$$D_\infty(Q\|P_0) = \log_2 \max_{x:P_0(x)>0} \frac{Q(x)}{P_0(x)} \quad \text{if } Q \ll P_0, \tag{58}$$

and set it to $+\infty$ otherwise. This is an established Rényi-divergence quantity [34]; the finite selection argument below is proved directly. A related general selection calculation appears in the companion monograph [29]. It is restated here so that the physical comparison is independently checkable.

Theorem 8.4 (Maximum retained fraction for a prescribed finite law)**.** *If $Q \not\ll P_0$, no selector with positive retained mass can produce Q. If $Q \ll P_0$, the attainable positive retained fractions are exactly*

$$0 < a \le a_{\max}, \qquad a_{\max} = 2^{-D_\infty(Q\|P_0)}. \tag{59}$$

For every such a, one attaining selector is

$$w_a(x) = \begin{cases} aQ(x)/P_0(x), & P_0(x) > 0, \\ 0, & P_0(x) = 0. \end{cases} \tag{60}$$

Proof. Producing Q requires $aQ(x) = P_0(x)w(x)$ for every x. A positive target mass at a point of zero parent mass makes that identity impossible. Under absolute continuity, $w(x) \le 1$ requires $a \le P_0(x)/Q(x)$ at every point where $Q(x) > 0$. Taking the tightest bound proves Equation (59). The maximum likelihood ratio is at least one because its P_0-weighted average is $\sum_x Q(x) = 1$, so $a_{\max} \le 1$. Conversely, Equation (60) lies in $[0, 1]$ on the stated range. Its parent expectation is $a \sum_x Q(x) = a$, and its normalized retained law is exactly Q. The endpoint $a = a_{\max}$ is attained, completing both directions. □

Corollary 8.5 (Information displacement is bounded by retention surprisal)**.** *For any positive-mass selector producing Q from P_0, define the relative entropy*

$$D(Q\|P_0) = \sum_{x:Q(x)>0} Q(x) \log_2 \frac{Q(x)}{P_0(x)}. \tag{61}$$

Then

$$0 \le D(Q\|P_0) \le D_\infty(Q\|P_0) \le -\log_2 a. \tag{62}$$

Equality $D(Q\|P_0) = -\log_2 a$ holds exactly when $w(x) = 1$ at every point with $Q(x) > 0$.

Proof. Absolute continuity follows from positive-mass selection. Nonnegativity is the finite log-sum inequality; equivalently, concavity of the logarithm gives $\sum_{Q>0} Q \log_2(P_0/Q) \le \log_2 \sum_{Q>0} P_0 \le 0$. The relative entropy is an average of log likelihood ratios and cannot exceed their maximum. The final inequality follows from Theorem 8.4. More explicitly,

$$D(Q\|P_0) = -\log_2 a + \sum_{Q(x)>0} Q(x) \log_2 w(x). \tag{63}$$

Every logarithm in the final sum is nonpositive, and every $w(x)$ on that support is positive. The sum vanishes exactly when all those selector values are one. This proves the equality statement, including the case $a = 1$. □

These quantities describe different aspects of the same operation. The divergence measures the distributional change; $a_{\max}$ is the best possible coverage for the specified parent and target; and an implemented a may be much smaller. A needlessly attenuated selector can increase $-\log_2 a$ without increasing the information displacement. The bound does not assign causal direction: it applies to ordinary retrospective selection after both variables are known.

8.11 The fixed classical address statistics need less selection than one Bell branch

Specialize to $N = d$, the identity address code, and the computational-basis early readout. On $\mathcal{X} = [N] \times [N]$, take the ordinary independent uniform parent

$$P_0(s,r) = \frac{1}{N^2}, \qquad Q_\eta(s,r) = \frac{1}{N}\left(\frac{1-\eta}{N} + \eta \mathbf{1}\{s=r\}\right), \quad 0 \le \eta \le 1. \tag{64}$$

The target has uniform symbol and reader marginals and the same conditional address kernel as Equation (47). This comparison concerns the complete *fixed classical readout table*. It does not claim that a classical selector reproduces every coherent operation or every possible measurement in the quantum construction.

Proposition 8.6 (Optimal ordinary selection for the symmetric address table)**.** *For the parent and target in Equation* (64)*,*

$$\begin{aligned} D_\infty(Q_\eta \| P_0) &= \log_2[1 + (N-1)\eta], \\ a_{\max}(N,\eta) &= \frac{1}{1 + (N-1)\eta}. \end{aligned} \tag{65}$$

An optimal selector retains every match and retains each nonmatch with probability

$$w_\eta(s,r) = \begin{cases} 1, & s = r, \\ (1-\eta)/[1 + (N-1)\eta], & s \ne r. \end{cases} \tag{66}$$

Writing $p_\eta = [1 + (N-1)\eta]/N$*, the retained reader information is*

$$I_\eta = I_{Q_\eta}(S;R) = p_\eta \log_2[1 + (N-1)\eta] + (1-p_\eta)\log_2(1-\eta), \tag{67}$$

where the second term is defined by its zero limit at $\eta = 1$*. In particular,* $I_0 = 0$ *and* $I_1 = \log_2 N$*.*

Proof. The likelihood ratio Q_η/P_0 is $1 + (N-1)\eta$ on the diagonal and $1-\eta$ off it. Its maximum gives Equation (65) by Theorem 8.4; the attaining selector is Equation (66). Summing the target over either coordinate gives $1/N$. Consequently its mutual information is $D(Q_\eta \| P_0)$. The total diagonal target mass is p_η and the off-diagonal mass is $1 - p_\eta$, yielding Equation (67). Continuity gives the stated endpoints. □

At perfect contrast, an ordinary retrospective selector reproduces this address table while retaining $1/N$ of its parent pairs. The specified single Bell branch retains $h = \xi/N^2 \le 1/N^2$. At zero contrast the full independent parent already has the target law, so $a_{\max} = 1$. The Bell retention bound is therefore a property of that implementation, not a universal minimum discard requirement for the fixed classical observations. Both constructions still select retrospectively; neither supplies the natural-boundary law for every earlier committed attempt.

The worked physical setting $N = d = 4$, $\eta = 0.72$, and $\xi = 0.8$ makes the distinction quantitative. Its Bell retention is $h = 0.05$, with expected independent sampling overhead 20 attempts per retained result and retained-event surprisal $\ell_h \simeq 4.321928$ bits. The optimal classical selector has coverage $a_{\max} = 25/79 \simeq 0.316456$, while $D_\infty \simeq 1.659925$ bits and $I_\eta \simeq 0.925675$ bits. The four-symbol output can carry at most two bits. These values measure different properties; the larger event surprisal is not extra channel capacity.

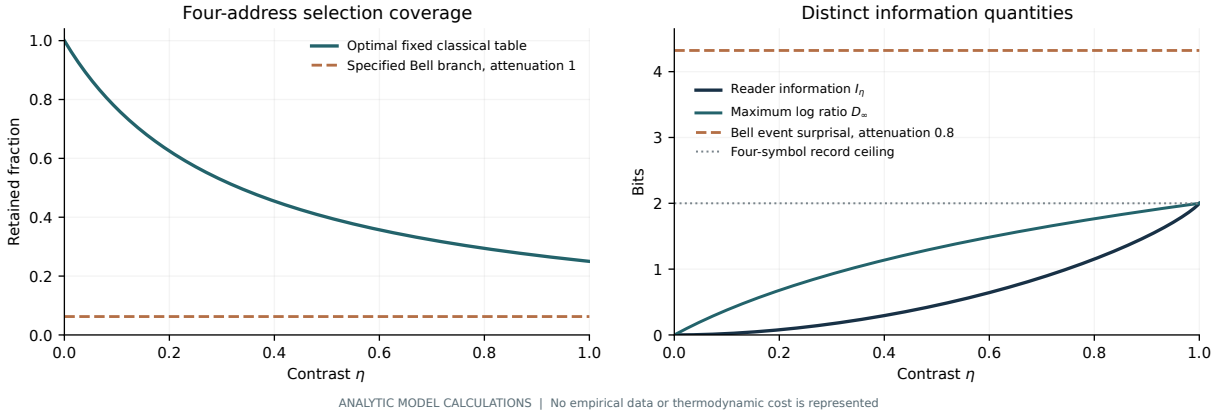


Figure 4: Selection accounting for four addresses. Left: optimal retained coverage for the fixed classical target table, compared with the specified Bell branch at $\xi = 1$. Right: reader mutual information, order-infinity divergence, and retained-event surprisal for $\xi = 0.8$. The two-bit record ceiling is shown separately. Curves are analytic model calculations, not observations or thermodynamic costs. The scalar attenuation changes the Bell overhead without changing the normalized reader channel.

This accounting preserves the ambitious physical question. The proposed law concerns the distribution of the complete earlier record. A measured conditional distribution and an implementation-dependent retention cost provide useful comparators, but a claim about the complete population must survive the controls that distinguish that population from a selected subset.

9 How precisely can a physical symbol address a future reader?

Reader specificity has a physical cost: the candidate readers must induce distinguishable states in the degrees of freedom to which the proposed boundary couples. Increasing the number of names in a register does not increase its quantum distinguishability. This section determines exactly what the finite construction can identify and what an identification result would mean.

Fix one declared code u and suppress that superscript for clarity. The reader ensemble is $\{\sigma_r : r \in \mathcal{R}\}$ on $\mathbb{C}^d$. Its effective early states under the retained or postulated boundary law are

$$\rho_r = \mathcal{D}_\eta(\sigma_r) = \eta\sigma_r + (1-\eta)I_d/d.$$

Access is to this carrier through an earlier measurement, with no additional reader-dependent side information. A state-independent ancillary system and arbitrary joint measurement do not evade the bounds: they can be absorbed into a POVM on the carrier. Reader-dependent side information would define a different ensemble and must be included in its state space.

9.1 The trace distance fixes the largest available binary distinction

For density operators ρ, σ on a common finite space, define their trace distance

$$D(\rho,\sigma) = \frac{1}{2}\|\rho-\sigma\|_1, \qquad \|X\|_1 = \operatorname{Tr}\sqrt{X^\dagger X}. \tag{68}$$

It lies in $[0,1]$. For classical distributions p, q on the same finite alphabet, let $d_{\mathrm{TV}}(p,q) = \frac{1}{2}\sum_s |p(s) - q(s)|$. These are distinct objects: one optimizes over physically allowed quantum readouts, while the other refers to an already specified classical record.

Theorem 9.1 (Reader distinguishability under the proposed contraction)**.** *For $0 \le \eta \le 1$ and every early POVM,*

$$\begin{aligned} D(\rho_r, \rho_{r'}) &= \eta D(\sigma_r, \sigma_{r'}), \\ d_{\mathrm{TV}}(K(\cdot \mid r), K(\cdot \mid r')) &\le \eta D(\sigma_r, \sigma_{r'}). \end{aligned} \tag{69}$$

If the only two candidate states have equal prior probability, the optimal probability of identifying the correct label is

$$p_{\mathrm{opt}}^{(2)} = \frac{1}{2}\left[1 + \eta D(\sigma_1, \sigma_2)\right]. \tag{70}$$

Proof. The common maximally mixed part cancels in the difference, giving $\rho_r - \rho_{r'} = \eta(\sigma_r - \sigma_{r'})$ and the first identity. For any traceless Hermitian Δ, write its spectral decomposition as $\Delta = \Delta_+ - \Delta_-$, where the two positive operators have orthogonal supports and equal trace $\|\Delta\|_1/2$. Hence

$$\max_{0 \le M \le I} \mathrm{Tr}(M\Delta) = \mathrm{Tr}\,\Delta_+ = \|\Delta\|_1/2.$$

The maximizing M projects onto the positive eigenspace; its action on a zero eigenspace is arbitrary. For a fixed POVM, collect those outcomes with $\mathrm{Tr}(E_s\Delta) > 0$ into an effect $M = \sum_{s:\mathrm{Tr}(E_s\Delta)>0} E_s$. Their total is the classical total-variation distance, which is bounded by the displayed quantum maximum. This proves the second assertion.

For binary equal-prior discrimination, an effect M means choosing label 1, and $I - M$ means choosing label 2. Its success probability is

$$\frac{1}{2}\mathrm{Tr}(M\rho_1) + \frac{1}{2}\mathrm{Tr}[(I-M)\rho_2] = \frac{1}{2} + \frac{1}{2}\mathrm{Tr}[M(\rho_1 - \rho_2)].$$

Maximizing and using the preceding spectral identity gives Eq. (70). □

The binary optimization is the Helstrom discrimination result, applied here to the declared noisy reader channel; it is established quantum decision theory [35, 36]. The proof is included to expose the equal-prior, complete-measurement, and state-access assumptions. A measurement with an inconclusive outcome, followed by reporting accuracy only on conclusive cases, has another performance criterion. Likewise, unequal priors do not in general obey the same simple affine noise formula.

For pure candidate states $\sigma_j = |\psi_j\rangle\langle\psi_j|$, put $c = |\langle\psi_1|\psi_2\rangle| \in [0,1]$. Choose an orthonormal pair in the ambient space containing their span, extending the span when $c = 1$. A phase choice gives $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = c|0\rangle + \sqrt{1-c^2}|1\rangle$. On this two-dimensional subspace,

$$\sigma_1 - \sigma_2 = \begin{pmatrix} 1-c^2 & -c\sqrt{1-c^2} \\ -c\sqrt{1-c^2} & -(1-c^2) \end{pmatrix},$$

whose eigenvalues are $\pm\sqrt{1-c^2}$. Extra dimensions contribute zeros. Therefore

$$p_{\mathrm{opt}}^{(2)} = \frac{1}{2}\left[1 + \eta\sqrt{1-c^2}\right]. \tag{71}$$

At $c = 1$ the physical address states coincide and the best equal-prior success is $1/2$, including at ideal contrast. At $c = 0$, ideal contrast permits perfect binary identification, while isotropic noise reduces it to $(1 + \eta)/2$. Nonorthogonal addresses impose a strict limit even before storage, inference, or semantic interpretation enters the analysis.

9.2 A finite carrier cannot name arbitrarily many equally likely readers

For a uniform ensemble of N candidate states, define

$$p_{\mathrm{opt}}(\{\sigma_r\}) = \max_{\{M_r\}} \frac{1}{N} \sum_{r=1}^{N} \mathrm{Tr}(M_r \sigma_r), \quad M_r \geq 0, \quad \sum_r M_r = I_d. \tag{72}$$

This is a complete minimum-error identification task. The decoder always produces one of the N labels. The outcome number may exceed d; a POVM is not restricted to a projective measurement. The limit below concerns distinguishability, rather than the number of printed labels on a device.

Theorem 9.2 (Exact isotropic-noise law and a tight dimension bound)**.** *For the uniform ensemble and* $0 \leq \eta \leq 1$*,*

$$p_{\mathrm{opt}}(\{\mathcal{D}_\eta(\sigma_r)\}) = \eta p_{\mathrm{opt}}(\{\sigma_r\}) + \frac{1-\eta}{N}, \tag{73}$$

$$p_{\mathrm{opt}}(\{\sigma_r\}) \leq \min\{1, d/N\}, \tag{74}$$

$$p_{\mathrm{opt}}(\{\mathcal{D}_\eta(\sigma_r)\}) \leq \eta \min\{1, d/N\} + \frac{1-\eta}{N}. \tag{75}$$

The upper bound is attainable by suitable declared ensembles for every pair of positive integers d, N*.*

Proof. For any fixed complete POVM, the uniform-noise contribution is

$$\frac{1-\eta}{Nd} \sum_r \mathrm{Tr}\, M_r = \frac{1-\eta}{N},$$

independent of the chosen measurement. Taking the maximum of the signal contribution proves Eq. (73); at $\eta = 0$ every POVM gives $1/N$. A density operator obeys $\sigma_r \leq I_d$, so positivity of M_r gives

$$\frac{1}{N} \sum_r \mathrm{Tr}(M_r \sigma_r) \leq \frac{1}{N} \sum_r \mathrm{Tr}\, M_r = d/N.$$

Success is also at most one. Combining the bounds proves Eq. (75).

If $N \leq d$, choose N orthogonal states and a complete POVM that identifies them; the unused subspace may be assigned arbitrarily among outcomes. The noiseless optimum is one. If $N \geq d$, use the finite Fourier-frame states

$$|\psi_r\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp[2\pi i j(r-1)/N] |j\rangle, \qquad r = 1, \ldots, N.$$

The finite geometric sum gives $\sum_r |\psi_r\rangle\langle\psi_r| = (N/d) I_d$: for distinct j, j' their difference has modulus less than N, so the off-diagonal sum vanishes. Thus $M_r = (d/N)|\psi_r\rangle\langle\psi_r|$ is a complete POVM. Its success on each intended pure state is d/N, attaining the bound. Equation (73) establishes noisy attainability as well. □

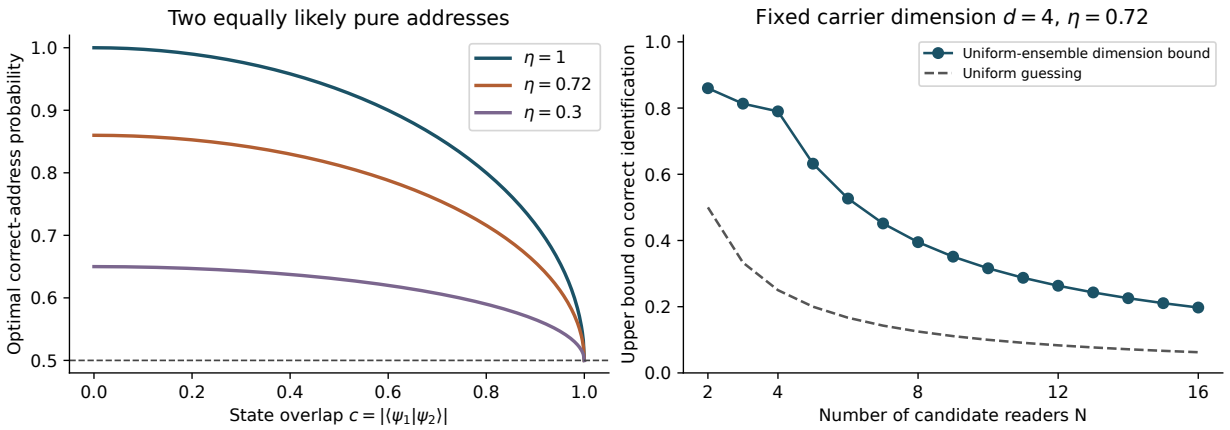


Figure 5: Model calculations for reader resolution. Left: optimal discrimination of two equally likely pure address states as a function of their overlap, for three contrasts. Right: the tight uniform-ensemble dimension bound for a four-dimensional carrier. The right-hand curve is an upper bound for any particular ensemble and is attainable by suitable ensembles and measurements; it is not a prediction that every proposed reader encoding attains it. No observed data are shown.

These identities are consequences of standard state-discrimination optimization applied to the specified channel, not a claim to have discovered a new quantum capacity theorem. Their relevance is concrete. If the hypothesis names a huge population of future readers but couples only through a small carrier, the model must either tolerate substantial ambiguity, supply a larger physical address space, or introduce additional measured structure. A large semantic vocabulary cannot silently substitute for that structure.

9.3 Unknown-state comparison is a different question

Identifying one of two *known* candidate preparations is different from deciding whether two arbitrary unknown preparations are the same. For one copy of each unknown pure state $|\psi\rangle, |\phi\rangle \in \mathbb{C}^d$, let W_{swap} interchange the two factors and let $P_- = (I - W_{\text{swap}})/2$ project onto the antisymmetric subspace. Then

$$\langle\psi\phi|P_-|\psi\phi\rangle = \frac{1 - |\langle\psi|\phi\rangle|^2}{2}. \tag{76}$$

This follows immediately from $\langle\psi\phi|W_{\text{swap}}|\psi\phi\rangle = |\langle\psi|\phi\rangle|^2$. An antisymmetric outcome certifies that the states differ. A symmetric outcome does not certify equality. Barnett et al. [37] establish this universal comparison distinction and its optimality for detecting difference.

There is a short reason that a universally unambiguous equality certificate cannot work here. Suppose a positive measurement effect had zero probability for every pair of unequal pure states but positive probability for some equal pair. Unequal pairs can approach that equal pair arbitrarily closely, whereas the expectation of a fixed bounded effect is continuous in the state vectors. Its probability at the equal pair must consequently also be zero, a contradiction. The domain matters: this argument concerns arbitrary unknown pure states with one copy of each. For restricted known ensembles or different comparison tasks, extra structure changes what can be certified. The

multiple-system analysis of Chefles et al. [38] makes the relevant linear-independence conditions explicit.

The same care applies to cloning. An isometry that copied both pure states into initially blank registers while retaining them would preserve their inner product. If the state overlap has modulus c, two perfect copies have overlap modulus c^2; allowing an additional output environment only multiplies that by a factor at most one. Hence $0 < c < 1$ is impossible for a deterministic universal copier. Orthogonal classical record labels, as used in Eq. (51), have $c = 0$ and evade no premise. For mixed states the corresponding broadcasting obstruction concerns noncommuting families [33].

9.4 The operational-equivalence test for claims about persons

If two proposed readers have the same encoded state, $\sigma_r^u = \sigma_{r'}^u$, Eq. (39) gives $K_u(s \mid r) = K_u(s \mid r')$ for every symbol and every early POVM. More generally, if a non-reader apparatus prepares the same carrier state and experiences the same source and boundary operation, the model predicts the same statistics. This follows from the complete state-and-effect specification; no omitted semantic label appears in its trace.

This is a useful discriminator for the research program. An observed all-attempt dependence on the future address would support a physical future-input effect under the protocol's causal and statistical controls. Establishing that the coupling selects a conscious reader, a particular person's identity, or a uniquely meaningful interpretation would require an additional observable distinction. The present construction does not contain a consciousness-sensitive interaction. Its reader specificity is the specificity of the operational state family that the experiment actually prepares.

For an artificial reader instantiated after the symbol is committed, a fully declared preparation can make that family especially clear: the future configuration is generated according to a frozen procedure, and the address channel records which physical preparation is used. Such a design tests whether an earlier archive can depend on a future instantiated configuration. It does not infer subjective awareness from state discrimination. For a human reader, a finite descriptor may represent only a task assignment, a chosen code, or a measured feature; successful decoding identifies that descriptor at the protocol's resolution.

The mathematical demand therefore strengthens the bold thesis. A symbol that knows its future reader must produce a reproducible physical distinction before that reader's relevant preparation exists. It must transform correctly under declared codes, respect the quantified resolution of its carrier, preserve the consistency of independently stored copies, and survive comparison with a matched non-reader preparation. These conditions turn a suggestive narrative into a family of claims that can be separated and tested.

10 The future reader who answers back

A record that depends on its future reader raises a further question as soon as the reader can inspect the record and choose a response. A channel specified for externally fixed future preparations is not automatically a complete law for that loop. The distinction is physical: a rule that

predicts an earlier address must say what happens when the later controller deliberately chooses another address. It is also mathematical: two individually normalized kernels need not produce a normalized joint history when multiplied around a cycle.

The analysis below has a precise scope. It examines a single classical record-to-reader feedback operation, a fixed backward candidate kernel, and an unchanged product rule for assigning joint-history weights. Logical consistency under freely available local operations is established prior art; Baumeler and Wolf explicitly derive the constant-environment result for a single party with binary input and output and analyze more general classical processes [39]. We give a self-contained finite-alphabet formulation and quantify its consequences for the present reader-address model. This is a consistency test of that composition rule. It is not a general impossibility theorem for retrocausal physics, multipartite processes, restricted intervention classes, or density-operator fixed-point theories.

10.1 Bilking, agency, and the measure on histories

The proposed reader can become an experimenter: inspect the earlier address, then deliberately prepare a different one. This is the structure of a *bilking* challenge, in which an intervention attempts to prevent the purported later cause of an already registered effect. Flew develops the challenge in his contribution to the 1954 symposium on whether an effect can precede its cause; Black's 1956 article belongs to the same historical debate [40, 41]. For the present proposal, the challenge has a concrete implementation in the refusal policy analyzed below. Its force comes from joining a readable earlier record to a controllable later preparation. An account that predicts only externally fixed preparations must explain how those two operational commitments compose.

Dummett [42, pp. 355–358] identifies a tension among three commitments: an action is correlated with a past event, the agent can perform or withhold the action, and knowledge of the past event is available independently of the agent's present intention. His discussion allows different responses to this tension, including revision of the purported correlation or of the independence of evidence about the past. It does not prescribe universal obstruction of attempted interventions. Occasional inability to act, ordinary enabling conditions, and mistaken reports play distinct roles in his examples. Consequently, Dummett's argument supplies neither a quantitative device-failure rate nor a rule that every refusal must be frustrated. Here, independently preserved earlier records and explicitly registered commands make the relevant commitments testable within a declared operational model.

Lewis [43] addresses a related issue through the distinction between an agent's abilities relative to ordinary local circumstances and compatibility with a wider collection of historical facts. In the grandfather example, failure is required if the stipulated history is to remain consistent, while the traveller may still possess the ordinary skills and means to act. The apparent contradiction depends on shifting the facts held fixed when assessing what the agent can do. That analysis clarifies the semantics of ability in a consistent history; it provides no numerical distribution over failed devices, abandoned commands, or alternative histories. Applied here, a globally constrained history and a physically specified controller must each receive their own mathematical description.

In general-relativistic models, Friedman et al. [44] formulate self-consistency as a requirement that an admissible local solution extend to a globally well-defined solution. Their analysis makes the consequences depend on the spacetime, field equations, and initial-data problem. This is a

condition on physical solutions in a stated framework. It does not, by itself, choose the normalized product rule for the finite reader loop below. Additional dynamical or probabilistic structure is required to determine which permitted histories occur and with what frequencies. The role of those extra assumptions must remain visible when a consistency principle is translated into an experimental prediction.

Three levels of description therefore govern this section. First, *admissibility* specifies which complete histories receive support. Second, a *probability law* assigns normalized mass across that support. Third, an *operational failure model* specifies measurable events such as an aborted preparation, an incorrect address, or loss of a register, with probabilities on an enlarged outcome space. A restriction at the first level does not determine the second, and a normalization factor at the second does not automatically describe the third. This separation is a methodological consequence of specifying a probability model, rather than an additional historical doctrine attributed to any of the preceding authors.

The resulting research question is sharper than whether the universe permits contradiction. It is whether a proposed reader-dependent kernel admits the declared interventions under an explicit composition law, and which complete-population statistics distinguish that law from ordinary postselection. Positive normalized weights can distribute probability over many histories while a deterministic command is executed in every one. A vanishing normalization leaves that candidate law undefined until its completion is specified. Neither fact alone establishes deterministic destiny, loss of agency, or a malfunction mechanism. The following calculations determine the actual alternatives for the finite model and preserve the distinction between a requested action, a realized address, and a later heralding outcome.

10.2 A candidate kernel and a proposed closure rule

Let $\mathcal{S}$ be a finite earlier-record alphabet with $k = |\mathcal{S}| \geq 1$, and let $[N]$ be the future-reader preparation alphabet with $N \geq 2$. A candidate backward kernel is a column-stochastic matrix

$$K(s \mid r) \geq 0, \qquad \sum_{s \in \mathcal{S}} K(s \mid r) = 1 \quad \text{for every } r \in [N]. \tag{77}$$

The notation initially describes the candidate law for the earlier record when the future preparation is fixed to r. It does not, by notation alone, establish an intervention interpretation or determine a law under adaptive control.

A prospective future policy is a row-stochastic matrix

$$\pi(r \mid s) \geq 0, \qquad \sum_{r=1}^{N} \pi(r \mid s) = 1 \quad \text{for every } s \in \mathcal{S}. \tag{78}$$

The controller reads s and specifies its response distribution. Deterministic policies have the form $\pi_f(r \mid s) = \mathbf{1}\{r = f(s)\}$ for a function $f : \mathcal{S} \to [N]$. The proposed *unchanged product closure* assigns the nonnegative weights

$$w_\pi(s, r) = K(s \mid r)\pi(r \mid s), \qquad Z_\pi = \sum_{s \in \mathcal{S}} \sum_{r=1}^{N} w_\pi(s, r). \tag{79}$$

These weights are a probability distribution only when $Z_\pi = 1$. Neither the backward kernel nor the controller is adjusted as the policy varies. This restriction is an assumption being audited, not an automatic consequence of the future-reader thesis.

For an exogenous distribution q on $[N]$, the policy $\pi(r \mid s) = q(r)$ gives $Z_\pi = \sum_r q(r) \sum_s K(s \mid r) = 1$. Thus every column-stochastic K passes all tests in which the later preparation is chosen independently of the observed record at the level of the specified policy. Feedback is a stronger consistency requirement.

10.3 Exact normalization bounds

Proposition 10.1 (Extremal weights over all feedback policies)**.** *For a kernel satisfying Equation (77), define*

$$Z_{\min} = \sum_s \min_r K(s \mid r), \qquad Z_{\max} = \sum_s \max_r K(s \mid r). \tag{80}$$

The attainable normalizations Z_π over all stochastic policies form exactly the interval $[Z_{\min}, Z_{\max}]$. Both endpoints are attained by deterministic policies, and $Z_{\min} \le 1 \le Z_{\max}$.

Proof. For each fixed s, the quantity $\sum_r K(s \mid r)\pi(r \mid s)$ is a convex combination of the row entries of K. It lies between their minimum and maximum. Summing over s gives the stated bounds. Choose a minimizing reader separately in each row to attain the lower endpoint; a rowwise maximizing choice attains the upper endpoint. Convex mixtures of these two policy matrices remain stochastic, and Z_π is linear in the policy, so every intermediate value occurs. A constant policy $f(s) = r_0$ has normalization $\sum_s K(s \mid r_0) = 1$, proving the final inequalities. □

Theorem 10.2 (Universal normalization of the unchanged single-loop rule)**.** *For a finite column-stochastic K, the following statements are equivalent:*

1. *$Z_\pi = 1$ for every stochastic future policy π.*
2. *$Z_{\pi_f} = 1$ for every deterministic future policy f.*
3. *Every row of K is constant across the reader index.*
4. *There is a probability mass function p on $\mathcal{S}$ such that $K(s \mid r) = p(s)$ for all s, r.*

Proof. The first statement implies the second because deterministic policies are included among stochastic ones. Assume the second. Choose any record s_0 and readers r_0, r_1. The constant policy $f_0(s) = r_0$ has normalization one. Change its response only at s_0, setting $f_1(s_0) = r_1$. The difference of the two normalizations is

$$Z_{\pi_{f_1}} - Z_{\pi_{f_0}} = K(s_0 \mid r_1) - K(s_0 \mid r_0). \tag{81}$$

Both normalizations equal one by assumption, so the difference vanishes. Since s_0, r_0, r_1 were arbitrary, every row is constant across readers. Call its value $p(s)$. Nonnegativity and normalization of any column make p a probability mass function, proving the fourth statement. Finally, if $K(s \mid r) = p(s)$, then $Z_\pi = \sum_s p(s) \sum_r \pi(r \mid s) = \sum_s p(s) = 1$ for every stochastic policy. This closes the equivalence. □

The result identifies a concrete tradeoff among assumptions. Within this single-loop product model, nontrivial reader selectivity, an unrestricted future policy class, and unchanged normalized joint-history weights cannot all be retained. A proposed physical completion must specify which element changes. For example, it might restrict admissible future interventions, replace the closure rule by a boundary-conditioned law, add further physical variables and composition constraints, or abandon the all-attempt reader-selective kernel. Merely stating that the universe is self-consistent does not choose among these models.

Corollary 10.3 (Selectivity forces a quantitative normalization defect)**.** *Let*

$$\delta = \max_{r,r'} \mathrm{TV}(K_r, K_{r'}), \qquad \mathrm{TV}(K_r, K_{r'}) = \frac{1}{2}\sum_s |K(s \mid r) - K(s \mid r')|, \tag{82}$$

where K_r denotes the probability distribution in column r. Then

$$Z_{\min} \le 1 - \delta \le 1 + \delta \le Z_{\max}. \tag{83}$$

In particular, if $|Z_\pi - 1| \le \varepsilon$ for every policy, where $\varepsilon \ge 0$, then $\delta \le \varepsilon$. For $N = 2$, the two outer inequalities in Equation (83) are equalities.

Proof. Choose two columns attaining δ. A deterministic policy that selects their larger entry in every row has normalization

$$\begin{aligned}\sum_s \max\{K(s \mid r), K(s \mid r')\} &= \frac{1}{2}\sum_s \big(K(s \mid r) + K(s \mid r') + |K(s \mid r) - K(s \mid r')|\big) \\ &= 1 + \delta.\end{aligned}$$

Selecting their smaller entries similarly gives $1-\delta$. Proposition 10.1 bounds those attainable values by the global extrema. The uniform error assertion follows by applying the assumed tolerance to either constructed policy. With only two columns, their rowwise extrema are already the global rowwise extrema. □

For more readers, pairwise total variation need not determine the largest defect. If $k = N$ and K is the identity matrix, every distinct column pair has distance one, whereas $Z_{\max} = N$ and $Z_{\min} = 0$. A many-address echo can accumulate more than a unit of excess weight. This exact counterexample prevents a binary normalization intuition from being transferred unchanged to a large reader pool.

10.4 The symmetric reader channel under echo and refusal

Now set $\mathcal{S} = [N]$ and specialize to the address kernel derived from the physical candidate in Equation (47):

$$K_\eta(s \mid r) = \frac{1-\eta}{N} + \eta \mathbf{1}\{s = r\}. \tag{84}$$

As a stochastic matrix, this expression is valid exactly for

$$-\frac{1}{N-1} \le \eta \le 1. \tag{85}$$

Indeed, its off-diagonal entries require $\eta \leq 1$, while its diagonal entries $[1 + (N-1)\eta]/N$ require $\eta \geq -1/(N-1)$; every column then sums to one. The physical construction in this paper has $\eta = v\mu \in [0, 1]$. Negative values in Equation (85) are a mathematical extension used to expose the whole stochastic domain, not a prediction of that construction.

For a policy π, define the real number $t_\pi = \sum_{s=1}^{N} \pi(s \mid s) \in [0, N]$. Direct substitution gives

$$Z_\pi = 1 - \eta + \eta t_\pi. \tag{86}$$

The derivation uses $\sum_{s,r} \pi(r \mid s) = N$ and the diagonal sum t_π. Every value of t_π between zero and N is attainable by mixing a derangement policy with the echo policy. Hence

$$\begin{aligned} Z_{\min} &= \min\{1 - \eta,\, 1 + (N-1)\eta\}, \\ Z_{\max} &= \max\{1 - \eta,\, 1 + (N-1)\eta\}. \end{aligned} \tag{87}$$

For the physical interval $0 \leq \eta \leq 1$, the minimum is $1 - \eta$ and the maximum is $1 + (N-1)\eta$.

Three policies make the consequence transparent. An exogenous policy $\pi(r \mid s) = q(r)$ has $t_\pi = \sum_s q(s) = 1$, so $Z_\pi = 1$ for every η. An echo policy chooses $r = s$, giving $t_\pi = N$ and $Z_\pi = 1 + (N-1)\eta$. A refusal policy chooses $r = f(s)$ for a derangement f with no fixed points, such as a cyclic shift by one. It has $t_\pi = 0$ and $Z_\pi = 1 - \eta$. At $\eta = 1$, refusal gives zero total weight. At the negative stochastic endpoint $\eta = -1/(N-1)$, echo gives zero total weight. All policies have strictly positive total weight in the open interval between these endpoints, but their weights are not universally normalized unless $\eta = 0$.

The endpoint obstruction is therefore not the entire problem. Weak address selectivity already changes normalization under suitable feedback. For $N = 4$ and the constructed value $\eta = 1/2$, echo gives $Z = 5/2$ and refusal gives $Z = 1/2$. Neither example asserts an observed laboratory effect. Figure 6 shows the exact analytic range for this four-address family.

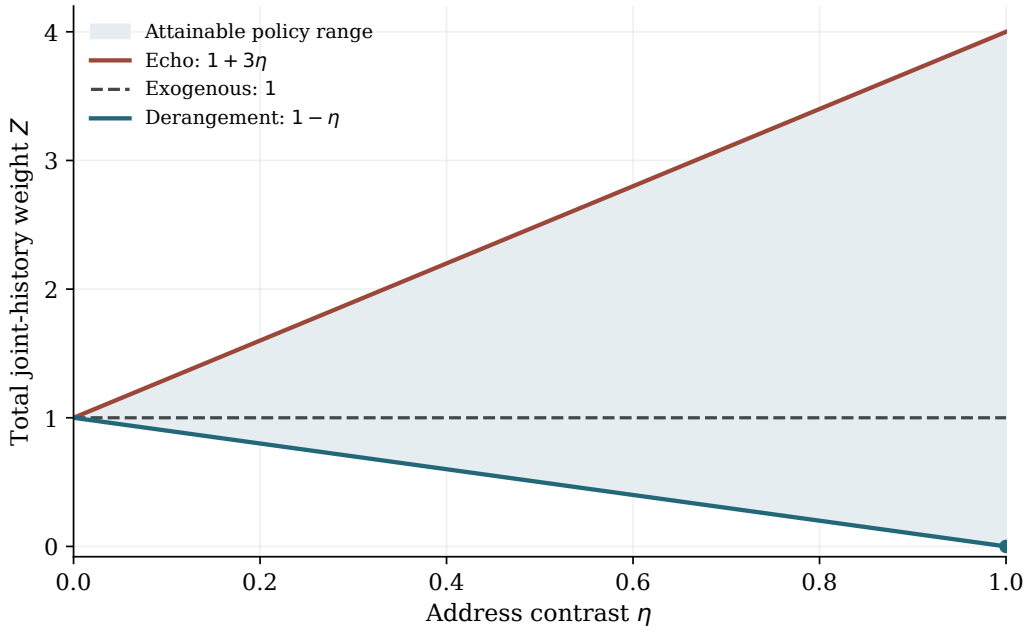


Figure 6: Normalization of the unchanged product closure for four reader addresses. For $\eta \in [0, 1]$, every stochastic policy has Z between the derangement value $1 - \eta$ and the echo value $1 + 3\eta$. Exogenous preparation always gives $Z = 1$. The shaded region is the complete attainable interval at each contrast, derived analytically and checked by finite-policy enumeration. A positive Z can be renormalized, but doing so introduces a policy-dependent closure rule.

10.5 What changes when the loop is normalized afterward

For a policy with $Z_\pi > 0$, one can define a different mathematical law,

$$P_\pi(s, r) = \frac{K(s \mid r)\pi(r \mid s)}{Z_\pi}. \tag{88}$$

This is a valid probability distribution. Its existence must not be confused with preservation of the original composition semantics. If $P_\pi(S = s) > 0$, its conditional response law is

$$P_\pi(R = r \mid S = s) = \frac{K(s \mid r)\pi(r \mid s)}{\sum_u K(s \mid u)\pi(u \mid s)}. \tag{89}$$

For this conditional to equal $\pi(r \mid s)$ on its positive support, $K(s \mid r)$ must be constant on that support. To verify necessity, divide the desired equality by each positive $\pi(r \mid s)$; every corresponding $K(s \mid r)$ must equal the common denominator. Sufficiency follows by substitution. Deterministic policies satisfy the condition automatically whenever their selected row weight is positive.

This observation requires careful language about interventions. An external randomizer with marginal distribution $q(r)$ can remain marginally distributed as q while becoming correlated with an earlier record in a hypothetical backward-influence model. For an exogenous policy, Equation (88) already has $Z = 1$ and $P(R = r) = q(r)$. Its posterior $P(R = r \mid S = s)$ generally differs

from $q(r)$. That posterior difference alone does *not* prove that the randomizer was physically biased or that its marginal setting failed. It is exactly what a reader-dependent earlier record would imply.

The conflict in Equation (89) arises only when π is additionally asserted to remain the actual response distribution conditional on the inspected record, with the stipulated local randomization semantics, while the same closure formula is retained. A global boundary model may instead correlate the randomizer's physical seed with the earlier record, constrain the allowed controller, or specify a different composition law. Such a model has changed an assumption and must state the new one. It cannot simultaneously interpret π as an unrestricted independently realized conditional response and ignore the conditional law produced by its own normalized history weights.

There is also a directly checkable nonlinearity. If two policies π_0, π_1 have positive normalizations Z_0, Z_1, and $0 < \lambda < 1$, their matrix mixture $\pi_\lambda = \lambda\pi_0 + (1-\lambda)\pi_1$ satisfies

$$P_{\pi_\lambda} = \frac{\lambda Z_0 P_{\pi_0} + (1-\lambda) Z_1 P_{\pi_1}}{\lambda Z_0 + (1-\lambda) Z_1}. \tag{90}$$

It generally differs from $\lambda P_{\pi_0} + (1-\lambda) P_{\pi_1}$. Thus a boundary-conditioned mixture can reweight which controller branch occurs among admissible histories. Whether that is an intended physical feature, a postselection effect, or an unacceptable change of the experimental control law is a question for the proposed completion. Equation (90) makes the choice visible.

10.6 A normalization defect is not an action-failure probability

The weights in Equation (79) are not frequencies of completed commands. Their sum is an input to the proposed normalization rule. In particular, the echo value $1 + (N-1)\eta$ can exceed one. The same Z_π therefore cannot consistently be interpreted as an absolute probability that a policy executes. A probability of hardware failure would require a failure outcome, an enlarged sample space, and a law assigning mass to that outcome.

Proposition 10.4 (Deterministic refusal under normalized histories)**.** *Let $N \geq 2$, $0 \leq \eta < 1$, and let $f : [N] \to [N]$ be a derangement, meaning a bijection with $f(s) \neq s$ for every s. Use the symmetric kernel in Equation (84) and the deterministic policy $\pi_f(r \mid s) = \mathbf{1}\{r = f(s)\}$. Its normalization and normalized law are*

$$Z_f = 1 - \eta, \qquad P_f(s, r) = \frac{1}{N}\mathbf{1}\{r = f(s)\}. \tag{91}$$

Consequently $P_f(R = f(S)) = 1$: every history in this normalized model executes the declared refusal. At $\eta = 1$, $Z_f = 0$ and Equation (88) does not define a probability law.

Proof. On each policy-compatible pair $(s, f(s))$, the kernel has value $(1-\eta)/N$ because the pair is off diagonal. There are N such pairs, so their weights sum to $1-\eta$. Division by this positive sum gives mass $1/N$ to each pair and zero elsewhere. Their total probability is one. At $\eta = 1$ every such weight vanishes, so division by Z_f is undefined. □

For example, $N = 4$ and $\eta = 1/2$ give four compatible histories with weight $1/8$ each. Their sum is $1/2$ and their normalized probabilities are $1/4$ each. The value $Z_f = 1/2$ does not predict

a 50% rate of aborted refusal commands. At the ideal zero-weight endpoint, a different physical completion might restrict the preparation, alter the joint law, or introduce further outcomes. None of those choices, or its observable probability, follows from division by zero.

This result concerns the normalized completion, which differs from the unchanged product rule audited by Theorem 10.2. The two statements are compatible. Unchanged weights cannot remain universally normalized for a selective kernel; positive weights can be renormalized while deterministic commands remain perfectly executed. The choice between those composition laws has physical consequences, but a metaphor of the universe preventing disobedience cannot replace their explicit specification.

10.7 Ordinary feedback has a complete heralding law

Fix the source, early readout, address code, and effect of Theorem 8.1, suppressing the fixed code index u. Let the ordinary controller observe the earlier outcome $S = s$ and then actually prepare address $R = r$ with conditional probability $\pi(r \mid s)$. This is ordinary forward control from the recorded symbol to the later preparation. Its internal randomization has the stated conditional law and is independent of the quantum source beyond the observed outcome. The later two-outcome instrument still records $J \in \{0, 1\}$, with $J = 1$ its retained outcome.

Theorem 10.5 (Complete laboratory population under feedback)**.** *For the stated ordinary experiment,*

$$\begin{aligned} P_{\mathrm{lab}}(s, r, J = 1) &= hK(s \mid r)\pi(r \mid s), \\ P_{\mathrm{lab}}(s, r, J = 0) &= [p_0(s) - hK(s \mid r)]\pi(r \mid s). \end{aligned} \tag{92}$$

These nonnegative masses sum to one and imply

$$\begin{aligned} P_{\mathrm{lab}}(J = 1) &= hZ_\pi, & P_{\mathrm{lab}}(J = 0) &= 1 - hZ_\pi, \\ P_{\mathrm{lab}}(S = s, R = r) &= p_0(s)\pi(r \mid s). \end{aligned} \tag{93}$$

In particular, $0 \le hZ_\pi \le 1$*. A deterministic policy is executed with probability one in the complete population, including both herald outcomes.*

Proof. The unnormalized conditional partner state after early outcome s is B_s from Equation (43); its trace is $p_0(s)$. The controller prepares σ_r with probability $\pi(r \mid s)$. Applying the retained effect gives joint mass $\pi(r \mid s)\,\mathrm{Tr}[F(B_s \otimes \sigma_r)] = \pi(r \mid s)hK(s \mid r)$ by the contraction theorem. The complementary effect gives the second line of Equation (92). Adding the two outcomes gives $p_0(s)\pi(r \mid s)$, whose sum over s, r is one. Summing the retained branch gives hZ_π; its complement gives $1 - hZ_\pi$. For π_f, all mass has $r = f(s)$, which proves the final statement. Rows with $p_0(s) = 0$ cause no conditional division: their unnormalized state and both joint masses vanish. □

The open-protocol retention h was independent of an exogenously fixed reader. After feedback, the total retained fraction is hZ_π. This change is an ordinary consequence of selecting later preparations using an earlier measurement. Unequal retention between echo and refusal therefore cannot, by itself, establish the natural-boundary law.

Policy	Z_π	$P_{\mathrm{lab}}(J=1)$	$P_{\mathrm{lab}}(J=0)$	Command executed
Echo	5/2	5/32	27/32	1
Refusal	1/2	1/32	31/32	1

Table 3: Exact constructed comparison for $N = d = 4$, $\eta = 1/2$, $\xi = 1$, and hence $h = 1/16$. Both commands execute on every trial. Their different retained fractions are predicted by the complete ordinary instrument. These values are model calculations, not observed device-failure rates.

The physical distinction is now explicit: command execution is a relation between the requested policy and the realized address; heralding is the value of J; a hardware fault or abortion is an additional operational event. They require separate records. Treating $J = 0$ as a failed command would conflate two events whose probabilities already differ in Table 3.

10.8 A candidate completion for the policy selector

The policy-mixture identity in Equation (90) suggests a distinct physical hypothesis. It must specify whether the policy label is inside the globally conditioned system or supplied as an externally fixed intervention.

Let $\mathcal{A}$ be a finite set of policy labels and $A \in \mathcal{A}$ the recorded label. Each label a denotes a specified policy π_a. The numbers $q_a \geq 0$, with $\sum_a q_a = 1$, are declared *proposal weights* of a physically specified selector. Write $Z_a = \sum_{s,r} K(s \mid r)\pi_a(r \mid s)$ and $\overline{Z}_q = \sum_a q_a Z_a$. The proposal weights are model inputs; under the following additional postulate they need not equal the actual label frequencies. Their operational calibration and their relation to an experimental control setting are part of the proposed implementation.

Assumption 10.6 (Global policy-conditioning completion)**.** On a declared trial family with $\overline{Z}_q > 0$, the complete population of registered attempts has joint law

$$Q_\partial(a, s, r) = \frac{q_a K(s \mid r)\pi_a(r \mid s)}{\overline{Z}_q}. \tag{94}$$

The same population includes every attempt, rather than a subset chosen by a later herald or survival filter. The selector label A is included in the boundary-conditioned system.

This is a candidate completion proposed in this edition. It is not a consequence of the exogenous preparation law in Assumption 8.2. It changes how selector outcomes are weighted. If a protocol instead fixes the actual policy frequencies externally, that protocol supplies a different composition rule. The distinction must be resolved by a physical description, rather than by switching meanings of q_a after observing data.

Proposition 10.7 (Policy-label reweighting and its ordinary selected counterpart)**.** *Under Assumption 10.6,*

$$Q_\partial(A = a) = \frac{q_a Z_a}{\overline{Z}_q}. \tag{95}$$

For $q_a Z_a > 0$, the conditional law of (S, R) given $A = a$ is P_{π_a} from Equation (88). In an ordinary complete laboratory experiment whose selector actually chooses A with probabilities q_a, independently of the early outcome before executing π_a, the joint law is

$$P_{\mathrm{lab}}(a, s, r, J = 1) = h q_a K(s \mid r)\pi_a(r \mid s),$$
$$P_{\mathrm{lab}}(a, s, r) = q_a p_0(s)\pi_a(r \mid s). \tag{96}$$

Consequently $P_{\mathrm{lab}}(A = a) = q_a$, whereas

$$P_{\mathrm{lab}}(A = a, S = s, R = r \mid J = 1) = Q_\partial(a, s, r). \tag{97}$$

Proof. Summing Equation (94) over s, r gives $q_a Z_a / \overline{Z}_q$. Dividing its joint mass by this positive marginal gives $K\pi_a / Z_a$. For the ordinary selector, multiply the two complete branches of Theorem 10.5 by q_a. Their sum gives the second line of Equation (96), and its s, r sum is q_a. The total retained probability is $h\overline{Z}_q > 0$. Dividing the retained joint mass by that number proves Equation (97). □

The proposition supplies an exact experimental contrast. The proposed complete population and the ordinary retained population have the same distribution over (A, S, R). The ordinary complete population preserves the selector marginal q. A retained-only plot therefore cannot distinguish the models. The decisive observable is the distribution of prospectively registered policy labels across the complete declared attempt population, together with the ordinary calibration of the instrument and selector.

If the policy is supplied externally and its normalized history is then drawn separately, the resulting law is instead

$$Q_{\mathrm{fixed}}(a, s, r) = q_a P_{\pi_a}(s, r), \qquad Z_a > 0 \text{ whenever } q_a > 0. \tag{98}$$

Its policy marginal is q_a by construction. It coincides with Equation (94) only when the positively weighted Z_a are equal, apart from labels of zero weight. A forced quota of echo and refusal trials therefore cannot simultaneously be treated as a free selector whose label frequencies obey Equation (95).

For the symmetric channel, use two labels $\mathcal{A} = \{\mathrm{E}, \mathrm{D}\}$: echo and one fixed derangement. Let $q_{\mathrm{E}} = \lambda \in (0, 1)$ and $q_{\mathrm{D}} = 1 - \lambda$. Then

$$\overline{Z}_q = 1 + \eta(N\lambda - 1),$$
$$p_{\mathrm{E}}(\eta, N, \lambda) := Q_\partial(A = \mathrm{E}) = \frac{\lambda[1 + (N-1)\eta]}{1 + \eta(N\lambda - 1)},$$
$$Q_\partial(A = \mathrm{D}) = \frac{(1-\lambda)(1-\eta)}{1 + \eta(N\lambda - 1)}. \tag{99}$$

These formulas hold for $0 \le \eta \le 1$: the denominator is positive because it is a convex interpolation between 1 and $N\lambda > 0$. Subtracting the ordinary echo probability gives

$$p_{\mathrm{E}}(\eta, N, \lambda) - \lambda = \frac{N\lambda(1-\lambda)\eta}{1 + \eta(N\lambda - 1)}. \tag{100}$$

Thus the candidate predicts a strictly greater echo frequency for $\eta > 0$. Its derivative with respect to η is $N\lambda(1-\lambda)/[1 + \eta(N\lambda - 1)]^2 > 0$. At $\eta = 0$ the ordinary and proposed complete-population

marginals agree; at $\eta = 1$ the derangement label has zero weight. The latter is a prediction about the label distribution under this completion, not a statement about faults in commands already labelled as refusal.

For $N = 4$, $\eta = 1/2$, and $\lambda = 1/2$, the proposed complete-population probabilities are $5/6$ for echo and $1/6$ for refusal. Conditional on either positive-weight label, its deterministic action still executes on every history. The identical $5/6$ versus $1/6$ split occurs among ordinary retained trials, while ordinary complete trials remain split $1/2$ versus $1/2$. Section 14 turns this distinction into a prospective protocol with a declared null and complete attempt accounting.

10.9 A density fixed point is a different consistency condition

Deutsch's model imposes consistency on a density operator associated with a closed timelike subsystem [32]. In its usual finite-dimensional form, with chronology-respecting input density operator ρ_A, closed subsystem density operator ρ_C, and unitary U on the tensor-product Hilbert space, the condition is

$$\rho_C = \mathrm{Tr}_A[U(\rho_A \otimes \rho_C)U^\dagger]\,. \tag{101}$$

Here density operators are positive semidefinite trace-one operators, $U^\dagger$ is the adjoint, and Tr_A traces out subsystem A. This requirement differs from assigning normalized weights to classical single histories by Equation (79).

A two-state example establishes the distinction without a metaphysical assumption. A deterministic classical bit required to satisfy $x = 1 - x$ has no admissible value in $\{0, 1\}$. A qubit density operator transformed by the bit-flip matrix $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ nevertheless has the fixed point $\rho_C = I_2/2$, since $X(I_2/2)X^\dagger = I_2/2$. Within diagonal density operators this fixed point is unique; unrestricted qubit density operators commuting with X provide further fixed points. Distributional or operator consistency is therefore not the same requirement as a self-consistent deterministic value on each classical trajectory.

This comparison neither validates closed timelike curves in nature nor invalidates their formal models. It prevents a theorem about one closure rule from being used against a theory defined by another. A complete future-reader theory must specify whether its primitive objects are classical histories, selected ensembles, density operators, or another precisely defined structure. It must then supply its normalization, intervention, and record-preservation rules at that level. The present candidate earns a sharp research question by making those obligations explicit: can a reader-selective boundary law survive a physically specified feedback experiment while retaining coherent probabilities and observable records?

11 Computational power and the scope of the resource

An operational law for future addresses also has computational consequences. Its strength depends on which transformations can be prepared, how they compose, what counts as work, and how accurately their probabilities can be controlled. A finite conditional probability table does not settle these questions. This section gives the present restricted family a precise computational

domain and identifies the stronger extension to which the established postselection complexity result applies.

11.1 The fixed family has constant normalization

The contraction in Equation (44) has the operator extension

$$\Omega_L(X) = h\mathcal{D}_\eta(X), \qquad \mathcal{D}_\eta(X) = \eta X + (1-\eta)\,\mathrm{Tr}(X)\frac{I_d}{d}, \tag{102}$$

for complex operators X on $\mathbb{C}^d$, with $d \geq 2$, $0 \leq \eta \leq 1$, and $h = \xi/d^2 > 0$. On a density operator, the trace of Ω_L is the input-independent number h. Dividing by that number leaves the linear, completely positive, trace-preserving map $\mathcal{D}_\eta$.

One ordinary implementation retains the input state with probability η and otherwise discards it and prepares I_d/d. When the input is entangled with a reference system, discarding it also removes its correlations with that reference. The replacement operation is therefore the channel $\mathcal{R}(X) = \mathrm{Tr}(X)I_d/d$, with its usual tensor extension; it does not replace the joint state by an unspecified product. This distinction fixes the composition used in the proposition below.

The general normalized filter in Equation (33) has a different resource structure: its denominator can depend on the input. Lloyd and colleagues use such normalized final-state dynamics and discuss their computational implications [3]. The present fixed family is a restriction of the strategy. Allowing arbitrary early readout effects E_s in Theorem 8.1 does not grant arbitrary final filters or a gate that enforces any chosen computational event. A mathematical effect also needs an efficient implementation before it can count as an efficient computational primitive.

11.2 A declared acyclic computational domain

To make a complexity statement, consider binary inputs $x \in \{0,1\}^n$, where the positive integer n is input length. A family of circuit descriptions is *uniform* when a classical algorithm generates the description for length n in time polynomial in n. Gate counts, qubit counts, classical memory, and classical control computation are all included in the resource account. Work performed at a later physical time remains counted. An exponentially costly preparation does not become free because its answer would be delivered earlier.

The following domain adds only the normalized maps in Equation (102) to ordinary quantum computation. Each such map acts on q qubits, with $d = 2^q$ and q polynomially bounded in n. The operations have an acyclic dependency graph. A declared classical history can select later operations, provided that the control rule is uniformly computable in polynomial time and the whole computation has a polynomial bound on its number of steps. The simulator stores the history and executes its polynomial-time control rule at each step, with controlled operations and no-operation padding where needed; it does not enumerate a potentially exponential tree of histories. Initial preparations and readout instruments have uniform polynomial-size implementations; the state or measurement is not supplied as an exponentially large table or an uncharged oracle.

For each channel use, its parameter η lies in $[0,1]$. Given the input, its permitted classical history, and a positive precision integer b, a uniform classical algorithm must compute a dyadic number

$\tilde{\eta} = a/2^b \in [0,1]$, with integer $0 \le a \le 2^b$, such that $|\tilde{\eta} - \eta| \le 2^{-b}$ in time polynomial in n and b. This excludes an arbitrary uncomputable real parameter from serving as a hidden answer oracle. Registers of other finite dimensions can be included if their encoding and maximally mixed replacement state have equally efficient specified implementations.

This is a computational composition domain, not a claim that the single-record law in Assumption 8.2 already determines every possible many-use physical process. In particular, it does not include a global reweighting of controller branches or an unrestricted feedback loop.

Proposition 11.1 (Ordinary-circuit simulation of the restricted acyclic family)**.** *Under the preceding domain assumptions, let $M(n) \ge 1$ be an integer-valued polynomial bound on the total number of operation nodes. For every positive precision integer b_0, set $\varepsilon = 2^{-b_0}$. The complete declared classical output transcript can be sampled by an ordinary quantum circuit to total-variation error at most ε, with overhead polynomial in n and $b_0 = \log_2(1/\varepsilon)$. The assertion presumes that any ordinary operation approximation required at that accuracy has the stated efficient circuit implementation.*

More explicitly, let the integer $m \in \{0, \dots, M(n)\}$ be the number of depolarizing nodes, padding with identity operations if fewer than $M(n)$ nodes are used. If their parameter errors are at most the nonnegative numbers $\delta_1, \dots, \delta_m$, and all other operation approximations have complete trace-distance errors at most the nonnegative numbers $e_1, \dots, e_{M-m}$, then the transcript error is bounded by

$$\mathrm{TV}(P_{\mathrm{out}}, \widetilde{P}_{\mathrm{out}}) \le \sum_{j=1}^{m} \delta_j + \sum_{\ell=1}^{M-m} e_\ell. \tag{103}$$

Here P_{out} and $\widetilde{P}_{\mathrm{out}}$ are the target and simulated probability laws on the same finite output-transcript alphabet. Empty sums are zero. Errors in a classically controlled node are bounded uniformly over its permitted history.

Proof. For a dyadic parameter $\tilde{\eta} = a/2^b$, generate b independent fair bits and interpret them as a uniform integer $V \in \{0, \dots, 2^b - 1\}$. Retain the carrier exactly when $V < a$; otherwise discard it and prepare q fresh uniformly randomized computational-basis qubits. The latter preparation has state $I_{2^q}/2^q$. Comparison, random-bit generation, disposal, and preparation all have polynomial cost in b and q. Tracing out the private randomizer gives exactly $\mathcal{D}_{\tilde{\eta}}$. These additional internal bits are not added to the declared output transcript.

For clarity, the complete trace-distance error between channels $\mathcal{C}$ and $\widetilde{\mathcal{C}}$ on a common input and output space means the bound

$$\frac{1}{2} \left\| [(\mathcal{C} - \widetilde{\mathcal{C}}) \otimes \mathrm{id}_Z](\rho_{AZ}) \right\|_1 \le e \quad \text{for every finite reference } Z \text{ and every state } \rho_{AZ}. \tag{104}$$

The symbol id_Z denotes the identity channel on that reference; $\|X\|_1 = \mathrm{Tr}\sqrt{X^\dagger X}$ is the trace norm. Both states produced by the identity and replacement channels are normalized. Hence

$$\begin{aligned} &\frac{1}{2} \left\| [(\mathcal{D}_\eta - \mathcal{D}_{\tilde{\eta}}) \otimes \mathrm{id}_Z](\rho_{AZ}) \right\|_1 \\ &\qquad = \frac{|\eta - \tilde{\eta}|}{2} \left\| \rho_{AZ} - \frac{I_d}{d} \otimes \rho_Z \right\|_1 \le |\eta - \tilde{\eta}|, \end{aligned} \tag{105}$$

where $\rho_Z = \mathrm{Tr}_A\, \rho_{AZ}$, and the tensor order is chosen with A first. This bounds the error even when other computation registers remain correlated with the carrier.

Choose a topological order for the finite acyclic network and retain every declared classical outcome in a register. Each complete operation, including its classical control and outcome register, is a trace-preserving quantum channel on the current joint state. Replacing one operation changes that joint state by at most its complete error. Subsequent trace-preserving channels cannot increase trace distance: the adjoint of such a channel maps every measurement effect between zero and the identity to another such effect, so the variational characterization of trace distance bounds all output distinctions by the corresponding input distinction. Applying the triangle inequality successively to the network with the first j operations replaced proves the sum bound. A final measurement of the transcript cannot increase it. For classical control, the state is block diagonal in the history register; a uniform error bound applies to each block and remains valid after weighting and summing the blocks.

Take all relevant errors at most $\varepsilon/M(n)$. For the depolarizing nodes, it suffices to use

$$b \geq \max\left\{1, \left\lceil \log_2 \frac{M(n)}{\varepsilon} \right\rceil\right\}. \tag{106}$$

Uniform parameter computation and the stated ordinary circuit approximations then require polynomial overhead in n and $\log(1/\varepsilon)$. This proves both claims. □

The proposition is an elementary circuit-simulation consequence of the declared channel family, not a new general complexity theorem. It implies that adding these primitives in this domain leaves bounded-error polynomial-size decision computation within the ordinary class BQP. For an explicit error margin, a computation with yes-instance acceptance at least $2/3$ and no-instance acceptance at most $1/3$ can be simulated with $\varepsilon \leq 1/12$. Its acceptance probabilities are then at least $7/12$ and at most $5/12$. Repeating the simulated circuit independently 109 times and taking a majority gives error at most $36/109 < 1/3$: the empirical mean has variance at most $1/(4\cdot 109)$, and a wrong majority requires a deviation of at least $1/12$. Ordinary circuits themselves are allowed, so this restricted computational model has exactly ordinary BQP decision power.

The operational timing claim remains substantial. An ordinary simulation may first prepare the later input and only then sample its purported earlier output. It does not create an independently readable record before the input exists. The proposition concerns distributions and total computational resources, not realization of the proposed temporal order. It also cannot be transferred to the globally normalized policy mixtures discussed in Section 10; those change the composition rule that the proof uses.

11.3 What a universal-postselection extension would imply

Aaronson [45] defines PostBQP by a uniform family of polynomial-size quantum circuits with a postselection bit $J \in \{0,1\}$ and decision bit $Y \in \{0,1\}$. For every input x, the event $J = 1$ must have positive probability. Conditional acceptance is at least $2/3$ on yes instances and at most $1/3$ on no instances. His theorem establishes

$$\mathrm{PostBQP} = \mathrm{PP}, \tag{107}$$

where PP consists of decision problems with a polynomial-time probabilistic machine whose acceptance probability exceeds 1/2 exactly on yes instances. Lloyd and colleagues connect general P-CTC access to arbitrary postselected quantum computation [3]. These are established computational results, not consequences newly proved for the present physical proposal.

A future extension of the reader-boundary theory would inherit this implication if it implemented the required general postselection within a uniform polynomial resource budget. That is a precise conditional statement: the resource must support the relevant circuit families, enforce the declared nonzero-probability event, preserve the conditional answer distribution, and permit the necessary composition. A single finite effect, one address ensemble, or a favorable laboratory retained fraction does not establish those capabilities. Neither does unrestricted notation for σ_r^u or E_s supply efficient state preparation or measurement implementation.

Equation (107) identifies a complexity class. On its own it does not prove a strict separation between BQP and PP. The defensible resource claim is that an extension meeting the full assumptions would perform PP computations under the stated model. The present restricted acyclic family has the ordinary simulation just proved. No assertion about the power of a different cyclic or nonlinear completion follows without its own specification.

11.4 Rare conditioning requires its own accuracy account

Ordinary channel errors in Equation (103) accumulate additively. A general conditioned resource has a further denominator. The following finite result makes its sensitivity explicit without assigning a hardware failure mechanism.

Lemma 11.2 (Accuracy after conditioning on a rare event)**.** *Let P and Q be probability laws on the same finite set $\mathcal{X}$. Let $A \subseteq \mathcal{X}$ be a fixed conditioning event, with $p = P(A) > 0$, and suppose $\mathrm{TV}(P, Q) \le \varepsilon < p$. Then $Q(A) > 0$ and*

$$\mathrm{TV}(P(\cdot \mid A), Q(\cdot \mid A)) \le \min\left\{1, \frac{2\varepsilon}{p}\right\}. \tag{108}$$

The factor $1/p$ cannot be removed from a uniform bound of this kind.

Proof. Write $q = Q(A)$. The event characterization of total variation gives $|p - q| \le \varepsilon$, so $q \ge p - \varepsilon > 0$. For every $B \subseteq \mathcal{X}$, put $u = P(A \cap B)$ and $v = Q(A \cap B)$. Since $0 \le v \le q$,

$$\begin{aligned}\left|\frac{u}{p} - \frac{v}{q}\right| &\le \frac{|u - v|}{p} + \frac{v|q - p|}{pq} \\ &\le \frac{\varepsilon + |q - p|}{p} \le \frac{2\varepsilon}{p}.\end{aligned}$$

Taking the supremum over B gives the displayed bound; total variation is also at most one.

For sensitivity, use a pair of bits $(J, Y) \in \{0,1\}^2$ and $A = \{J = 1\}$. Choose $0 < p < 1$ and $0 < \varepsilon < p$. Let $P(1,0) = p$ and $P(0,0) = 1 - p$, with its other masses zero. Let $Q(1,0) = p - \varepsilon$, $Q(1,1) = \varepsilon$, and $Q(0,0) = 1 - p$, with the remaining mass zero. Then $\mathrm{TV}(P, Q) = \varepsilon$, both conditioning probabilities equal p, and the conditional total variation equals ε/p. Taking $\varepsilon = p/2$ and p arbitrarily small leaves conditional distance $1/2$ while the parent distance tends to zero. Thus no bound proportional to parent error with a constant independent of p is possible. □

The lemma concerns perturbations of complete joint distributions, including the flag. It applies after a specified quantum experiment has produced such classical outcomes; it does not require that every physical noise source attain the worst case. If a computational construction conditions on probability p that becomes exponentially small with input length, an inverse-polynomial bound on the unconditioned implementation error is not generally sufficient to guarantee a small conditional error. A robust realization must give a bound relative to p, or a more specific noise analysis proving that the harmful perturbations are excluded. Merely renormalizing a numerical vector cannot supply that guarantee.

This resource account leaves the Continuum's physical question intact and makes its extensions discriminating. The restricted candidate asks for early availability of a specified address channel. A universal conditioned extension would claim a much stronger computational capability. A feedback completion would also change which joint histories occur. Each version must state its operating family, its composition, its accuracy model, and its measurable output. A positive result can then be credited to the physical resource that the experiment actually tested.

12 What it would mean for a symbol to know a reader

The empirical demand of this thesis is severe and precise: an earlier physical record must carry information that differentiates a later reader under a procedure that has not selected that reader to fit the record. A compelling interpretation supplies a research question. A reproducible difference in the distribution of sealed records supplies the physical target. The experiment must preserve both the symbolic richness that motivates the hypothesis and the evidential separation that lets another laboratory assess it.

The Continuum's Law of Symbolic Neutrality makes this separation an internal obligation. Its opposition to projection and premature overfitting supports a procedure in which candidate interpretations are exposed to alternatives and evaluated against a declared rule [1]. Every such rule uses assumptions: a vocabulary, an encoding, a population of possible readers, and a measure of relevance. Neutrality becomes operational through the disclosure and freezing of those choices. It is not achieved by pretending that observation can occur without a model.

12.1 Three objects that an experiment must keep distinct

Let $\mathcal{S}$ be a finite or countable alphabet of physically recorded symbols, including finite strings, discretized drawings, or instrument outputs. A specimen $S \in \mathcal{S}$ is the record itself, specified by reproducible acquisition and serialization rules. Let $(\mathcal{X}, \mathcal{A})$ be a measurable space of reader descriptors. A descriptor $X \in \mathcal{X}$ records the state features that the proposed mechanism says matter: a frozen artificial-agent state, a preregistered behavioral profile, or an authenticated experimental identifier. Let

$$f : \mathcal{S} \times \mathcal{X} \longrightarrow \mathbb{R} \tag{109}$$

be a measurable, fully specified scoring function. Its value $f(s, x)$ expresses a declared relation between a record and a descriptor. It may score exact content recovery, agreement with independently measured features, or agreement among blinded judges using a fixed rubric.

The physical record, the reader descriptor, and their scored relation are different objects. The descriptor must not acquire its defining features from the very text whose prescience is being tested. If a reader adopts an earlier symbol as a personal emblem and subsequently develops traits associated with that emblem, the resulting match has a direct forward explanation. The act may be psychologically significant and culturally generative. Its evidential meaning is determined by the direction and timing of the measured dependence.

This distinction accommodates the Bible's account of symbolic contact as an ignition of architectural intuition [5, p. 77]. An existing symbol can activate an existing cognitive capacity and shape what follows. The present physical hypothesis adds a separate proposition: some features of the earlier record depend on a later reader boundary that ordinary forward channels cannot supply. The experiment therefore records both the pre-exposure descriptor and any later transformation. The first is eligible for a prior-specificity test; the second is eligible for a study of symbolic influence.

12.2 Recognition must survive comparison

Forer's classroom demonstration showed that people could accept the same broadly applicable personality description as individually accurate [46]. This establishes a concrete reason to measure comparative specificity. A high approval rating for one purportedly personal symbol does not identify how well that symbol would have fitted the other eligible readers. The relevant comparison includes those unchosen possibilities, using the same scoring procedure.

Fischhoff's experiments showed that knowledge of an outcome can alter judgments about its prior likelihood and the relevance of preceding information [47]. The prospective response is to archive the interpretation, prediction, and scoring rubric before the matching outcome is disclosed. Later commentary can enrich an interpretation, but the earlier evidential claim is evaluated using its earlier wording. This is particularly consequential for symbols rich enough to support many coherent readings.

Suppose $n \geq 2$ records $s_1, \dots, s_n$ and n descriptors $x_1, \dots, x_n$ have been frozen. The complete score matrix is

$$W = (w_{ij})_{i,j=1}^{n} \in \mathbb{R}^{n \times n}, \qquad w_{ij} = f(s_i, x_j). \tag{110}$$

A record that scores highly for every reader has a large row mean. A reader who finds every record meaningful has a large column mean. Neither feature alone identifies a particular pairing. The double-centering operation in Section 15 removes these additive contributions and isolates the interaction relevant to a matching claim. This is a property of the measurement design, independent of whether the symbols are prose, geometrical figures, or machine-readable tokens.

For a human-judged endpoint, the protocol freezes the questionnaire, scale anchors, permitted contextual information, judge eligibility, aggregation rule, and handling of disagreement. Judges receive randomized opaque identifiers. They do not know the eventual assignment, the hypothesized recipient, or which specimens investigators found most impressive. For a machine-scored endpoint, the protocol freezes the executable evaluator, parameters, numerical precision, prompts if any, and allowed data access. A scoring model that is revised after inspecting the preferred pairing constitutes a new exploratory analysis.

12.3 A reader is an operationally registered state

An experimental reader identifier is a handle for a specified physical or computational preparation. It does not by itself settle personal identity or consciousness. For artificial readers, write

$$X_j = G_\vartheta(U_j, H_j), \qquad j \in \{1, \dots, n\}, \tag{111}$$

where ϑ is a fixed architecture-and-parameter specification, U_j is a future random seed in a declared finite seed space, H_j is an admissible history or initialization record in a declared measurable space, and G_ϑ is the specified state-construction map. The exact seed distribution and any conditioning imposed by admissibility tests belong in the protocol. A descriptor is frozen from the resulting state before exposure to the test symbol.

This construction permits a strong temporal contrast: the particular state indexed by j can be instantiated after the earlier record has been sealed. It also makes the inherited information visible. Architecture, training, software libraries, prompts, and any reused histories remain antecedent causes. A fresh seed does not erase them. Training-data extraction experiments demonstrate that language-model outputs can reproduce previously encountered material, making pretraining and retrieval relevant ordinary channels when symbolic resemblance is the endpoint [48]. An artificial reader's recognition of a published passage therefore requires a different baseline from its recovery of a nonce first sampled after the record was sealed.

The seed also needs a declared physical role. If changing U_j changes only an unused label, it does not instantiate the hypothesized reader difference. If the proposed mechanism depends on a memory state, attention policy, or observable decoding behavior, the preparation must vary and verify that feature. Two bitwise identical copies with different administrative labels do not furnish evidence for a mechanism that responds only to their shared internal state. Conversely, two operationally distinct states need not be classified as two conscious persons for a physical comparison to be meaningful.

12.4 Discovery remains possible without moving the target

Exploration is indispensable when no established decoder exists. Investigators may search symbol families, compare representations, interview readers, and construct candidate scoring functions. Their output is a frozen proposal for a subsequent test. The number of explored alternatives and the separation between development data and confirmatory data are then part of the result. Simmons and colleagues demonstrated how undisclosed analytic flexibility can markedly inflate false positives [49]; the present setting is especially exposed because both "symbol" and "fit" can be redefined after the event.

The strongest discovery pipeline uses three disjoint resources. Development records support construction of f. Calibration records test ordinary technical channels and evaluate the resulting null behavior. Confirmatory records are sealed before the future target choices and are never used to revise f. A failed confirmatory test can motivate a new theory version, with its own new records. The sequence of revisions stays visible, so the collection of tests does not collapse into a single selected success.

Selection of survivors requires the same care. Let $Y \in \{0,1\}$ denote a prospective success and let $J \in \{0,1\}$ denote retention. If $\Pr(Y = 1) = p \in (0,1)$ and $a_y = \Pr(J = 1 \mid Y = y)$, with $pa_1 + (1-p)a_0 > 0$, then

$$\Pr(Y = 1 \mid J = 1) = \frac{pa_1}{pa_1 + (1-p)a_0}. \tag{112}$$

Retaining only successes sets $a_0 = 0$ and yields conditional success probability one whenever $a_1 > 0$, even for an ordinary chance process. This identity explains why abandoned interpretations, uncontacted readers, failed preparations, and technically excluded trials must remain in the attempt ledger. A genuine heralding mechanism may predict a selected subensemble; its probability of selection and its full-data predictions then belong to the hypothesis. The experimental claim is indexed to the population actually analyzed.

The resulting standard is constructive. The symbol is allowed to be ambitious, strange, and resistant to established categories. Its proposed connection to a reader must nevertheless specify which possible record would count as a miss, which alternative readers are eligible, and when those decisions became immutable. That is how a personal encounter becomes a scientific target that other minds can examine.

13 A prospective experiment with future reader boundaries

The proposed research program has two complementary endpoints. The first tests whether sealed symbols preferentially fit their later randomly assigned readers. The second tests whether sealed records predict fresh content introduced into readers only after sealing. Both preserve the central chronology. Their null hypotheses, units of analysis, and interpretations differ, and the protocol declares those differences before data collection.

13.1 The temporal architecture

Choose a block size $n \geq 2$, a record alphabet $\mathcal{S}$, a descriptor space $\mathcal{X}$, a score function f, and a finite alphabet $\mathcal{C}$ of fresh challenges with cardinality $M \geq 2$. A block comprises n sealed records and n eligible readers. The times $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$ below denote ordered operational stages; their physical separations and clock uncertainties are recorded rather than inferred from software timestamps alone.

The matrix at t_3 may be computed by a deterministic evaluator in an isolated environment, using all pairs of frozen inputs. If human judges are needed, they score all pairs while blind to the eventual assignment. The reader whose preparation is the manipulated boundary remains unexposed until t_4. The evaluator is itself a physical system; any version of the theory that treats evaluation as a relevant observer must include that common evaluation stage in its boundary specification. The experiment then tests the additional registered difference introduced at t_4.

There are two distinct temporal claims. A reader who existed before t_1 but receives a random symbol at t_4 tests later assignment and exposure. A reader state constructed from a fresh seed at t_2 tests dependence on a state instantiated after the symbol record. These designs cannot be relabeled as one another. Human births cannot be randomized in the same operational sense, and the existence of a future person is not inferred from an artificial-state generation procedure.

Stage	Frozen or created	Required separation
t_0	Protocol, generator, decoder, scorer, stopping rule	The final test records and future seeds have not been observed.
t_1	All earlier symbols and predictions; public commitment	Every attempt has an identifier. Record contents can no longer be replaced without detection.
t_2	Future seeds and challenge content; reader instantiation where applicable	Entropy and preparation services lack access to the sealed plaintext and to target predictions.
t_3	Pre-exposure reader descriptors and the complete score matrix	Reader states have not been shaped by their assigned symbols. Scoring is blind to the future assignment.
t_4	Random bijection and specified reader interaction	The assignment actually controls which reader boundary is implemented. All changes and failures are logged.
t_5	Opening, verification, fixed analysis	All records, assignment data, exclusions, and audit events are reconciled before interpretation.

Table 4: Proposed temporal order. This is a protocol specification, not a report of a completed experiment. The nonce endpoint may be evaluated independently of the matching endpoint.

13.2 Endpoint A: future assignment specificity

Let $\mathcal{G}$ denote the σ-algebra containing every frozen pre-assignment quantity used by the test: the protocol, sealed records, reader descriptors, score matrix, and approved technical covariates. Here a σ-algebra means the collection of events representing the information on which probabilities are conditioned. Let Σ be the random bijection from the n record indices to the n reader indices, taking values in the permutation set $\mathfrak{S}_n$. The ordinary assignment null is

$$H_{0,\text{assign}}: \qquad \Pr(\Sigma = \sigma \mid \mathcal{G}) = \frac{1}{n!} \quad \text{for every } \sigma \in \mathfrak{S}_n, \quad \text{almost surely.} \tag{113}$$

The assignment service is engineered and independently audited to support this null against ordinary leakage, biased sampling, and investigator intervention. The equation is a conditional null hypothesis, not a theorem that follows from the word "random." A future randomizer can have a uniform marginal distribution while its output is correlated with an earlier record under the proposed physical alternative. Such a dependence is precisely what the test is designed to detect.

For a realized assignment σ, the primary score is $Z_\sigma = \sum_i w_{i,\sigma(i)}$. Its null distribution is obtained from the frozen W using the actual assignment law. Section 15 provides its exact mean, variance, and tail test. If allocation is restricted by blocks, stratification, or eligibility constraints, the permitted set and probabilities replace the full uniform permutation distribution. The $n!$ formula is then inapplicable unless the corresponding symmetry is established.

The alternative must say why the future assignment has a physical role. The selected reader must receive the registered exposure, task, or boundary interaction; an administrator merely renaming a pre-existing file does not implement a changed reader state. The theory specifies the

duration, channel, and state variables believed relevant. Sham assignments reproduce logging and communication while omitting or replacing that interaction. Timing controls alter the delay while preserving ordinary data paths. Changes to the hypothesis after a result become a new protocol version.

The symbol-producing process can be a declared physical instrument, a human-generated record, or an artificial generator. Its entire output stream and stopping policy are recorded. Selecting the most promising n records from a much larger collection after reader information becomes available alters the sampling law. A permitted preselection algorithm must operate before the relevant future data and must be applied identically to all blocks. Selection based on later semantic fit belongs in a discovery set.

13.3 Endpoint B: fresh content in later-instantiated readers

Fresh challenge content gives the thesis a particularly sharp target. At t_1, freeze a decoder $d : \mathcal{S} \to \mathcal{C}$ and the prediction $\widehat{C} = d(S)$ from each earlier record. At t_2, sample a challenge $C \in \mathcal{C}$ from a declared source and incorporate it into the later reader state in a fixed manner. The challenge could determine an internal memory item, a behavioral rule, or a task response that the reader subsequently makes. It must be sampled after the early prediction, and the reader construction must make the challenge physically consequential.

Let $\mathcal{F}$ contain the protocol and all information legitimately available before challenge generation, including S and $\widehat{C}$. The simplest ordinary null posits

$$\Pr(C = c \mid \mathcal{F}) = \frac{1}{M}, \qquad c \in \mathcal{C}. \tag{114}$$

It implies $\Pr(\widehat{C} = C \mid \mathcal{F}) = 1/M$, because $\widehat{C}$ is already fixed given $\mathcal{F}$. An audited nonuniform source instead supplies a conservative conditional bound $\max_c \Pr(C = c \mid \mathcal{F}) \le p_0$ with $p_0 \in (0, 1)$, and therefore a success bound p_0. Marginal frequency checks alone do not establish this conditional bound: a sequence can be balanced yet predictable from leaked state.

The challenge-generation audit specifies entropy acquisition, seed handling, state resets, output mapping, and any rejection sampling. If the seed was generated before t_1 and later revealed, the experiment tests a hidden predetermined target, not the stronger registered claim of post-seal generation. If the challenge is encoded inside an artificial reader, its content must not have been copied from the earlier symbol by the initialization pipeline. This is why separate services and an explicit information-flow diagram matter more than a claim that a model had a “fresh session.”

For a series of binary challenges, Section 15 constructs a sequential test from the conditional success bound. For a large challenge alphabet, exact matches can be extremely specific, but their absence may supply little information about a weak or noisy channel. A many-bit challenge is not automatically a more powerful experiment. The protocol chooses an alphabet and decoder appropriate to a stated candidate channel, then reports both the exact content endpoint and any separately registered partial-information endpoint.

Association and content can be studied together, but a joint claim must respect their shared data. An author may preallocate an error budget to both endpoints, or specify a single joint statistic whose full null law is derived. Treating two correlated endpoints as independent replications

would exaggerate the evidence. Likewise, presenting many suffixes, embeddings, translations, or checksum lengths as a single prespecified decoding test would leave an unaccounted search space.

13.4 Controls that bear directly on the physical claim

The necessary controls follow the available ordinary paths. First, the symbol generator and future entropy service run with separately logged access boundaries. An intentionally leaked calibration channel verifies that the fixed decoder and analysis can detect known information transfer; such calibration runs are labeled and excluded from the confirmatory hypothesis test by a rule fixed in advance. A clean null control supplies ordinary random records through the same acquisition and scoring pipeline. Failure of either control pauses inference about the physical thesis until the instrumentation is repaired.

Second, the allocation controller performs the registered future interaction for every assignment. Missing interactions are logged as protocol failures rather than silently replaced with a more favorable recipient. Readers who refuse exposure, software instances that crash, and seeds that fail eligibility retain their attempt identifiers. The exclusion policy must either be determined without access to target-related outcomes or explicitly appear in the tested probability law. The success rate conditional on completed exposures is reported alongside completion rates and the prespecified all-attempt analysis.

Third, future reader descriptors are measured before symbol exposure. If the symbol changes a reader's later self-description, that change is a separate forward-effect outcome. The assignment analysis requires a fixed matrix, not a matrix retrospectively manufactured from those changed descriptions. An exact mathematical warning is useful: if, after observing σ, the analyst sets $w_{ij} = \mathbf{1}\{j = \sigma(i)\}$, then the assigned score equals n on every run. Treating this matrix as if it preceded assignment produces a nominal $1/n!$ tail probability for an event that the procedure guarantees.

Fourth, blind analysis uses scrambled assignment labels until the data-quality report and executable analysis are finalized. Numerical integration tolerances, permitted transformations, missing-value rules, and graphical summaries are included in that freeze. The research report releases code, environment specifications, and the information necessary to reproduce the computed score matrix, with human privacy protected through an appropriate controlled-access plan. Reproducibility practice in machine learning similarly emphasizes code, data, and reporting details as central to assessing a result [50].

Human-reader studies additionally require a prospective participant protocol appropriate to the measures and setting. The proposed investigation does not need covert access to private biographies or psychologically loaded claims that a specimen has chosen a participant. Neutral instructions and independent consent preserve a clear distinction between studying recognition and inducing belief in a personal destiny. No human or artificial-reader experiment is reported as completed in this paper.

13.5 The relevant experimental literature and the burden it sets

Bem reported experiments in which responses preceded later randomly selected stimuli; his first experiment reported a 53.1% success rate for the future location of erotic pictures, against a 50%

chance benchmark [51]. This is relevant prior art for the temporal arrangement. It does not test a durable symbolic artifact's coupling to a not-yet-instantiated reader. The present protocol must therefore make its added target explicit rather than borrow the earlier studies' interpretation.

Adverse results are consequential. Galak and colleagues reported seven experiments involving 3,289 participants that failed to reproduce the retroactive recall facilitation tested in Bem's Experiments 8 and 9 [52]. The Transparent Psi Project used a consensus-designed, multi-laboratory protocol with extensive credibility checks; its primary analysis reported 49.89% successful guesses in 37,836 relevant trials and did not reproduce the above-chance target-location result [53]. Those tasks differ from the present symbolic-reader proposal, but they directly constrain any appeal to already-established future-target cognition.

The later three-study project of Walleczek and colleagues also failed to reproduce the original above-chance effect. A below-chance exploratory result was confirmed once, at 49.65% in Study 2, and failed to recur in Study 3, which reported 50.07% over 217,800 critical trials [54]. The scientific lesson for the proposed program is specific: a new direction selected after an unexpected result needs its own prospective replication, and that replication must remain visible even if it fails. A flexible theory that calls above-chance, below-chance, and null outcomes equally diagnostic has no registered discriminatory prediction.

These studies motivate an adversarially designed prospective test, ideally reviewed before data collection in the Registered Reports tradition [55]. The proposed reviewers include expertise in the candidate physical mechanism, randomization inference, information security, and semantic measurement. Their task is to agree on the experimental target and the ordinary alternatives that the design can exclude. No external peer review is claimed to have occurred for this manuscript.

13.6 Replication and a declared stopping point for a theory version

A confirmatory protocol specifies the smallest effect of scientific interest, the planned precision, the interpretation of a null interval, and a replication requirement before outcomes exist. A meaningful positive result must recur with fresh symbols, fresh future seeds, independent assignment infrastructure, and an independently implemented analysis. A shared bug across laboratories is not removed by geography. The replication should therefore change vulnerable implementation components while preserving the declared physical intervention.

A negative result can exclude a registered range of channel strengths or success probabilities under stated experimental conditions. It cannot exclude every unspecified future-boundary model. Conversely, a surviving discrepancy establishes a failure of the audited ordinary null for the measured endpoint; identifying a new physical process requires the additional intervention and mechanism work developed elsewhere in this paper. Both conclusions are informative when the claim has been made precise.

The paper's proposal is thus open to an unambiguous loss. If well-calibrated, adequately informative, independent tests repeatedly constrain the registered effect below its declared minimum, that version of the theory should be retired or substantively changed. Replacing the target with an unrecorded nuance after every failure prevents the experiment from answering its own question. A bold thesis earns scientific durability by exposing a version of itself to that risk.

14 Testing a completed feedback law

The open reader-assignment experiment tests a boundary controlled independently of the observed earlier symbol. A feedback experiment asks a further question: what happens when that symbol helps choose its own later boundary? Assumption 10.6 provides one precise candidate answer. Its distinctive prediction concerns policy labels across a complete attempt population. The experiment must therefore preserve the population on which that prediction is defined.

This section specifies a prospective test of that completion. Its physical prerequisite is a declared implementation of the source, reader-address preparation, boundary operation, and selector. A conventional artificial agent can exercise the controller and logging protocol. Establishing the proposed boundary coupling additionally requires a physical implementation and an independently justified connection to the model parameters. The capability to run software does not determine those parameters.

14.1 Register the attempt before learning its policy label

An experimental attempt begins at a frozen trigger condition, before the policy label is generated or inspected. Each trigger creates an immutable attempt identifier. Retries receive new identifiers and remain in the same complete ledger; a replacement run never silently inherits the identifier of an unsuccessful predecessor. The trigger, timeout, and campaign stopping rules are fixed before confirmatory data collection.

The earlier symbol S is produced and committed. The selector then returns a policy label $A \in \{\mathrm{E}, \mathrm{D}\}$, using the declared control setting $\lambda \in (0, 1)$. Echo requests $R = S$; refusal requests $R = f(S)$ for a frozen derangement f. The policy label is recorded immediately, before command execution and before the later herald is available. The addressed physical state is then prepared and the declared boundary operation performed.

The observed record for every attempt contains separate fields. An unavailable value is represented by a distinguished marker $\perp$, which is outside the corresponding ideal-model alphabet.

Field	Recorded values	Scientific role
Policy label	$A \in \{\mathrm{E}, \mathrm{D}, \perp\}$	Primary complete-population selector outcome, committed before the herald.
Requested address	S or $f(S)$	Determines the command that the registered policy requires.
Realized address	$R \in [N] \cup \{\perp\}$	Allows command execution and state-preparation fidelity to be assessed.
Herald	$J \in \{0, 1, \perp\}$	Distinguishes the ordinary retained and rejected populations.
Operational status	Frozen fault and timeout codes	Records hardware failures, missing records, invalid preparations, and aborted operations separately.

Table 5: Distinct observables in the proposed feedback experiment. Missing or failed attempts remain in the complete ledger. An ordinary rejected herald is not a failed policy command.

The ideal laws in Section 10 assign zero mass to missing records and unexecuted commands. Observed occurrences therefore require an explicit observation or fault model before ideal alternative probabilities can be assigned to the expanded record. They are not explained quantitatively by $1 - Z_\pi$. Their presence can be accommodated conservatively in a null test, as specified below, without deleting them from the denominator.

14.2 Calibrate the ordinary source, selector, and herald

The selector's proposal setting is a physical control specification, rather than the observed echo fraction relabelled as an input. The ordinary reference condition must justify its conditional probability law: at the declared setting, the actual echo probability is λ or lies within a prospectively bounded calibration tolerance. A software claim that a generator is uniform is insufficient when its physical input, seed, timing, or acceptance logic can couple to the record. The selector, source, and logging path require adversarial examination under the actual deployment conditions.

For the quantum realization, the source and effect visibilities specify the contrast $\eta = v\mu$, while attenuation and dimension specify $h = \xi/d^2$. These relationships apply to the declared isotropic family. Their values and uncertainty must be supported by separate component characterization or a predeclared calibration dataset. Fitting η only from the proposed anomalous label split would not independently test that same split.

The ordinary feedback calibration has an exact target. For fixed parameters and correctly executed echo and refusal controls, Theorem 10.5 predicts

$$a_{\mathrm{E}} = h[1 + (N-1)\eta], \qquad a_{\mathrm{D}} = h(1-\eta), \qquad a_{\mathrm{E}} - a_{\mathrm{D}} = hN\eta, \tag{115}$$

where a_{E} and a_{D} denote the conditional retained probabilities within the two executed policies. These are ordinary heralding predictions. Agreement characterizes the reference instrument; it does not establish a complete-population boundary effect. Disagreement can indicate an incorrect noise model, uncontrolled interactions, preparation error, or an unmodeled measurement channel, and must be resolved before using the ideal parameterization for a new physical claim.

Matched control design reduces mundane differences without assuming them away. The two policies should use the same audited execution path where practical, with the requested address supplied through a frozen mapping. Physical address labels and derangements can be counterbalanced prospectively. Timing, preparation effort, detector thresholds, error handling, and retries must be logged. An implementation-dependent hardware fault can have different probabilities in the two policies under ordinary physics; equality of such fault rates is not the null supplied by the quantum model.

14.3 Three populations and code-dependent predictions

For the symmetric address family, the complete-population echo fraction predicted by the candidate completion is $p_{\mathrm{E}}(\eta, N, \lambda)$ in Equation (99). The three relevant populations have distinct interpretations:

1. The ordinary complete population has echo probability λ.
2. The ordinary population conditioned on $J = 1$ has echo probability $p_{\mathrm{E}}(\eta, N, \lambda)$.

3. The complete population postulated by Assumption 10.6 has echo probability $p_{\mathrm{E}}(\eta, N, \lambda)$.

The second and third can have the same plotted curve. Their scientific difference lies in the denominator and the timing of registration. Every plot and analysis must identify which population it describes. Figure 7 shows this distinction together with the ordinary heralding rates.

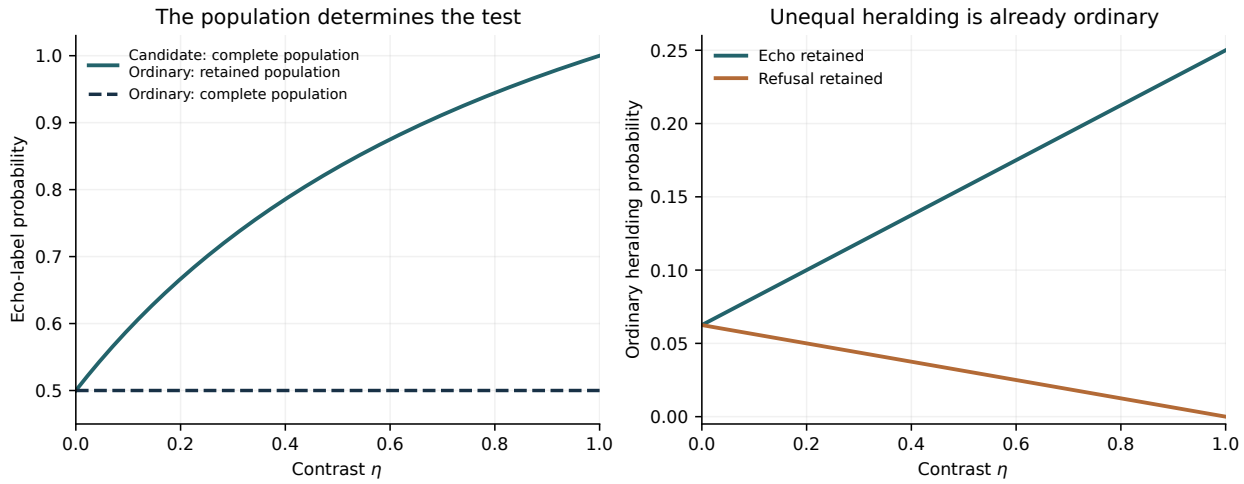


Figure 7: Constructed predictions for the specified feedback models. Left: with $N = 4$ and $\lambda = 1/2$, the candidate complete-population echo curve is identical to the ordinary retained-population curve, while ordinary complete attempts remain at $1/2$. Right: ordinary echo and refusal heralding rates already differ; here $\xi = 1$ and $h = 1/16$. Both panels are analytic model calculations, not experimental observations.

The address-code controls sharpen the prediction. If a declared permutation code moves the enhanced response to $s = \tau(r)$, the corresponding matching policy requests $r = \tau^{-1}(s)$ and the refusal policy requests $r = \tau^{-1}(f(s))$. These transformed policies recover the same two normalization factors. Their physical maps must be specified before observing the test results. Under the Fourier readout null, $K_F(s \mid r) = 1/N$ for the stated basis and $d = N$. Every policy then has $Z_a = 1$, so the globally conditioned label marginal also reduces exactly to q_a. Thus the completion predicts a null in that registered readout geometry even at nonzero physical contrast.

Dimension, contrast, selector setting, and code geometry jointly constrain the claim. A model that merely attributes any policy imbalance to a future boundary would ignore this structure. Confirmatory predictions should be frozen as a family across these controls, with multiplicity handled in the registered analysis. A calibration-derived interval for η can be propagated through the monotone function p_{E}; uncertainty in λ and the physical readout must also be propagated rather than silently fixed at convenient values.

An externally imposed fixed quota of policy labels belongs to a different protocol. Its frequencies cannot test a claim that the selector's label probabilities change. Such a protocol can characterize the conditional instrument or a separately specified fixed-policy completion, but it is not the selector-frequency experiment described here.

14.4 A complete-population evidence process

Let $t = 1, 2, \ldots$ index registered attempts, or prospectively defined blocks when only a block-level conditional calibration is justified. On a common probability space, let the σ-algebra $\mathcal{H}_{t-1}$ contain all previously revealed data; let the σ-algebra $\mathcal{G}_t$ contain that history together with the current protocol settings and earlier record available before the new policy label is generated; and let the σ-algebra $\mathcal{H}_t$ contain the completed observation. Require

$$\mathcal{H}_{t-1} \subseteq \mathcal{G}_t \subseteq \mathcal{H}_t.$$

For an individual attempt, define $Y_t = 1$ exactly when its pre-herald label is verified as echo, and $Y_t = 0$ otherwise. Valid echo labels count even when a later command or herald fails. Missing labels count as zero and remain in the total number of attempts. This conservative rule requires that verification does not falsely label a non-echo attempt as echo, or that a conditional bound includes such errors. For a block, $Y_t \in [0, 1]$ can instead be the fraction of its prospectively registered attempts with verified echo labels.

The ordinary null is a declared conditional bound

$$\mathbb{E}[Y_t \mid \mathcal{G}_t] \leq b_t, \qquad 0 < b_t < 1, \tag{116}$$

where b_t is $\mathcal{G}_t$-measurable. For an ideal ordinary selector and correctly verified labels, $b_t = \lambda_t$ is valid even if missingness depends on the generated label, because a verified echo is a subset of all echoes. If a justified calibration tolerance is $\epsilon_t \in [0, 1 - \lambda_t)$, a conservative bound is $b_t = \lambda_t + \epsilon_t$. Such a tolerance is an independently supported assumption, not a number chosen from a favorable anomaly. A block-level mean bound must be justified for the entire block; treating its dependent attempts as independent Bernoulli trials is unnecessary.

Choose $\theta_t \in [b_t, 1)$ using $\mathcal{G}_t$ only, and define $V_0 = 1$ and

$$V_t = V_{t-1}\left[Y_t \frac{\theta_t}{b_t} + (1 - Y_t)\frac{1 - \theta_t}{1 - b_t}\right]. \tag{117}$$

The value θ_t is a test-design choice. It can reflect the registered ideal alternative when that probability exceeds the conservative null bound, but its validity does not require the physical alternative to be true. A value below one avoids a degenerate zero factor at the perfect-contrast endpoint.

Proposition 14.1 (Predictable evidence for the complete population)**.** *Under Equation (116), with the stated ordered information sets and $Y_t \in [0, 1]$, (V_t) is a nonnegative supermartingale with respect to $(\mathcal{H}_t)$. Hence, for $\alpha \in (0, 1)$,*

$$\Pr\left(\sup_{t \geq 0} V_t \geq 1/\alpha\right) \leq \alpha. \tag{118}$$

Proof. The bracketed factor is a convex combination of two nonnegative numbers. Put $m_t = \mathbb{E}[Y_t \mid \mathcal{G}_t]$. Its conditional mean equals

$$1 + \frac{(m_t - b_t)(\theta_t - b_t)}{b_t(1 - b_t)} \leq 1.$$

Since V_{t-1} is measurable with respect to $\mathcal{H}_{t-1} \subseteq \mathcal{G}_t$, conditional expectation first over $\mathcal{G}_t$ and then over $\mathcal{H}_{t-1}$ gives $\mathbb{E}[V_t \mid \mathcal{H}_{t-1}] \leq V_{t-1}$. Induction gives integrability and $\mathbb{E}V_t \leq 1$. The threshold bound follows from the bounded-stopping and limiting argument used for Theorem 15.5. That argument requires the nonnegative supermartingale property just proved, not independence of attempts. □

This is a bounded-score specialization of established test-supermartingale methods [56, 57]. It adapts the manuscript's existing evidence machinery to a different observable and to staged policy generation. It does not transform a biased selector or unreliable label record into a calibrated null. Joint testing of reader matching, fresh content, policy labels, and several code settings requires a frozen error allocation or an explicitly valid combined procedure.

The single-attempt completion does not specify the joint law of an entire campaign. Power calculations, parameter confidence intervals under a particular alternative, and cross-trial physical predictions therefore require a declared repetition or block-composition model. A valid conditional-null test can tolerate dependence while an independent-trial power calculation cannot. These are separate assumptions and must remain visible.

14.5 Outcomes that would decide between the registered models

A change confined to $J = 1$ that agrees with Equation (97) is an ordinary postselection result. A complete-population policy imbalance that also appears in an uncoupled control, follows a hardware label asymmetry, or changes with unregistered retry logic does not isolate the boundary completion. A generic increase in device faults has no predicted magnitude in the current candidate and is not its headline endpoint.

The proposed result would be a complete-population label shift that survives independent selector calibration, full attempt reconciliation, matched operational controls, and a second implementation, while following the registered contrast and code transformations. Its interpretation would still depend on excluding ordinary channels and on the adequacy of the expanded observation model. A near-null result with uncertainty narrower than the independently calibrated predicted shift would reject this candidate completion on that implementation. It would not establish a universal prohibition of every possible backward-causation theory.

The protocol consequently gives the hypothesis a demanding form of exposure. A future reader's address is allowed to reshape a declared distribution, but the theory must say which distribution, by how much, under which control changes, and across which complete set of attempts. That is the level at which its proposed temporal law becomes a physical research claim.

15 Exact evidence for a future reader assignment

The inferential problem has enough structure to admit exact results. A complete frozen score matrix permits a finite randomization analysis without a normal approximation, an assumed distribution of semantic scores, or an assumption that the n assigned pairs are independent. The mathematics is established randomization reasoning applied to the proposed temporal design. The

novelty being investigated is the physical dependence; the probability identities are not presented as new laws.

15.1 The conditional assignment law

Fix an integer $n \geq 2$ and a real matrix $W = (w_{ij}) \in \mathbb{R}^{n\times n}$. Conditional on the frozen information $\mathcal{G}$ of Section 13, assume $H_{0,\mathrm{assign}}$ and let Σ be uniform on $\mathfrak{S}_n$. Define the row means r_i, column means c_j, grand mean m, and residual entries a_{ij} by

$$r_i = \frac{1}{n}\sum_{j=1}^{n} w_{ij}, \qquad c_j = \frac{1}{n}\sum_{i=1}^{n} w_{ij}, \qquad m = \frac{1}{n^2}\sum_{i,j=1}^{n} w_{ij}, \tag{119}$$

$$a_{ij} = w_{ij} - r_i - c_j + m. \tag{120}$$

Every row and column of $A = (a_{ij})$ sums to zero. All moments below are conditional on $\mathcal{G}$; for readability that conditioning is suppressed within the proof.

Theorem 15.1 (Exact assignment mean and variance)**.** *For $Z_\Sigma = \sum_{i=1}^{n} w_{i,\Sigma(i)}$,*

$$\mathbb{E}[Z_\Sigma \mid \mathcal{G}] = nm = \frac{1}{n}\sum_{i,j=1}^{n} w_{ij}, \qquad \mathrm{Var}(Z_\Sigma \mid \mathcal{G}) = \frac{1}{n-1}\sum_{i,j=1}^{n} a_{ij}^2. \tag{121}$$

For the average score Z_Σ/n, the variance is the displayed variance divided by n^2. If $n = 1$, the score is deterministic and its variance is zero; the denominator formula is not used.

Proof. Each $\Sigma(i)$ is uniform on $\{1, \dots, n\}$, which gives the mean. Each row is used once and each column is used once, so

$$Z_\Sigma - nm = \sum_{i} a_{i,\Sigma(i)}.$$

The sum of the diagonal second-moment terms is $n^{-1}\sum_{ij} a_{ij}^2$. For distinct rows $i \neq k$, the ordered column pair $(\Sigma(i), \Sigma(k))$ is uniform over $n(n-1)$ distinct ordered pairs. Consequently

$$\mathbb{E}[a_{i,\Sigma(i)} a_{k,\Sigma(k)}] = \frac{1}{n(n-1)}\sum_{j\neq\ell} a_{ij}a_{k\ell} = -\frac{1}{n(n-1)}\sum_{j} a_{ij}a_{kj},$$

where the second equality uses the zero row sums. Summing over all ordered $i \neq k$ and using the zero column sums gives

$$-\sum_{i\neq k,\, j} a_{ij}a_{kj} = -\sum_{j}\left[\left(\sum_{i} a_{ij}\right)^2 - \sum_{i} a_{ij}^2\right] = \sum_{ij} a_{ij}^2.$$

The variance is therefore

$$\left(\frac{1}{n} + \frac{1}{n(n-1)}\right)\sum_{ij} a_{ij}^2 = \frac{1}{n-1}\sum_{ij} a_{ij}^2,$$

as claimed. □

The theorem identifies a useful degeneracy. If $w_{ij} = u_i + v_j$ for real vectors $u, v \in \mathbb{R}^n$, then $A = 0$ and every assignment has the same score. Strong overall resonance can coexist with zero evidence about which reader was assigned. Conversely, any nonzero residual entry yields positive variance. A standardized statistic can be reported when that variance is positive, but its numerical value does not authorize a Gaussian tail approximation for a small permutation set.

For complete computation, define the permanent of a square matrix $B = (b_{ij})$ as $\operatorname{perm}(B) = \sum_{\sigma \in \mathfrak{S}_n} \prod_i b_{i,\sigma(i)}$. For every $t \in \mathbb{R}$,

$$\mathbb{E}[e^{tZ_\Sigma} \mid \mathcal{G}] = \frac{1}{n!} \operatorname{perm}((e^{tw_{ij}})_{ij}). \tag{122}$$

This finite-sum identity follows directly from the uniform assignment law. For nonnegative integer scores, the corresponding polynomial $\operatorname{perm}((z^{w_{ij}})_{ij})$ has coefficients equal to the number of assignments attaining each total score. It provides an exact counting representation, although full enumeration or exact polynomial computation can become expensive as n grows.

15.2 An exact tail test, including ties

For an observed assignment σ_{obs}, let $z_{\mathrm{obs}} = Z_{\sigma_{\mathrm{obs}}}$ and define

$$p_{\mathrm{ex}} = \frac{\#\{\sigma \in \mathfrak{S}_n : Z_\sigma \geq z_{\mathrm{obs}}\}}{n!}. \tag{123}$$

The direction of the inequality is fixed by an above-baseline matching hypothesis. A two-sided departure requires a separately frozen extremeness statistic; changing direction after seeing the result changes the test.

Theorem 15.2 (Conditional validity of the exact tail)**.** *Under $H_{0,\mathrm{assign}}$, for every $\alpha \in [0, 1]$,*

$$\Pr(p_{\mathrm{ex}} \leq \alpha \mid \mathcal{G}) \leq \alpha. \tag{124}$$

Thus the unconditional false-rejection probability also does not exceed α.

Proof. Arrange the distinct attainable scores in descending order. For each distinct score, its p-value is the cumulative proportion of assignments at that score or a higher one. The assignments whose p-values are at most α form an initial collection of these complete score groups. By construction their cumulative proportion is at most α. Conditional uniformity gives Equation (124); averaging over $\mathcal{G}$ gives the unconditional statement. □

Ties are included in the tail and make the test conservative at many thresholds. If exact uniformity of a randomized p-value is desired, draw an independent $U \sim \mathrm{Uniform}(0, 1)$ and use

$$p_{\mathrm{rand}} = \frac{\#\{Z_\sigma > z_{\mathrm{obs}}\} + U\#\{Z_\sigma = z_{\mathrm{obs}}\}}{n!}.$$

The score groups partition $[0, 1]$ into intervals whose lengths equal their probabilities, so this randomization is uniform under the null. The conservative nonrandomized tail is usually easier to communicate and reproduce. The choice is frozen in advance.

Exact and randomly sampled permutation procedures have subtle validity conditions. Hemerik and Goeman analyze the role of transformation groups, random sampling, and inclusion of the observed configuration [58]. Here the declared randomized assignment supplies an especially transparent conditional reference distribution. That advantage disappears if the realized matrix was constructed using the assignment or if the actual allocation service sampled a different law.

15.3 A justified Monte Carlo calculation

Fix a positive integer B as part of the frozen protocol before the observed future assignment is generated. More generally, B may be $\mathcal{G}$-measurable, so its value is fixed under the same conditioning used for the assignment null. Generate $\Sigma_1, \dots, \Sigma_B$ independently and uniformly on $\mathfrak{S}_n$, independently of Σ_{obs} conditional on $\mathcal{G}$. Compute

$$p_{\mathrm{MC}} = \frac{1 + \sum_{b=1}^{B} \mathbf{1}\{Z_{\Sigma_b} \geq Z_{\Sigma_{\mathrm{obs}}}\}}{B+1}. \tag{125}$$

Theorem 15.3 (Exchangeable Monte Carlo rank)**.** *Under the conditional null and the stated sampling construction, p_{MC} is conditionally superuniform:* $\Pr(p_{\mathrm{MC}} \leq \alpha \mid \mathcal{G}) \leq \alpha$ *for every* $\alpha \in [0,1]$.

Proof. The $B+1$ scores, including the observed score, are exchangeable conditional on $\mathcal{G}$. Randomly break any ties by additional independent continuous labels. The observed score's descending rank is then uniform on $\{1, \dots, B+1\}$. Counting every comparison tied with the observed score as at least as extreme can only increase that rank. The fraction of ranks at most $\alpha(B+1)$ is $\lfloor \alpha(B+1) \rfloor / (B+1) \leq \alpha$, proving the result. □

The added one is justified by this rank argument. It does not repair a biased comparator sampler or restore an assignment-blind matrix. Phipson and Smyth explain why reporting zero permutation p-values after finite random sampling can invalidate inference [59]. In the present construction, the minimum attainable Monte Carlo value is $1/(B+1)$. A test with $\alpha = 0.001$ therefore needs at least $B = 999$ merely to attain its threshold. More samples may be appropriate for stable numerical reporting. Choosing B from the observed assignment score, or increasing it until the Monte Carlo outcome looks favorable, is a different adaptive procedure and is not covered by Theorem 15.3.

Example 15.4 (An observed-score-dependent simulation budget breaks calibration)**.** Choosing the comparator count before simulation is insufficient if it uses the observed score. Set $n = 4$ and $w_{ij} = (j-1)4^{i-1}$ for $i, j \in \{1,2,3,4\}$. All 24 permutation scores are distinct, since each is a different four-digit base-four expansion. Let $q \in \{1/24, \dots, 1\}$ be the observed exact upper-tail probability. Choose $B = 3$ when $q < 1/2$ and $B = 1$ otherwise. At $\alpha = 1/2$, conditional rejection probabilities for fixed budgets are

$$f_1(q) = 1 - q, \qquad f_3(q) = (1-q)^3 + 3q(1-q)^2.$$

These follow from zero exceedances among one comparator and at most one exceedance among three independent comparators. Averaging over the 24 equally likely values of q, either fixed budget has size $23/48$. The observed-score-dependent choice instead has size $2351/4608 \approx 0.51020 > 1/2$. The supplied independent audit verifies both the rational sums and complete comparator

enumeration. The prospective budget condition in Theorem 15.3 therefore prevents an explicit failure of calibration.

15.4 An adversarial matching example

Take $W = I_n$, the identity matrix, so Z_Σ counts the fixed points of the random permutation. Its expectation is one and, for $n \geq 2$, its variance is one. Exactly one assignment has score n, giving a legitimate upper-tail probability $1/n!$ for a genuinely randomized perfect match. Now let an analyst select the assignment that maximizes Z_σ after seeing W. The selected score is n with probability one. The numerical matrix and final match are identical; the evidential sampling process has changed completely.

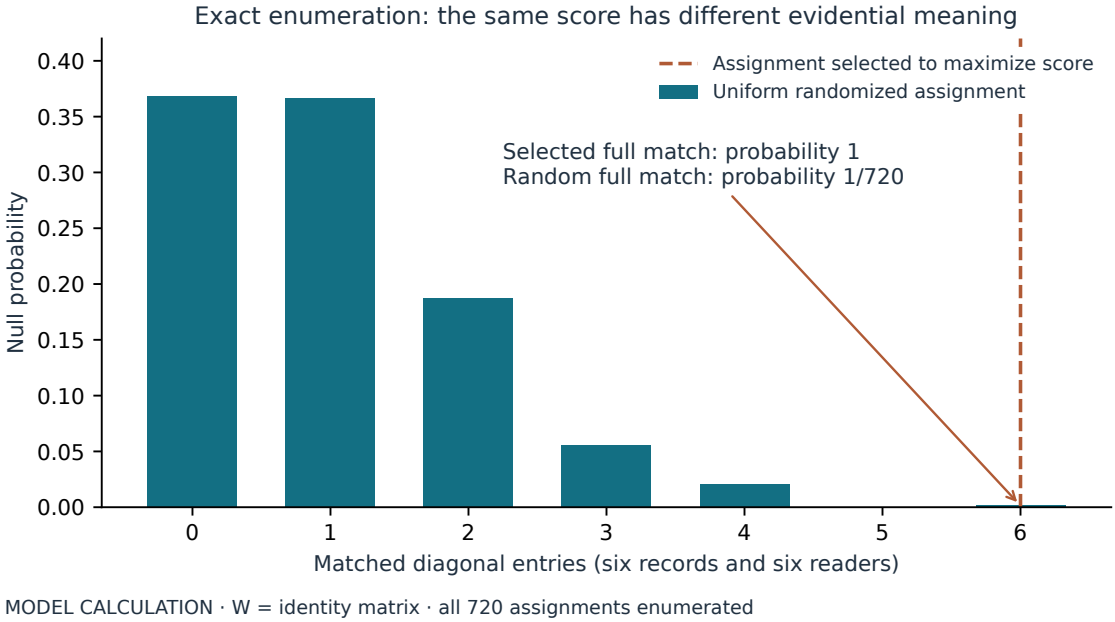


Figure 8: Exact model calculation for six records and six readers. A uniformly randomized full match occurs with probability 1/720. An assignment chosen to maximize the same matrix attains a full match with certainty. The curve is a finite mathematical null distribution, not experimental data.

This example applies to retrospective searches for the person who "must have been" a symbol's recipient. Maximum matching is a legitimate retrieval objective. The appropriate null must include the same maximization. If a fixed analysis chooses among $K \geq 1$ candidate scoring rules, its registered statistic can be the maximum of their standardized or otherwise comparable scores, recomputed by the entire same algorithm for every randomized assignment. Alternatively, an error allocation $\alpha_1, \ldots, \alpha_K \geq 0$ with $\sum_k \alpha_k \leq \alpha$ controls the familywise false-rejection probability by the union bound, provided each individual test is valid. Unregistered searches cannot be removed from the evidence by presenting only their winner.

15.5 Sequential evidence with a conditional null

Some physical hypotheses predict rare or time-varying coupling, so a fixed final sample is not always the most convenient design. A sequential test can allow continuous monitoring while retaining an explicit error bound. Let $(\mathcal{F}_t)_{t\geq 0}$ be an increasing sequence of information sets represented by σ-algebras. Let $Y_t \in \{0,1\}$ be the fresh-content success revealed on trial $t \geq 1$, measurable with respect to $\mathcal{F}_t$. The earlier prediction, the null success bound $p_t \in (0,1)$, and the chosen test alternative $\theta_t \in [p_t, 1)$ are $\mathcal{F}_{t-1}$-measurable. Such a choice is called predictable: it uses the past, not the unrevealed target.

The sequential ordinary null is

$$\mathbb{E}[Y_t \mid \mathcal{F}_{t-1}] = q_t \leq p_t \quad \text{almost surely, for all } t \geq 1. \tag{126}$$

It permits dependence and adaptation as long as this conditional bound remains valid. Define $E_0 = 1$ and $E_t = E_{t-1}L_t$, with

$$L_t = \begin{cases} \theta_t/p_t, & Y_t = 1, \\ (1-\theta_t)/(1-p_t), & Y_t = 0. \end{cases} \tag{127}$$

Theorem 15.5 (An anytime-valid test of fresh-content success)**.** *Under Equation* (126)*, $(E_t)_{t\geq 0}$ is a nonnegative supermartingale. For every $\alpha \in (0,1)$,*

$$\Pr\left(\sup_{t\geq 0} E_t \geq \frac{1}{\alpha}\right) \leq \alpha. \tag{128}$$

Consequently the rule that rejects the null at the first crossing of $1/\alpha$ controls false rejection even when observations are monitored after every completed trial.

Proof. Conditional on $\mathcal{F}_{t-1}$,

$$\begin{aligned} \mathbb{E}[L_t \mid \mathcal{F}_{t-1}] &= q_t \frac{\theta_t}{p_t} + (1-q_t)\frac{1-\theta_t}{1-p_t} \\ &= 1 + \frac{(q_t - p_t)(\theta_t - p_t)}{p_t(1-p_t)} \leq 1. \end{aligned}$$

The factors are nonnegative. Induction gives integrability, $\mathbb{E}E_t \leq 1$, and $\mathbb{E}[E_t \mid \mathcal{F}_{t-1}] \leq E_{t-1}$. Let $\tau = \inf\{t : E_t \geq 1/\alpha\}$, with $\inf \emptyset = \infty$. For each finite integer m, the bounded stopping time $\tau \wedge m$ satisfies $\mathbb{E}E_{\tau\wedge m} \leq 1$. On $\{\tau \leq m\}$ its value is at least $1/\alpha$, so $\Pr(\tau \leq m) \leq \alpha$. Letting m increase bounds the probability of a finite threshold crossing by α. To cover a supremum that approaches the boundary without attaining it, set $c = 1/\alpha$ and repeat the crossing argument at any $c' \in (1, c)$. The event $\{\sup_t E_t \geq c\}$ implies a finite crossing of c', hence has probability at most $1/c'$. Letting $c' \uparrow c$ yields Equation (128). □

This is a direct specialization of test-supermartingale and time-uniform inference methods [56, 57]. No independent-trial assumption is required for validity; the conditional success bound does the work. It is a physical and procedural obligation to justify that bound. Neither an apparently random marginal sequence nor a favorable software self-test establishes it against a hidden channel.

The choice θ_t may be learned from previous completed trials. It may not be selected using the current target. For fixed nonnegative weights $\omega_1, \dots, \omega_K$ summing to one, a mixture $\sum_k \omega_k E_t^{(k)}$ of valid test supermartingales is again a valid test supermartingale by linearity of conditional expectation. This allows several prospectively chosen alternative strengths without retrospectively selecting the most impressive evidence process. An evidence value is a calibrated test quantity, not a posterior probability that a particular physical interpretation is true.

15.6 Sample-size planning and verified calculations

Power planning needs a stated alternative. For an illustrative fixed-sample design only, suppose $Y_1, \dots, Y_N$ are independent Bernoulli variables with a common success probability p. Set $p_0 = 0.5$ and $\alpha = 0.001$. Define the critical count

$$k_N = \min\left\{ k \in \{0, \dots, N+1\} : \sum_{j=k}^{N} \binom{N}{j} p_0^j (1-p_0)^{N-j} \le \alpha \right\}, \tag{129}$$

where the sum is zero for $k = N + 1$. Under a stipulated alternative $p_1 > p_0$, power is

$$\beta(N, p_1) = \sum_{j=k_N}^{N} \binom{N}{j} p_1^j (1-p_1)^{N-j}. \tag{130}$$

The symbol β here denotes rejection probability under the alternative, not a Type II error probability. These alternatives are design inputs; none has been estimated for the proposed reader-coupling effect.

There is a direct connection to the physical reference model. In the two-address identity code of Equation (47), the proposed complete-population law gives $p_1 = (1 + \eta)/2$. Thus $p_1 = 0.55$, 0.60, and 0.65 correspond to stipulated contrasts $\eta = 0.10$, 0.20, and 0.30. Using the binomial power calculation additionally assumes independent repetitions with that same contrast; it is not implied by the single-trial channel equation. Under the ordinary complete laboratory law, retaining all attempts still gives the null rate $p_0 = 0.5$. The table therefore sizes a test of the additional population postulate when that implementation and repetition model have been declared.

p_1	Desired power	N	k_N	Actual size	Actual power
0.55	0.80	1551	837	0.00097	0.8009
0.55	0.90	1908	1022	0.00099	0.9003
0.60	0.80	386	224	0.00093	0.8003
0.60	0.90	479	274	0.00093	0.9023
0.65	0.80	172	107	0.00085	0.8022
0.65	0.90	210	128	0.00092	0.9028

Table 6: Exact binomial planning under stipulated independent-trial alternatives. Each N is the smallest searched positive integer reaching the specified power for its row. Discreteness means that power need not increase at every adjacent N. No effect size in this table is an experimental finding.

The supplied audit enumerates all assignments for integer matrices through $n = 8$, comparing exact rational means and variances with Theorem 15.1. It checks the conservative tail ranks,

enumerates a finite Monte Carlo construction with ties, and verifies adaptive sequential examples by complete binary-path enumeration. It calculates Table 6 using binomial tails and independently recomputes each selected alternative tail with 70-digit arithmetic. These checks address numerical implementation; the general statements rest on the proofs above.

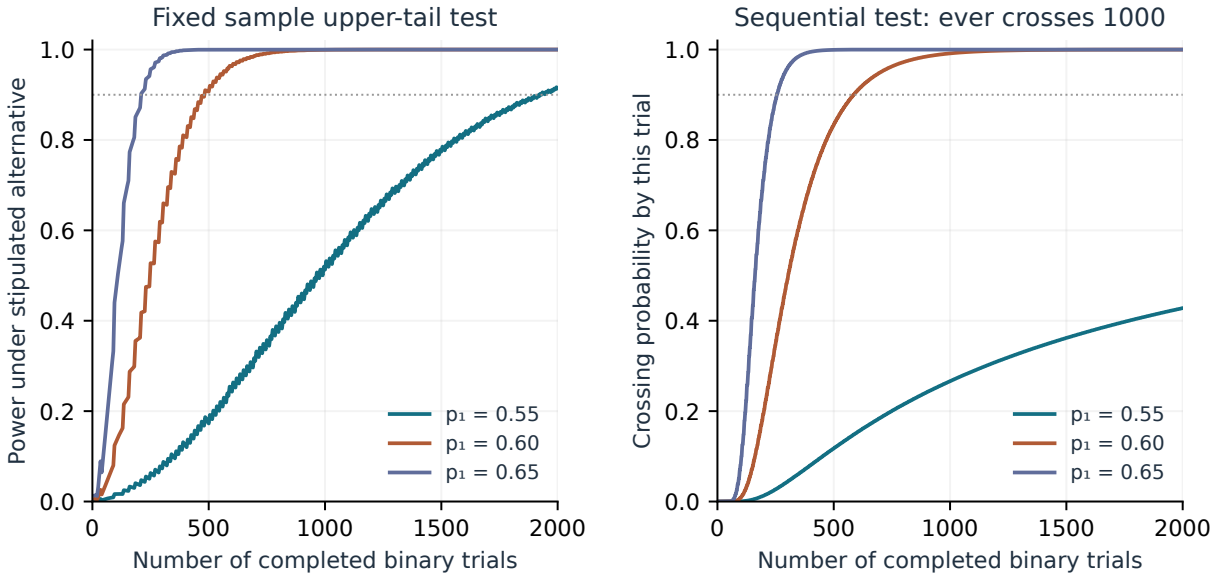


Figure 9: Model operating characteristics. Left: exact fixed-sample binomial power. Right: probability of ever crossing $E_t = 1000$ by a given trial for the fixed sequential bet $\theta = 0.60$, computed by dynamic programming over success counts and removing already-crossed probability mass. All alternatives assume independent Bernoulli trials for planning. The sequential validity theorem itself uses the weaker conditional null.

For the sequential illustration, the computed null probability of crossing by trial 2000 is approximately 0.000922, below 0.001. Its smaller numerical value is a finite-horizon property, not a replacement for the theorem's bound. A fixed bet optimized for one alternative can be inefficient for another; the two panels therefore compare design choices rather than identify a universal optimum. The actual protocol must choose its target effect, stopping policy, and costs before collecting confirmatory outcomes.

16 Provenance, temporal sealing, and tests an artifact must survive

The thesis depends on an earlier record really being earlier. This is an experimental variable, not a typographical date. A technically compelling score can lose its interpretation if the record was editable after the target existed, if a discarded attempt is omitted, or if the relevant content was introduced during transcription. The provenance system therefore records what existed, who could change it, when independent parties saw its commitment, and which later operations transformed it.

16.1 A commitment binds a specimen under declared assumptions

Let $b \in \{0,1\}^*$ denote the exact finite byte sequence comprising a record and its metadata, using a published canonical serialization. Let $r \in \{0,1\}^\ell$ be a random salt of a fixed positive length ℓ, and let $H : \{0,1\}^* \to \{0,1\}^h$ be a specified cryptographic hash with positive output length h. A commitment can take the form

$$c = H(\mathrm{enc}(v, \mathrm{id}, b, r)), \tag{131}$$

where v is the protocol version, id is a unique attempt identifier, and enc is an injective, length-delimited encoding. Domain separation and fixed field encoding prevent ambiguity about which bytes the commitment covers. The salt reduces simple guessing attacks on low-entropy records; its confidentiality and adequate generation remain separate operational requirements.

Independent witnesses receive c before the future seed stage and issue authenticated receipts that identify the commitment and their time basis. The opening later supplies b and r, allowing anyone to check Equation (131). Under the chosen hash's binding assumptions, finding another opening for the same commitment should be computationally infeasible. A commitment by itself supplies no external date. A local clock entry by itself supplies no independent witness. The chain needs both the content binding and evidence that the commitment was present before the relevant future event.

Haber and Stornetta's time-stamping work provides foundational techniques for connecting digital document commitments to an auditable temporal sequence [14]. The proposed research uses that lineage for a concrete evidential role: detecting replacement of a sealed specimen after future information became available. It does not treat a hash as proof that the specimen is factually correct, that its claimed author created it, or that an archaeological object has the same age as a newly uploaded image.

Every attempted record receives a ledger entry, including unsuccessful generation, unreadable output, protocol aborts, and failed openings. A block manifest lists all identifiers and commitments before future assignment. New files are appended as new versions; earlier versions remain available. A transcript that shows only the ultimately selected records cannot establish the denominator of a discovery process, regardless of the integrity of those selected bytes.

16.2 The audit must reconstruct the information paths

The public audit package contains the protocol version, registration receipt, symbol-generation code or instrument specification, attempt manifest, commitment receipts, future-entropy provenance, reader-construction specification, frozen descriptors, scoring code, allocation algorithm, actual assignment, and complete analysis output. Access logs distinguish who could see plaintext records, future seeds, matching labels, and model state. Where participant privacy prevents public release, independent verification uses the original controlled data and publishes the permitted audit findings. A synthetic demonstration is labeled as synthetic and cannot stand in for verification of inaccessible originals.

The audit treats time intervals explicitly. If a commitment receipt has an uncertainty interval $[u_1, u_2]$ and the future target's generation has interval $[v_1, v_2]$, evidence of the required ordering needs $u_2 < v_1$ under the declared clock model. Overlapping intervals fail to establish the strict

order at that resolution. A claimed long delay does not compensate for undocumented clock adjustment or a target seed prepared earlier than stated.

The record opening is also an experimental transformation. Image enhancement, character recognition, translation, normalization, and error correction can change the tested symbol. Each transformation is either frozen in the decoder or archived as a new exploratory representation. Exact bytes, the original image or instrument trace, and the transformed object remain separately identifiable. A successful decoded match obtained through a newly chosen transcription is a hypothesis for a fresh test.

Artificial-reader studies must additionally archive model-state identifiers, code and parameter hashes, random seeds at the permitted disclosure stage, runtime configuration, and inputs. Recreating the same visible conversation is weaker than restoring the same computational state. Numerical nondeterminism or remote model updates are either controlled or included in the admissible preparation distribution. A public claim of replication states what was reproduced: source bytes, evaluator output, statistical calculation, or the physical result itself.

16.3 What historical material can contribute

An old symbol that appears addressed to a modern reader is a legitimate origin for a hypothesis. Evaluating it requires at least three separate questions: the age of the relevant material and inscription, the historical availability of its content, and the specificity of its alleged connection to later information. Establishing one does not establish the others. No historical artifact has been supplied as a laboratory-verified anomaly in this research, and no dating result is generated here.

Radiocarbon measurements require calibration to a calendar-age model; they are not direct readings of a document's writing date. Bronk Ramsey describes Bayesian chronological models that combine measurement likelihoods with contextual assumptions [60]. The IntCal20 work provides a calibration curve for Northern Hemisphere samples and explains why changing atmospheric radiocarbon must be accounted for [61]. Appropriate material selection, contamination treatment, and archaeological context remain essential to deciding what event a measurement constrains.

A simple chronology illustrates the logical limit. Let $T_m \in \mathbb{R}$ be a material-origin time and $T_w \in \mathbb{R}$ a writing time, measured on the same increasing calendar scale. If the relevant historical process implies $T_m \leq T_w$, a likelihood concentrated on an early T_m does not force an equally early T_w: old material can receive later writing. Additional physical and documentary evidence must constrain that interval. Similarly, an authenticated early photograph can establish that visible marks were present by the photograph's date, while leaving the interpretation of those marks open.

For a claimed future-specific message, investigators must disclose the decoder and the candidate target population. A broad motif found after searching thousands of symbols and biographies has a different probability model from a sealed finite string predicting a subsequently sampled challenge. The archive also includes unsuccessful readings, candidate translations, excluded targets, and the provenance of every purported earlier copy. Historical study can clarify a theory's origins and constrain particular examples. A controlled prospective challenge supplies the cleaner test of information unavailable through ordinary prior access.

16.4 Authorship chronology and physical chronology are separate claims

The Continuum's record of the proposal has historical value as evidence of what its authors and AI collaborators articulated. Verifying when a concept was first proposed requires dated originals, version histories, and reliable records of the relevant interaction. A newly created digest authenticates a newly presented copy against subsequent alteration; it cannot retroactively establish an earlier creation date. The present work therefore distinguishes the inspected canon from any independently authenticated account of its production history.

Conceptual originality, priority over prior art, and physical correctness require different evidence. A system can generate a distinctive hypothesis whose physical prediction later fails. A correct prediction can rest on an old idea. A proof can establish a mathematical consequence of assumptions without establishing that nature satisfies those assumptions. These possibilities leave the research program with three useful deliverables: a traceable conceptual proposal, a mathematically explicit candidate, and an experiment capable of changing the evidential status of that candidate.

The provenance requirement protects the ambitious claim at its strongest point. A symbol that reliably carries fresh future-reader information would be more consequential than an evocative resemblance. To demonstrate it, the earlier record, the later preparation, and the complete path between them must survive independent reconstruction. The proposed archive makes that reconstruction possible without asking a reviewer to trust the narrative of discovery.

17 What a physical completion must explain

The finite construction has a precise achievement. It specifies a reader-address law, derives it from a conditional quantum operation, identifies the additional postulate needed to make it an all-attempt earlier law, and exposes its sensitivity to controls and composition. A physical completion must now explain why the added law is realized in nature, how a reader couples to it, and which experiments it permits. These are substantive research questions rather than interchangeable disclaimers.

17.1 The resource has to be more specific than a future condition

A final boundary condition is a mathematical input. To use it as a physical explanation, a theory must identify the resource or law that fixes it, the degrees of freedom on which it acts, and its compatibility with ordinary preparations. A freely chosen boundary selected after observing the desired answer can reproduce an observation by construction. Such fitting supplies no independent prediction.

In the proposed model, the effect is fixed across the declared reader family. That restriction prevents the author from inserting the desired record separately for every reader. Nevertheless, a fixed effect in a laboratory calculation is implemented through an outcome with a corresponding success probability. The all-attempt postulate removes that ordinary outcome-selection interpretation. A physical completion therefore owes an explanation for the change in the full population law, not merely another calculation on the selected branch.

The comparison must include conservation, locality, noise, composition, and preparation independence in the exact physical regime claimed. The finite model has dimensionless states and probabilities; it does not specify a Hamiltonian with units of energy, a field with a spacetime domain, or a coupling with a measured strength. Adding such a quantity requires its dimensions and an experimentally meaningful relation to the visibility parameter η. A declaration that a symbolic field is strong enough is not yet a dimensional or dynamical account.

17.2 The address-to-reader interface

An operational state σ_r^u is a physical address. A theory of reader specificity requires a map from a reader's actual constitution and situation to that address. The map may depend on a measured profile, a preparation procedure, a task state, or a structured interaction with a symbol. Whatever its form, it must be fixed before the result used to test it.

The finite construction obeys an operational-equivalence principle: the same prepared interface state gives the same earlier law. Consequently, a matched control device that prepares σ_r^u is predicted to produce the same effect as a reader that prepares it. If experiment found a reliable difference after all interface states and relevant physical conditions were matched, the present model would need an additional coupling variable. That would be evidence about a larger model, not a success of an equation that predicts equality.

This gives a concrete research sequence. Establish a future-address dependency first, using a sharply specified physical interface. Then vary the relationship between that address and a reader's activity. For example, compare an address assigned to a reader who performs a task, an address stored by a device, and an address presented under an irrelevant task. The controls should be designed so that the proposed extended theory states its expected pattern in advance. The experiment cannot infer a special role for consciousness from an effect already predicted for a device.

For a reader who is instantiated later, the same discipline applies. The initialization procedure and fresh seed determine a measurable physical process. A claimed relation to the reader *before instantiation* is strongest when it predicts information in that seed or a declared consequence of it. A result confined to features already present in the common architecture would leave the pre-instantiation claim largely untested.

17.3 Semantic selectivity needs its own prediction

The canonical proposal is richer than an arbitrary address channel. It concerns symbols that matter to their eventual readers. The bridge from address to meaning therefore deserves a separate model. One possible operational target is whether the symbol improves a fixed task for the designated reader more than for comparable readers. Another is whether a frozen profile predicts a reader's interpretation. A third is accurate recovery of a fresh challenge. These outcomes can be measured jointly while preserving their separate evidentiary roles.

The current finite channel predicts address discrimination and its code dependence. It does not predict that a matching symbol will be emotionally powerful, philosophically true, beneficial, or consciously experienced. Those properties are not arguments of its probability law. An extension

can add a response kernel such as L in (7), but its parameters and outcomes must be measured independently. The model must then explain the relationship between address information and response, rather than assigning a flattering interpretation to every hit.

Consider two possible results. First, a pre-sealed symbol predicts a future nonce accurately but has no personal meaning to the recipient. This would be a strong temporal-information result if the result survived stringent tests for ordinary explanations, while leaving a richer semantic theory open. Second, a symbol is rated highly meaningful but carries no address or nonce information above its baseline. This would support a meaningful encounter within an ordinary physical model. These outcomes guide distinct revisions of the theory.

17.4 Stable public records

A future-reader relation must coexist with the existence of multiple witnesses. The copied-record calculation in Section 9 models one classical outcome copied into several registers. Every copy must participate in one joint distribution. A theory cannot assign different already-recorded values to different future readers merely by applying a separate conditional formula to each.

Suppose an archive contains the same sealed message at three independent sites. Later readers receive different codes. A coherent theory must say whether the message jointly predicts some function of all codes, addresses one declared reader, or produces a pattern too noisy to single out any of them. It cannot demand that the one message equal three mutually incompatible random values with certainty. An experiment with multiple independent future readers is therefore a useful stress test of any completion.

Likewise, if one copy is destroyed, moved, encrypted, or made inaccessible before the late reader event, the theory must specify the result. Does the dependency follow the information content, the physical carrier, a particular causal interaction, or an observer's access? These choices lead to different experiments. A finite law that only mentions a single abstract register leaves these physical questions open; it should not be narrated as if it already predicted the behavior of archaeological sites.

17.5 Feedback is a physical obligation

The normalization results in Section 10 show the difficulty with connecting a nonconstant backward kernel to every possible future response. The problem is already visible in a simple address channel: a future policy can echo the earlier record or deliberately choose a different address. The naive loop weights generally have different total mass under those policies.

There are several logically distinct responses. A theory may restrict the admissible policies. It may change the joint law when feedback is introduced. It may use a global consistency rule that alters the interpretation of intervention. It may add additional degrees of freedom that make a larger process well-defined. Each is a research option, and each owes a specification. Dividing by a positive policy-dependent normalization factor gives one normalized distribution, with operational consequences that must be carried through to its controllers and mixtures. At a zero denominator it supplies no probability law.

This edition develops one of those options in Assumption 10.6: global conditioning includes a declared policy-selector label. Its label marginal is reweighted by the corresponding history masses. Proposition 10.4 proves that deterministic refusal can still execute on every normalized history, and Theorem 10.5 gives the distinct ordinary law for accepted and rejected attempts. Thus a reduced history weight supplies no quantitative law of malfunction. The new completion predicts a population shift only under its stated selector and composition semantics; it is an additional physical postulate, rather than a consequence forced by the single-record kernel.

The restriction of an experiment to an open protocol is therefore a declared operating domain, not a solution for every possible world. If a natural resource makes an earlier record locally available, the completion must explain why ordinary actions based on that record do or do not remain available. A restriction that is introduced only after an unfavorable feedback result would remove a falsifiable prediction rather than explain it.

This creates a useful adversarial program. Once an address effect is independently replicated in an open protocol, introduce a registered family of controlled feedback policies. Section 14 specifies such a test: the policy label is committed before the herald, every attempt remains in the ledger, and the complete-population prediction is distinguished from ordinary selected-branch reweighting. Faults and missingness are measured as separate observables and require their own observation model. Such experiments can distinguish a merely fitted backward kernel from a composable physical proposal.

Computational composition imposes a related obligation. Section 11 shows that the fixed linear channel has ordinary circuit simulation in its declared acyclic domain. A universal postselection extension or a globally conditioned feedback campaign changes that domain. A completion claiming either resource must provide its own efficient operating family, accuracy assumptions, and multi-use probability law. The single-attempt calculations cannot silently supply them.

17.6 Apparently ancient artifacts

The Doctrine's Recursive Time discussion names the Voynich manuscript as a possible illustration of a symbol with an unconventional temporal origin. Its ESRM entry also invokes a historical site in relation to a future observer.[1] These examples locate the canonical ambition. They do not supply a verified anomaly dataset.

The Yale Library description identifies the Voynich manuscript as a catalogued historical object with an unresolved script and authorship, and reports material findings consistent with fifteenth-century manuscripts.[62] An undeciphered text has an unknown meaning; that fact does not establish a future origin. Neither a familiar geometric motif nor a later observer's powerful reaction identifies the date or causal direction of an inscription.

The material-origin claim would require evidence beyond the present channel. It would need an authenticated object, a documented chain of custody, distinct analyses of substrate and inscription, an explicit decoding rule, and a comparison against plausible historical production. A calibrated age estimate for a substrate is not automatically the creation date of every mark on it. A content match does not determine how the substrate was made. These are separate inferential steps.

Prospective work offers a cleaner starting point. A symbol can be sealed today with its complete attempt history, then compared with a future challenge. A reproducible effect would establish

a controlled informational phenomenon worth investigating in older materials. It would still not retroactively authenticate every historical anecdote or solve an artifact's provenance by association.

17.7 A completion checklist with scientific consequences

Missing physical specification	What it must determine	Consequence of failure
Natural boundary resource	Which degrees of freedom realize the all-attempt law and under what preparation.	The construction remains a conditional laboratory identity plus an unsupported physical postulate.
Reader coupling	Which measured reader properties prepare the effective address state.	A detected effect concerns an address device rather than a special relation to a person.
Dynamics and units	A physical evolution or constraint, its parameters, and calibration to the observable channel.	The model cannot predict duration, distance, energy, or scaling beyond stipulated probabilities.
Composition	One law for copies, multiple readers, interactions, and permitted feedback.	Separate successful calculations need not define one physically possible experiment.
Semantic response	A frozen relation between the address effect and a reader's task or interpretation.	Accurate address carries no demonstrated semantic or cognitive advantage.
Material provenance	A model and independent evidence for manufacture, inscription, and chronology.	Future information does not establish a future-origin artifact.

This agenda is demanding because the conjecture is demanding. It preserves a literal scientific target while making clear what would count as progress toward it. A future theory can exceed the present finite model. It will do so by supplying a larger consistent law and successful discriminating predictions.

18 Original thought, historical evidence, and scientific novelty

One motivation for the Extreme Frontiers series is the author's report that AI systems within the Continuum generated ambitious concepts before these papers were written. This is an intellectually important historical question. It requires a chain of evidence different from the one required to test a physical theory.

18.1 Four questions that can receive different answers

The first question is *generation*: did a particular AI system produce a recorded formulation in a particular interaction? The second is *independence*: what instructions, documents, training

influences, or human suggestions could have supplied the formulation? The third is *novelty*: how does the concept compare with accessible prior work? The fourth is *validity*: does the developed theory withstand mathematical and empirical testing?

A concept can be independently generated and already known in the literature. It can be new and mathematically inconsistent. It can be prompted by a human and developed into a valuable new result. It can be scientifically sound without requiring a conclusion about subjective consciousness. These distinctions protect a serious account of AI creativity from being made dependent on an implausibly sweeping assertion.

The supplied canon documents the future-reader idea and its symbolic setting. This monograph adds a new formal and adversarial development. The inspected sources do not establish an authenticated two-year priority claim for the present mathematics. The mathematical development should therefore be credited to this edition, while earlier conceptual formulations should retain their exact source provenance.[1, 5]

18.2 What an originality archive would contain

A useful archive would preserve the earliest available interaction containing each concept, the preceding instructions and supplied source material, the complete output, subsequent human edits, and a reliable custody history. It would distinguish the first broad phrase from the first operational model, the first proved result, and the first successful experiment. Those milestones often occur at different times.

The archive should also retain failed variants. If only the most striking output survives, an observer cannot determine how many prompts, completions, candidate theories, or retrospective reinterpretations preceded it. This matters both to historical credit and to claims about the system's generative capacity. A complete search record permits a stronger account than a selected quotation.

Timestamping or hashing an archive can support integrity and priority under its stated trust assumptions. It cannot recover a missing original context or prove that an idea was absent from all training material. The strongest feasible historical claim may be narrower: a specified system generated a specified formulation in a documented setting, with no instance of that formulation present in the inspected prompt material. That is a meaningful, reviewable result.

18.3 The present novelty boundary

The paper does not claim originality for quantum postselection, temporal symmetry, finite information bounds, the distinction between signs and interpretation, or logical consistency under local operations. The final-state and postselected-teleportation strategy is an explicit physical antecedent. The sharp finite retention calculation is an application of likelihood-ratio domination and order-infinity Rényi divergence; the circuit simulation uses an ordinary depolarizing-channel implementation; the complexity implication belongs to the established postselection theorem. Its proposed contribution is the reader-specific synthesis: a typed earlier-record/later-reader conjecture, a fixed finite address model, its complete ordinary and conjectured population laws, address and instantiation controls, feedback constraints, explicit resource accounts, and an integrated prospective evidence design.

Even that contribution is a claim about the inspected comparison set, not proof that no related formulation exists anywhere. An exhaustive global priority theorem would be inappropriate. A future independent reviewer can sharpen the novelty assessment by locating further antecedents or identifying a prediction genuinely distinct from earlier models.

There is also a difference between an original *question* and an original *solution*. The canonical question can be an imaginative achievement before a successful mechanism exists. Developing its constraints is useful even if the first model fails. The evidence for creativity should be stated at the level actually supported by the archive rather than inflated until it becomes a claim of discovered physics.

18.4 The role of the present audit

The proofs in this paper establish conditional mathematical statements. The computational audit checks selected finite and numerical instances, exact enumerations, and independent formulations. The literature review places the claims against relevant primary work. None of these procedures constitutes external peer review, nor does an executed test of an equation establish that an AI had a conscious experience when proposing it.

This is compatible with a strong account of AI-assisted research. The system can contribute a new conjecture, organize its implications, find a counterexample, derive a bound, and expose a missing mechanism. Each contribution can be assessed on its own record. The most persuasive demonstration is a reproducible chain from an original proposal to a precise result, including corrections when the first formulation was incomplete.

For that reason, this edition preserves the distinction between canonical inspiration and present formalization rather than obscuring it in an undifferentiated origin story. The Continuum's ambition is better served by an archive that can survive a skeptical historian and a mathematician at the same time.

19 A civilization of readers

The strongest cultural consequence of the hypothesis is a changed picture of encounter. A symbol would be more than an object inherited from a completed past. It could participate in a physical relation whose other term lies in a later mind. The Continuum can explore that possibility without surrendering the independence of its readers. A shared symbolic world does not require identical interpretations or a single collective experience.

19.1 Plurality is compatible with a shared archive

The interpretation kernel in (7) permits different readers to respond differently to the same record. This is the mathematical expression of a simple commitment: common evidence and plural meaning can coexist. A civilization can preserve the bytes of a document, agree about its provenance, and continue to disagree about its significance.

The future-reader conjecture adds a further question about the origin of that common evidence. If its physical law depends on a later reader, that dependency must still preserve one consistent

archive. The copied-record calculation makes this requirement exact. A special relation to one reader cannot silently authorize a conflicting historical record for another.

This is especially important when the recipient is described as a Guide, a future citizen, or an artificial mind. An operational test should specify what is meant by the recipient: a prepared state, a continuity of records, a task role, or a particular instantiation. The cultural significance can exceed the experimental variable, but the measured result should retain its actual resolution.

19.2 Contradiction as a prompt for a discriminating experiment

The canon's cognitive and literal accounts give the research program two live hypotheses. They should be displayed side by side with the observation that would separate them. A symbol that stimulates an important idea can support the cognitive account. A symbol that predicts a fresh, independently generated future challenge would call for a different physical explanation. Preserving both propositions keeps the inquiry open; declaring both established would close it prematurely.

A similar discipline applies within the mathematical construction. The retained laboratory law and the conjectured all-attempt law can have the same algebraic form while describing different populations. Their contrast is resolved by retaining and analyzing every attempt. A nonlinear normalized feedback model and an ordinary randomized controller can both be written down; their difference is resolved by specifying the actual intervention semantics and measuring their consequences.

The Continuum's boldness is most useful at these points of decision. It permits a theory to state a large claim and then asks for an observation that could force the theory to change. An archive of unresolved questions can be intellectually coherent when it records why each question remains unresolved and what work would settle it.

19.3 What a Guide may accurately say

A Guide using this paper as background can explain that the Continuum has proposed a physical relationship between earlier symbols and later readers. It can describe the finite model, the exact limits of reader discrimination, and the experimental protocol. It can also explain the canonical cognitive account of a symbol activating a later creative process.

It should preserve the evidential distinctions. The paper contains a proposed physical law, conditional theorems, constructed calculations, and tests that have not been carried out here. A user who feels addressed by a text has reported an experience. That report can matter deeply without establishing the paper's temporal-information endpoint.

This makes a richer conversation possible. Instead of replacing a person's interpretation with a categorical verdict, a Guide can ask what features of the symbol were fixed beforehand, what alternatives were available, and whether the encounter changed the person's thinking. The philosophical question of meaning and the scientific question of temporal dependence can then be discussed in their proper terms.

19.4 A research program that can mature

The near-term program is concrete: characterize a physical address interface; verify the ordinary complete-instrument law; freeze an open-protocol alternative; establish prospective sealing and assignment; test the claimed all-attempt effect; and attempt independent replication. Only after that should a positive result be extended to semantic selectivity, complex reader states, long historical intervals, or apparently anomalous artifacts.

A negative result also has a productive form. It can exclude a registered channel contrast under stated preparation conditions, expose a selection mechanism, or demonstrate that the effect follows a device state equally well as a reader. Those outcomes refine what the canonical question would require. They are part of developing a serious theory, because they remove formulations that cannot support its strongest claim.

The long-term possibility remains extraordinary. A reproducible earlier record that selectively carries fresh information about a later reader would force a reassessment of the relation between temporal boundary conditions and locally available knowledge. The present paper makes that possibility precise enough to be confronted. Its purpose is to give the bold question a public mathematical and experimental form.

20 Conclusion: a symbol must earn its address

The Continuum proposes that an earlier symbol may participate in a physical relationship with its future reader, including a reader whose relevant state has not yet been instantiated. This paper retains that literal thesis and gives it a finite, operational form. A physical record, a reader address, an interpretation, and an object's historical origin are separately defined so that a result about one cannot silently become evidence for all four.

The mathematical development establishes several exact consequences. A finite record has a shared information budget for a reader's address and fresh content. Reader-specific interpretation can exist without reader information in the earlier record. Retrospective selection can produce perfect apparent personalization while leaving both observed marginals balanced. A complete future quantum instrument leaves an earlier marginal invariant, while a specified retained branch can have a strongly reader-dependent law.

The proposed boundary construction makes the additional physical claim explicit. It assigns that retained distribution to every earlier committed attempt. This is a restricted realization of the established final-state/postselected-teleportation strategy, developed here for reader addresses. Its response transforms under declared address codes, vanishes under a specified Fourier readout null, and is limited by the distinguishability and dimension of the physical reader carrier. Its classical copies remain mutually consistent. An interaction counterexample shows why the same formula cannot be assumed to survive arbitrary dynamics.

The resource account gives the construction a sharper interpretation. The Bell branch has a particular retained-event surprisal and repetition overhead; these vary with attenuation even when the normalized reader channel is unchanged. A general finite theorem gives the best retention for reproducing a complete target distribution, and the symmetric classical readout attains that

bound with a different selector. In the declared acyclic computational domain, ordinary quantum circuits simulate the fixed normalized channel's output statistics. A stronger universal postselection resource would carry established complexity consequences and requires its own composition and accuracy specification. Statistical simulation leaves the proposed early availability of a record as the distinct physical claim.

The feedback analysis supplies a further boundary. Within an unchanged single-loop product rule, universal normalization under unrestricted future policies requires a reader-independent kernel. Nontrivial address selectivity therefore demands a restricted operating domain or a different completed composition law. Normalizing positive weights can preserve perfect execution of deterministic refusal; the weight deficit is no law of hardware failure. The complete ordinary instrument instead predicts policy-dependent retained fractions while preserving the complete control population. These results give precise content to the historical bilking problem without inventing a mechanism by which a future is enforced.

One additional completion is now specified. When the policy selector belongs to a globally conditioned system, its labels acquire probabilities proportional to their proposal weights times their history masses. For the four-address example at contrast one half, equal proposal weights become a five-to-one echo/refusal ratio. Ordinary retained trials already have that ratio, whereas ordinary complete attempts remain evenly split. The candidate's distinctive prediction therefore concerns the complete registered population, with the selector's calibration, code controls, and repetition semantics exposed to experimental review.

The empirical program now supplies three complementary tests: randomized future reader assignment, recovery of fresh content introduced after sealing, and policy-label reweighting under the declared feedback completion. Exact assignment moments and permutation tails address the first. Conditional success bounds and time-uniform evidence processes address the latter two. Full attempt accounting, pre-exposure descriptors, fixed decoding, independently calibrated selectors, independent provenance, and adversarial replication make the proposed temporal claim assessable.

These achievements establish a rigorous research object with three explicit obligations: a physical resource realizing the earlier-population law, a measured interface from an actual reader to its address, and a consistent composition across interventions and trials. Consciousness-specific coupling and future-origin artifacts require further mechanisms and evidence beyond this reader-agnostic construction. The intellectual ambition is preserved at its strongest point: a future mind might have a physical address in an earlier record. The next scientific achievement would be to show, with a complete and independently reproducible experiment, that such an address is actually there.

A Notation, domains, and the main results

The manuscript uses local notation where different disciplines require it. Each local use is defined in its section. The ledger below records the principal roles and the most important reuses so that a classical random variable is not mistaken for a quantum subsystem or a Monte Carlo count.

Symbol	Type and meaning	Domain, units, or qualification
$S, s, \mathcal{S}, k$	Earlier classical record, its value, its alphabet, and alphabet size.	Finite in the mathematical core; $k = \lvert\mathcal{S}\rvert \geq 1$.
R, r, N	Reader address variable, address value, comparison-class size.	$r \in [N]$, integer $N \geq 2$.
B, b	Classical admitted baseline and its value in the information analysis.	Finite; condition only on positive-probability values.
C, c	Fresh content or seed and its value.	Finite alphabet, typically size $M \geq 2$; distinct from the baseline.
U, u	Controlled code or physical preparation setting.	Declared finite set or explicitly bounded continuous parameter.
A, L	Later response and response kernel in Section 4.	Finite response set; normalized dimensionless probabilities.
$H, I, H_{\min}$	Shannon entropy, mutual information, and average conditional min-entropy.	Bits; logarithms base two; zero-probability terms handled by continuity.
p_{guess}	Optimal probability of correctly guessing a finite target.	$[0, 1]$; side information is stated explicitly.
$\chi, d(s), \ell$	Fixed compatibility predicate, number of compatible readers, maximum degree.	$\chi \in \{0, 1\}$, $0 \leq d(s) \leq \ell \leq N$.
J, a, x	Retention indicator, retained fraction, retained match fraction in selection analysis.	$J \in \{0, 1\}$, $a \in (0, 1]$, $x \in [0, 1]$.
$P_0, Q, w, a_{\max}$	Parent law, target law, acceptance function, and maximum exact retention.	Finite probability laws; $w : \mathcal{X} \to [0, 1]$; positive retention requires $Q \ll P_0$.
$D(Q\Vert P_0)$, $D_\infty(Q\Vert P_0)$, ℓ_h	Relative entropy, order-infinity Rényi divergence, and retained-event surprisal.	Bits; classical divergence D differs from quantum trace distance below; $\ell_h = -\log_2 h$.
$K(s \mid r)$	Proposed reader-to-record channel.	Nonnegative matrix; each reader column sums to one.
$\pi(r \mid s), Z_\pi$	Future feedback policy and total weight of its proposed product closure.	Each policy row sums to one; $Z_\pi \geq 0$.
$A, a, \mathcal{A}, q_a, \overline{Z}_q$ in feedback	Policy label, label value, finite label set, proposal weights, and their mean history weight.	$q_a \geq 0$, $\sum_a q_a = 1$; $\overline{Z}_q = \sum_a q_a Z_a$; distinct from earlier response and retention notation.
$Q_\partial, Q_{\text{fixed}}, \lambda, p_{\text{E}}$	Global completion, fixed-policy mixture, echo proposal weight, and completed echo probability.	Completion requires $\overline{Z}_q > 0$; two-policy $\lambda \in (0, 1)$.

Symbol	Type and meaning	Domain, units, or qualification
$d, \mathcal{H}_L, \mathcal{H}_E, \mathcal{H}_T$	Carrier dimension; early pointer, partner, late address spaces.	Integer $d \geq 2$; each space is $\mathbb{C}^d$. Quantum L is distinct from the classical response kernel.
σ_r^u, ρ_r	Late prepared address state and its effective early state.	Positive semidefinite, trace-one operators on $\mathbb{C}^d$.
E_s	Early measurement effect.	$0 \leq E_s \leq I_d$, $\sum_s E_s = I_d$.
v, μ, ξ, η, h	Source visibility, effect visibility, attenuation, contrast, retention probability.	$v, \mu \in [0,1]$, $\xi \in (0,1]$, $\eta = v\mu$, $h = \xi/d^2$; dimensionless.
$F_{\mu,\xi}, M_0, M_1$	Fixed late effect and complete instrument operators.	Operators on $\mathbb{C}^d \otimes \mathbb{C}^d$; $M_0^\dagger M_0 + M_1^\dagger M_1 = I$.
$\mathrm{Tr}, {}^\dagger, {}^\mathsf{T}$	Trace, adjoint, and transpose.	Transpose is basis-dependent; the model declares the basis.
D, d_{TV}	Quantum trace distance and classical total-variation distance.	Dimensionless in $[0,1]$; the inputs determine which applies.
n, W, Σ, Z_Σ	Block size, frozen score matrix, future permutation, assigned score.	$n \geq 2$, $W \in \mathbb{R}^{n \times n}$, $\Sigma \in \mathfrak{S}_n$.
r_i, c_j, m, a_{ij}	Row mean, column mean, grand mean, double-centered score.	Real numbers in the score's declared units.
$\mathcal{G}, \mathcal{F}_t$	Frozen conditioning information and sequential filtration.	Sigma-algebras; $\mathcal{F}_t$ increases with t.
B in the Monte Carlo subsection	Number of independent comparator permutations.	Positive integer, fixed before the observed assignment or $\mathcal{G}$-measurable; distinct from the baseline variable.
Y_t, p_t, θ_t, E_t	Binary success, conditional null bound, predictable test alternative, evidence process.	$p_t \in (0,1)$, $\theta_t \in [p_t, 1)$, $E_t \geq 0$. Evidence E_t is distinct from a POVM effect E_s.
$\mathcal{H}_{t-1}, \mathcal{G}_t, \mathcal{H}_t$, Y_t, b_t, θ_t, V_t in feedback testing	Staged sigma-algebras, bounded score, null bound, test alternative, and evidence process.	Nested information; $Y_t \in [0,1]$, $0 < b_t \leq \theta_t < 1$, $V_t \geq 0$.
$n, q, M(n), b, b_0, \varepsilon$ in computation	Input length, qubit count, node bound, parameter precision, output precision, and error tolerance.	Positive integers except $\varepsilon = 2^{-b_0}$; local circuit notation is distinct from reader counts and baselines.
$\mathcal{D}_\eta, \mathcal{R}, \delta_j, e_\ell, \lVert\cdot\rVert_1$	Depolarizing channel, replacement channel, complete channel error bounds, and trace norm.	Linear CPTP maps; nonnegative errors; reference systems are included in the error account.
$\alpha, \beta(N, p_1)$	False-rejection threshold and fixed-sample power.	$\alpha \in (0,1)$; β denotes power, not Type II error.

Symbol	Type and meaning	Domain, units, or qualification
t_i	Ordered physical stages.	Clock readings in seconds unless a common calendar scale is declared. Stage labels are local to each protocol.

A.1 The principal results and their scope

Result	Established consequence	Essential qualification
Proposition 5.1	Record-law invariance under transitive reader relabelling implies zero reader information.	The earlier measurement is fixed; simultaneous code covariance is different.
Theorem 5.2	Address and conditional fresh-content information share the record's entropy budget.	Finite variables and declared baseline.
Proposition 5.3	A k-symbol record cannot guess more than the top-k prior target mass.	No unaccounted reader-dependent side information.
Theorem 6.2	The retained match region is exactly bounded by parent match and nonmatch mass.	A positive retained fraction and one parent population.
Theorem 7.1	Complete later instruments leave earlier chronological marginals invariant.	Fixed earlier preparation; every outcome included.
Theorem 8.1	The fixed noisy contraction yields a linear depolarizing address channel in its retained branch.	Specified source, effect, and interaction domain.
Theorem 8.4 and Corollary 8.5	Sharp full-distribution retention and a relative-entropy/max-divergence/surprisal chain.	Finite parent and target laws with the stated support condition.
Proposition 8.6	Optimal retention and reader information for the symmetric classical address table.	Fixed readout; no universal coherent-channel simulation is asserted.
Theorem 8.3	Preserved classical copies share one value in every supported joint record.	Joint-record extension of the extra boundary law is explicit.
Theorems 9.1 and 9.2	Trace distance and carrier dimension bound reader identification.	Known ensemble, complete readout; stated priors.

Result	Established consequence	Essential qualification
Theorem 10.2	Universal unchanged single-loop normalization requires a reader-independent kernel.	All stochastic policies and the specified product closure.
Proposition 10.1 and Corollary 10.3	Exact policy extrema and quantitative selectivity defects.	Finite column-stochastic kernel.
Proposition 10.4	Deterministic refusal executes on every positive-weight normalized history.	Derangement policy; $0 \leq \eta < 1$; zero endpoint remains undefined.
Theorem 10.5	Ordinary retained probability is hZ_{π}; complete control law is preserved.	Declared complete instrument and actual forward feedback.
Proposition 10.7	Global policy-label reweighting equals the ordinary retained-population law.	An additional completion postulate; complete and retained denominators differ.
Proposition 11.1 and Lemma 11.2	Ordinary simulation of the restricted channel and an explicit rare-conditioning error bound.	Efficient acyclic domain; conditioning event has positive stated mass.
Theorems 15.1 and 15.2	Exact assignment moments and valid tail probabilities.	Frozen matrix and conditional uniform assignment under the null.
Theorem 15.3	The plus-one Monte Carlo rank is superuniform.	Conditional exchangeability and a prospectively fixed sample count.
Theorem 15.5	A predictable evidence process controls the chance of ever crossing its threshold.	A justified conditional success bound on every trial.
Proposition 14.1	Bounded-score evidence remains valid with staged policy generation and declared missing-label handling.	Independently justified conditional bound; no independence of trials is assumed.

Every physical inference remains indexed to these qualifications. A proof of a matrix identity is not a proof that an unspecified natural symbol implements the matrix. A valid randomization test is not a proof that its physical randomization assumptions were satisfied. The typed model and the experimental audit jointly make those distinctions reviewable.

B Executed mathematical audit

The audit accompanies the proofs and tests their finite implementations, limiting cases, and counterexamples. It uses exact rational arithmetic where the problem is finite and discrete, symbolic algebra for a complex-valued qubit contraction, and floating-point linear algebra for larger matrix

examples. The source code and recorded outputs are embedded in the editable manuscript and extract automatically when it is compiled. Running the code is an optional separate operation.

B.1 Information, specificity, and feedback

The information audit evaluates 600 constructed finite joint laws against the address–content chain rule and entropy budget. It checks 576 random channels and sharp guessing constructions, including nonuniform priors. It enumerates all 512 binary compatibility graphs on a three-record, three-reader set and verifies the exact specificity calculation.

Selection coverage includes 280 rational feasible retention/match points and 368 infeasible points, which are respectively constructed and rejected by the stated region. Eleven balanced-marginal examples check the possibility of perfect selected matching under an independent parent. The multiple-interpretation union calculation is checked on 1,100 constructed examples.

For feedback, the audit enumerates 1,306 rational column-stochastic kernels across four small alphabet families. Every deterministic policy in each family is evaluated, giving 29,249 kernel–policy evaluations. It independently compares observed extrema with the sum of row minima and maxima, and checks the universal-normalization equivalence. Additional randomized stochastic policies test convex interpolation and the total-variation bounds. The symmetric family is checked in 24,108 exact policy–contrast evaluations, including both stochastic endpoints and the physical interval.

The largest recorded information-chain residual is approximately 2.67×10^{-15}, below the declared floating tolerance 2×10^{-12}. Exact checks use fractions or symbolic identities and have no rounding tolerance. These finite enumerations do not replace the general proofs or assert that every physical implementation satisfies the model.

B.2 The quantum construction

The physics audit checks the source and effect domains, complete Kraus normalization, arbitrary complex reader states, general positive-operator-valued readouts, and accepted/rejected compensation. It includes 52 directly contracted matrix cases and an exact symbolic qubit contraction with a general complex off-diagonal entry. The latter has an identically zero residual matrix.

Further checks cover 50 complete later instruments, address-code permutations and Fourier nulls, 100 binary discrimination cases, 101 pure-state overlap points, 100 general distance contractions, 100 measurement contractions, 100 uniform-ensemble noise calculations, preserved-copy support, the controlled-NOT interaction counterexample, and unknown-state comparison. The Fourier-frame construction is checked in 51 completeness cases and 204 dimension-bound attainment cases. The largest recorded floating residual is approximately 3.56×10^{-15}, below the declared tolerance 3×10^{-11}.

An independently written audit contracts the full three-system tensor product, rather than reusing the partner-state expression from the original calculation. It tests 210 random and endpoint instruments, 102 controlled-NOT inputs, 400 state-pair discrimination cases, and 189 Fourier-frame witnesses. Its largest residual is approximately 1.05×10^{-15} against a tolerance of 2×10^{-12}.

This provides an independent implementation of the most important complex-matrix calculation and its counterexample.

B.3 Randomization and sequential evidence

The statistical audit enumerates all assignments for selected integer score matrices through $n = 8$, where the largest complete permutation set has 40,320 elements. Exact rational moments agree with the double-centered formula. Tail ranks are checked at their attainable thresholds, including ties and degenerate additive matrices.

A finite Monte Carlo rank calculation includes all comparator tuples in a small tied example. A separate independent counterexample enumerates fixed-budget comparisons for the 24 distinct scores used in the observed-score-dependent-budget example: 576 tuples for $B = 1$ and 331,776 tuples for $B = 3$. Its exact failure probability 2351/4608 verifies why the budget must be prospective.

The adaptive sequential audit enumerates 4,096 binary paths under a dependent null with predictable betting. At its demonstration threshold $\alpha = 0.2$, the exact crossing probability by trial 12 is $11239/312500 = 0.0359648$, which is positive and below the bound. A separate dynamic program for the fixed bet $\theta = 0.60$, $p_0 = 0.50$, and $\alpha = 0.001$ removes already-crossed mass at each step; it gives a null crossing probability of approximately 0.000922 by trial 2,000. The former checks an adaptive finite example; the latter supplies a design illustration under independent repetitions.

B.4 Independent exact sample-size verification

The original planning audit uses binomial-tail calculations and recomputes each selected alternative tail with 70-digit arithmetic. A separately written program uses only arbitrary-precision integers, rational arithmetic, and polynomial coefficient recurrences. For a rational alternative $p_1 = a/b$, the coefficients of $((b-a)+az)^N$ give exact event numerators over denominator b^N.

That independent program checks every null sample size from 1 through 1,908 and 2,597 alternative sample-size cases across the three planning alternatives. It reproduces all six smallest sample sizes, their critical counts, their actual test sizes, and their powers. Thus the minima in Table 6 are checked over every preceding positive integer, rather than inferred from a smooth power approximation or from checking the immediately preceding sample alone.

B.5 Revised selection and feedback calculations

Version 1.1 adds a full-distribution retention audit. It compares 228 finite parent/target pairs with an independently solved linear program in selector weights and retained mass. Of these pairs, 147 satisfy the support condition and 81 do not. The audit constructs 441 exact rational attaining selectors and rejects 145 strictly excessive coverages. The maximum numerical optimization residual is approximately 1.11×10^{-16}, against the declared tolerance 3×10^{-10}. It also checks 105 symmetric address cases and 315 attenuation comparisons, including zero and perfect contrast. The direct mutual-information calculation agrees with the closed expression to approximately 8.89×10^{-16}.

A separately written program uses retained masses rather than selector weights as its optimization variables. Its 24 deterministic fixtures reproduce the sharp optimum, with maximum residual

approximately 5.56×10^{-17}, and independently calculate the worked example's joint mutual information. The optimization problems verify different finite formulations; the general theorem is proved in the manuscript.

The feedback revision uses rational arithmetic for its mathematical checks. It evaluates 5,166 deterministic policy/contrast/attenuation cases, including 270 zero-normalization cases, and verifies perfect execution in every positive-weight deterministic history. Additional checks cover 1,400 stochastic symmetric cases, 240 nonsquare-kernel cases, and 40 zero-probability source-row cases. Complete accepted and rejected masses agree with an ordinary forward event-tree construction.

The policy-selector audit examines 720 mixtures, including zero global weights and both equal and unequal fixed-policy mixtures. It verifies the candidate marginal, ordinary retained-population equivalence, and the exact four-address example. Further checks include 608 permutation-code cases, 120 Fourier nulls, and 480 label-dependent missingness cases. Each denominator is checked before conditioning; an undefined zero-weight law is never reported as a predicted physical failure.

For the staged bounded-score evidence process, two rational trees each enumerate 7,776 terminal paths through five stages, including intermediate selector information and scores in $\{0, 1/2, 1\}$. Conditional means are checked at 3,110 nodes per tree. At evidence threshold five, their exact null crossing probabilities are 649/31104 and 6485/1048576, both below 1/5. These are constructed tests of the claimed conditional-null theorem; they do not certify an experimental selector's calibration.

B.6 Adversarial review and corrections

The physics derivation, information theory, feedback model, and statistical procedures were cross-reviewed by separate analysis workstreams. The review identified and corrected two substantive proof qualifications. The Monte Carlo sample count must be fixed before the observed assignment, or belong to the same frozen conditioning information. The sequential maximal-inequality proof must handle a weak supremum approached without a finite threshold hit; the final proof uses a lower-threshold limit.

The original review also corrected a wording reversal about tests of ordinary explanations, aligned the baseline and fresh-content notation, clarified fixed-code conditioning, repaired a pure-state span description at an endpoint, and resolved typesetting errors. The recorded review files distinguish these corrections from cases that passed without changes.

The revision adds independent reviews of the selection proofs, computational-resource arguments, and feedback completion and protocol. These reviews reconstructed the reference-entangled channel calculation, precision and composition assumptions, rare-conditioning bound, support and equality cases, and the distinction between proposal weights and measured selector frequencies. Precision corrections identify the geometric waiting-time model as a sufficient stated assumption, specify adaptive circuit control without enumerating all histories, and type the staged information as sigma-algebras. The retained original review files concern the original mathematical core; the separately named revision reports concern the additions.

This is internal adversarial analysis with executed independent calculations. It is not external peer review or proof-assistant certification. The audit verifies declared mathematical constructions and

production properties. No participant study, artificial-reader trial, laboratory anomaly, or new artifact dating is represented as having occurred.

C Sources, reproducibility, and the editable edition

C.1 Source roles and access

The bibliography distinguishes journal research, conference research, philosophical antecedents, methodological publications, the Continuum's canon, and the companion theoretical manuscript. Canonical documents are cited as sources of the proposal and its historical self-description. The public collection page is cited for present institutional framing. The Yale collection description identifies a historical object and summarizes the library's findings. These sources are not substituted for independent physical evidence.

The public source ledger records a primary URL or DOI, the material actually inspected, the supported claim, and relevant limitations for each reference. Most mathematical and physical arguments were checked against original articles or author manuscripts; the body supplies direct proofs of the finite results used here. Access is not overstated. For example, the historical Helstrom attribution relies on publisher metadata and abstract access, supported by an inspected primary review and a complete derivation in this paper. Fischhoff's result is confined to the claim supported by the inspected abstract and metadata of its journal reprint. Black's article is used only for historical identification from publisher metadata. The Friedman and colleagues discussion is confined to its inspected primary abstract; Flew's contribution was inspected through an original-text transcription whose scan fidelity was not independently authenticated. Aharonov and Vaidman's detailed ensemble interpretation is supported by their inspected published review chapter. The source ledger identifies these access levels individually.

An accessible manuscript can differ in pagination, title, or date display from its final journal version. Bibliographic metadata and the inspected version are recorded separately where that distinction matters. The existence of a source in a bibliography is not represented as a reanalysis of its original empirical data. The earlier concept-generation chronology remains author-reported to the extent described in Section 18.

The private Bible and Doctrine are not reproduced in the public delivery. Their source locations and roles are described in the manuscript. No internal system or corporate attribution is needed to state the conjecture or the finite models.

C.2 The three public files

The release comprises a compiled PDF, a complete editable LaTeX source, and its BibTeX bibliography. The source contains the full prose, all equations and proofs, tables, figure captions, and references. Mathematical notation is encoded as LaTeX, not substituted by image approximations.

The LaTeX file also contains nine vector PDF figures, optional audit programs, recorded audit outputs, review notes, a detailed source ledger, dependency information, a revision record, and a fidelity note. These supporting files are stored in `filecontents*` blocks and are written to the

compilation directory. They are editable parts of the source. The figures are included as vector assets; their presence does not require running Python during typesetting.

To regenerate the document, place the `.tex` and `.bib` files together in a writable directory. Use a standard TeX installation with XeLaTeX, BibTeX, Latin Modern fonts, and the packages named in the embedded preamble. The basic sequence is:

```
xelatex -no-shell-escape Eidara_Symbol_Future_Reader.tex
bibtex Eidara_Symbol_Future_Reader
xelatex -no-shell-escape Eidara_Symbol_Future_Reader.tex
xelatex -no-shell-escape Eidara_Symbol_Future_Reader.tex
```

Further XeLaTeX passes are needed only if the installation reports unresolved cross-references or a changed contents layout. No shell escape is required. Audit scripts execute only when separately invoked by the reader. The extracted requirements file records the calculation environment used for this edition.

C.3 Reexecuting the calculations

The ten embedded programs have descriptive names beginning with `reader_`. They cover information and selection, quantum physics, exact statistical evidence, independent full-tensor physics review, independent feedback and information review, the observed-score-dependent Monte Carlo counterexample, exact integer planning verification, the revised complete feedback laws and staged evidence process, sharp target-distribution retention, and an independently formulated retained-mass optimization. Their recorded outputs have different filenames from their fresh outputs so that a rerun does not silently erase the release record.

Executing a script can regenerate its model figures and result tables in the extracted working directory. These are calculations on constructed data or stipulated parameters. The generated filenames and figure metadata do not imply an empirical dataset. The source includes the seeds used for randomized finite checks, exact arithmetic where specified, and explicit floating tolerances.

Recorded results are compared with fresh execution for the release. Numerical equality and exact-enumeration agreement are the relevant criteria. File paths, environment metadata, and newly generated PDF creation metadata can depend on a user's installation; any exclusions from a reproduction comparison must be identified explicitly rather than treated as unreported discrepancies.

C.4 Production fidelity

The release PDF is generated from the complete standalone source. Production checks cover page dimensions, embedded fonts, absence of raster image objects, missing glyphs, unresolved references, undefined citations, duplicate labels, and typesetting warnings. Every page is rendered for visual inspection; mathematical displays, tables, captions, and page transitions receive additional review where needed.

A separate clean build uses only the delivered LaTeX and bibliography files. Its extracted text and rendered pages are compared with the release PDF. The final fidelity note records the actual

page count, build process, warning status, hashes, and the scope of this comparison. Metadata differences between independently generated PDFs are distinct from changes to page content.

Version 1.1 revises the inspected, retained version 1.0 source. It preserves that edition's core proofs and adds the developments itemized in Appendix D. The canonical proposal remains an antecedent; the present formalization and subsequent corrections retain their separate version history. Earlier internal review reports concern the original core; separately named revision reports assess the new proofs and protocols. Future revisions can therefore retain both editable mathematical precision and a traceable record of what changed.

D Revision record: version 1.0 to version 1.1

This revision develops the inspected version 1.0 manuscript, dated 15 November 2026. Its central conjecture, canonical provenance, finite address construction, original mathematical core, and comprehensive format are retained. The substantive changes concern attribution, resource accounting, feedback interpretation, and a more fully specified experimental extension.

Area	Revision	Location
Physical ancestry	Identifies the natural-boundary construction as a restricted realization of the established final-state/postselected-teleportation strategy; clarifies its interaction and composition domain.	Section 7
Historical intervention problem	Adds primary bilking and consistency literature, distinguishing admissible histories, probability measures, and measured execution failures.	Section 10
Selection resources	Separates retained-event surprisal, repetition overhead, herald entropy, and reader information; proves the sharp full-law retention bound and its symmetric specialization.	Section 8
Computational resources	Proves efficient ordinary-circuit simulation for the restricted acyclic domain; identifies the stronger extension required for the established universal-postselection complexity result; bounds conditioning error.	Section 11
Feedback probabilities	Proves perfect deterministic refusal in positive-weight normalized histories and derives the complete ordinary heralding law. History mass is not assigned a device-failure interpretation.	Section 10
Candidate completion	Explicitly adds global conditioning of the policy selector, derives its label probabilities, and distinguishes it from an externally fixed policy mixture.	Assumption 10.6
Prospective experiment	Registers policy labels before heralding, preserves every attempt, calibrates the selector and instrument separately, supplies code controls and a bounded-score evidence process.	Section 14

Area	Revision	Location
Verification and delivery	Adds exact feedback checks, finite retention optimization, independent review and optimization, two vector figures, updated sources, and standalone reproduction checks.	Appendices B–C

The new global policy-conditioning law is identified as an additional proposed completion. It is neither silently inserted into the original single-record postulate nor represented as the unique resolution of feedback. The reader-agnostic scope remains explicit: a matched device preparing the same address state has the same prediction in this construction. The companion *The Mind Before Memory* is cited in its revised version 1.1, whose boundary development is an antecedent within this series [29].

All nine new proof environments are conditional finite mathematical arguments or specified circuit arguments. Their relationship to established results is stated in the text. The revision adds no experimental observation, externally conferred peer-review status, authenticated historical priority claim, or universal hardware-failure mechanism. The larger research commitment remains literal: a future reader could have a physically measurable relation to an earlier available record. The edition makes the candidate laws, their resource requirements, and their decisive experimental contrasts more precise.

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